MULTICHANNEL BLIND SIGNAL DECONVOLUTION USING HIGH ORDER STATISTICS

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ABSTRACT

The problem of multichannel blind signal deconvolution is considered. We show that input signals can be restored (or separated) using only the condition that they are statistically independent. Two main necessary and sufficient conditions involving high order cumulants are given and proved. Hence, a class of criteria for multichannel signal deconvolution are obtained. Self adaptive gradient based algorithms are derived in order to optimize the proposed criteria and computer simulations are presented in order to demonstrate that the proposed algorithm works.

1. INTRODUCTION

The problem of multichannel blind signal deconvolution (or blind equalization) of Linear Time Invariant (LTI) systems is currently receiving a lot of attention, see [1]-[8] and references therein. The problem finds numerous applications in diverse fields of engineering and applied sciences, e.g. data communication, sonar processing, seismic exploration, antenna processing, speech processing.

In the past ten years most of the proposed approaches consider a restrictive model known as source separation [9]-[12]. Indeed in that case the coupling channels are assumed (unknown) constant gains. Here we consider the more general model in which the coupling channels are unknown LTI systems. It can be simply formulated as follows. Several linear (temporal and spatial) mixtures of certain independent signals called sources are observed. We want to recover the unknown original sources without knowing the mixing filter. Hence, this must be realized from the only knowledge of the observations. This is the reason why this kind of approach is often qualified as "blind" or "unsupervised". In this paper the case of complex signals is considered.

2. PROBLEM FORMULATION

We consider the multichannel LTI and generally non-causal system described by

$$\mathbf{x}(t) = \sum \mathbf{G}(k)\mathbf{a}(t-k) \tag{1}$$

where $\mathbf{a}(t)$ is the $(N,1)$ vector of *statistically independent* sources, $\mathbf{x}(t)$ is the $(N,1)$ vector of observations and $\{\mathbf{G}(.)\}$ is a sequence of (N,N) matrices which describes the impulse response of the LTI mixing filter.

The multichannel blind deconvolution problem consists in estimating a LTI filter (equalizer) $\{\mathbf{H}(.)\}$ thanks to the only observations $\mathbf{x}(t)$ of an unknown LTI system $\{\mathbf{G}(k)\}$ and such that the vector

$$\mathbf{y}(t) = \sum \mathbf{H}(k)\mathbf{x}(t-k) \tag{2}$$

restores the N input signals a_i. We define the global LTI filter $\{\mathbf{S}(.)\}$ according to

$$\mathbf{y}(t) = \sum \mathbf{S}(k)\mathbf{a}(t-k) \ . \tag{3}$$

It is necessary to make the two following assumptions.
A1 Each source a_i is a sequence of zero-mean complex independent and identically distributed (i.i.d.) continuous or discrete random variables. Without any loss of generality they are assumed unit power. Moreover we shall assume that non-zero cumulants of random variables exist and are finite whenever they are introduced. In particular, this implies that sources must be non-Gaussian. Finally we assume that the p-th order joint cumulant of the real and imaginary parts of each source are equal.
A2 The unknown LTI system $\{\mathbf{G}(.)\}$ is assumed stable and invertible.

Notice that assumption **A1** is not very restrictive e.g. in digital communication since most signals have a symmetric constellation, e.g. 4-QAM, 16-QAM, V27.

Because sources are assumed inobservable, there are some inherent indeterminations in their restitution. That is, in general, we cannot identify the order, the power and the time origin of each sources. Indeed this combines the inherent indeterminations of the source separation problem together with those of the classical blind scalar deconvolution problem. Hence signals are said separated if and only if (iff) the global LTI system $\{\mathbf{S}(.)\}$ reads

$$\mathbf{S}(z) \stackrel{\triangle}{=} \sum_k \mathbf{S}(k)z^{-k} = \mathbf{D}(z)\mathbf{D}_1\mathbf{P} \tag{4}$$

where $\mathbf{D}(z)$ is a diagonal matrix such that its entries are $d_{ii}(z) = z^{-n_i}$, $i = 1, \ldots, N$, n_i integers, $\mathbf{D}_1$ an invertible constant diagonal matrix and $\mathbf{P}$ a permutation matrix.

3. DECONVOLUTION CRITERIA

Contrast functions as defined in [2] constitute blind deconvolution criteria in the sense that they are maximum iff the relation in (4) holds for $\mathbf{S}(z)$. In the following "white" vectors $\mathbf{y}$ are considered, i.e. vectors such that

$$\mathsf{E}[\mathbf{y}(t)\mathbf{y}^H(t-\tau)] = \mathbf{I}\delta(\tau) \tag{5}$$

where $\mathbf{I}$ is the (N,N) identity matrix, $\delta(.)$ the dirac distribution and E the mathematical expectation operator. White vectors $\mathbf{y}$ are deduced from sources $\mathbf{a}$ thanks to (3) if

$$\mathbf{S}(z)\mathbf{S}^H(\frac{1}{z^*}) = \mathbf{I} \ . \tag{6}$$

Let us define the two functions $\mathrm{I}_p^{\mathcal{R}}$ and $\mathrm{I}_p^{\mathcal{I}}$ according to

$$\mathrm{I}_p^{\mathcal{R}}(\mathbf{y}) \triangleq \sum_{i=1}^{N} |\mathsf{C}_p\mathcal{R}(y_i)| \ , \quad \mathrm{I}_p^{\mathcal{I}}(\mathbf{y}) \triangleq \sum_{i=1}^{N} |\mathsf{C}_p\mathcal{I}(y_i)| \tag{7}$$

where $\mathsf{C}_p u$ is the p-th order joint cumulant of real random variable u, p an integer greater or equal to 3 and $\mathcal{R}(y_i)$ (resp. $\mathcal{I}(y_i)$) stands for the real (resp. imaginary) part of complex random variable y_i. The following theorems are proved in the paper.

Theorem 1 *The function $\mathrm{I}_p^{\mathcal{R}}(.)$ (resp. $\mathrm{I}_p^{\mathcal{I}}(.)$) for $p \geq 3$ is a contrast over the set of white random vectors having at most one null cumulant of order p of its real part (resp. imaginary part) .*

Proof: We only consider $\mathrm{I}_p^{\mathcal{R}}(.)$ because the proof for $\mathrm{I}_p^{\mathcal{I}}(.)$ is completly similar. Clearly, $\mathrm{I}_p^{\mathcal{R}}(.)$ is symmetrical and invariant under scale change. Let us show that if $\{\mathbf{S}(.)\}$ is such that (6) holds then

$$\mathrm{I}_p^{\mathcal{R}}(\mathbf{Sa}) \leq \mathrm{I}_p^{\mathcal{R}}(\mathbf{a}) \ . \tag{8}$$

From (3) one has

$$y_i(t) = \sum_{j,k} s_{ij}(k) a_j(t-k) \ . \tag{9}$$

Thus thanks to the independence of the sources

$$\mathsf{C}_p\mathcal{R}(y_i) = \sum_{j,k} \mathcal{R}^p(s_{ij}(k))\mathsf{C}_p\mathcal{R}(a_j) + (-1)^p\mathcal{I}^p(s_{ij}(k))\mathsf{C}_p\mathcal{I}(a_j) \ . \tag{10}$$

Since $\mathsf{C}_p\mathcal{R}(a_j) = \mathsf{C}_p\mathcal{I}(a_j)$ one has

$$\sum_{i=1}^{N} |\mathsf{C}_p\mathcal{R}(y_i)| \leq \sum_{j=1}^{N} |\mathsf{C}_p\mathcal{R}(a_j)| A_j \tag{11}$$

where

$$A_j = \sum_{i,k} (|\mathcal{R}(s_{ij}(k))|^p + |\mathcal{I}(s_{ij}(k))|^p) \ . \tag{12}$$

Now from (6), $\forall j$, $\sum_{i,k} |s_{ij}(k)|^2 = 1$, thus $\forall j$, $A_j \leq 1$ and (8) is realized.

Let us consider the equality in (8). If one source, say a_N, is such that $\mathsf{C}_p\mathcal{R}(a_N) = 0$ then equality in (8) requieres equality $A_j = 1$ for $j = 1, \ldots, N_1$ which holds if it exists one and only one (i,j), $i = 1, \ldots, N$; $j = 1, \ldots, N_1$ and $\forall k$ such that $|\mathcal{R}(s_{ij}(k))| = 1$ or $|\mathcal{I}(s_{ij}(k))|$. Because $\{\mathbf{S}(.)\}$ is such that (6) then $\mathbf{S}(z)$ is of the form (4) and $\mathrm{I}_p^{\mathcal{R}}(.)$ is a contrast over the set of white random vector having at most one null cumulant of order p of its real part. •

Hence by the theorem, for white random vectors $\mathbf{y}$ deduced from eq.(3), necessary and sufficient condition for blind deconvolution is

$$\sum_{i=1}^{N} |\mathsf{C}_p\mathcal{R}(y_i)| = \sum_{i=1}^{N} |\mathsf{C}_p\mathcal{R}(a_i)| \ , \tag{13}$$

or

$$\sum_{i=1}^{N} |\mathsf{C}_p\mathcal{I}(y_i)| = \sum_{i=1}^{N} |\mathsf{C}_p\mathcal{I}(a_i)| \ . \tag{14}$$

This leads to the two following constrained blind deconvolution criteria

$$\max \sum_{i=1}^{N} |\mathsf{C}_p\mathcal{R}(y_i)| \quad \text{subject to} \quad \mathbf{y} \text{ white} \tag{15}$$

$$\max \sum_{i=1}^{N} |\mathsf{C}_p\mathcal{I}(y_i)| \quad \text{subject to} \quad \mathbf{y} \text{ white} \tag{16}$$

Now in the specific case of sources a_i with identical sign ε_p of the p-th order cumulant of $\mathcal{R}(a_i)$ and $\mathcal{I}(a_i)$ for all i, we have the following theorem.

Theorem 2 *For even integer $p > 3$, the functions*

$$\mathrm{J}_p^{\mathcal{R}}(\mathbf{y}) \triangleq \varepsilon_p \sum_{i=1}^{N} \mathsf{C}_p\mathcal{R}(y_i) \quad \text{and} \quad \mathrm{J}_p^{\mathcal{I}}(\mathbf{y}) \triangleq \varepsilon_p \sum_{i=1}^{N} \mathsf{C}_p\mathcal{I}(y_i)$$

are contrasts over the set of white random vectors having non zero cumulant of order p of its real and imaginary part.

The proof is easily deduced from Theorem 1 and eq.(10) where if p is even then $\mathrm{sign}(\mathsf{C}_p\mathcal{R}(y_i)) = \mathrm{sign}(\mathsf{C}_p\mathcal{R}(a_i)) = \varepsilon_p$.

If we consider the value $p = 4$, we have the following simplified theorem.

Theorem 3 *The functions*

$$\mathrm{K}^{\mathcal{R}}(\mathbf{y}) \triangleq \varepsilon_4 \sum_{i=1}^{N} \mathsf{E}\mathcal{R}^4(y_i) \quad \text{and} \quad \mathrm{K}^{\mathcal{I}}(\mathbf{y}) \triangleq \varepsilon_4 \sum_{i=1}^{N} \mathsf{E}\mathcal{I}^4(y_i)$$

are contrasts over the set of white random vectors having non zero cumulant of order 4 of its real and imaginary part.

Proof: We only consider $\mathrm{K}^{\mathcal{R}}$. One has $\mathsf{C}_4\mathcal{R}(y_i) = \mathsf{E}\mathcal{R}^4(y_i) - 3\mathsf{E}^2\mathcal{R}^2(y_i)$. Since white vectors are considered $\mathsf{E}\mathcal{R}^2(y_i)$ is constant $\forall i$. Thus $\mathrm{K}^{\mathcal{R}}(\mathbf{y}) = \mathrm{J}_4^{\mathcal{R}}(\mathbf{y}) + \mathrm{cst}$ where cst is a certain constant. Then the theorem is proved. •

As previously we can deduce necessary and sufficient conditions for blind deconvolution and the corresponding maximization criteria.

4. SELF-ADAPTIVE ALGORITHM

In order to achieve the deconvolution, we have to find a filter $\{\mathbf{H}\}$ such that the proposed contrasts are maximum. A stochastic gradient based adaptive algorithm is proposed in this section. The set of definition of the proposed contrast is the set of white vectors. Hence in the following we consider that a first stage realize a multichannel spectral prewhitening of the observations. This "classical" stage will not be discussed here. In order to ensure the whiteness of $\mathbf{y}$, $\{\mathbf{H}\}$ must be such that

$$\mathbf{H}(z)\mathbf{H}^H(\frac{1}{z^*}) = \mathbf{I} \tag{17}$$

that is the filtering transfer matrix is lossless or all-pass. Such transfer admits a special parametrization thanks to planar (Givens) rotations, see e.g. [4]. In the simplest case ($N = 2$, $k = 0, 1$) one has

$$\mathbf{H}(z) = \mathbf{Q}_1(\theta_1, \phi_1) \begin{pmatrix} z^{-1} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{Q}_2(\theta_2, \phi_2) \tag{18}$$

where

$$\mathbf{Q}_i(\theta_i, \phi_i) = \begin{pmatrix} e^{j\phi_i}\cos\theta_i & \sin\theta_i \\ -\sin\theta_i & e^{-j\phi_i}\cos\theta_i \end{pmatrix}. \tag{19}$$

Using this parametrization, we have now to find the angles θ_i and ϕ_i in order to maximize one contrast. Denoting p anyone of parameters (θ_i, ϕ_i), a deterministic procedure is to reach the maximum of a contrast $\mathcal{C}$ thanks to an iterative algorithm which updates p with the increment

$$\Delta p = \mu \frac{\partial \mathcal{C}}{\partial p} \tag{20}$$

where μ is a small positive constant. Hence the optimum is found as the limit of the sequence

$$p(n) = p(n-1) + \mu \left.\frac{\partial \mathcal{C}}{\partial p}\right|_{p=p(n-1)}. \tag{21}$$

In cases of the contrast in this paper, it is possible to express the criteria as the expectation of some random variable. We use a loss complex version of the gradient algorithm (21) by dropping the expectation. It will be called a "stochastic algorithm". For $N = 2$ and contrast $\mathrm{K}^{\mathcal{R}}(.)$, one easily has the stochastic increment

$$\Delta p = 4\mu\varepsilon_4(\mathcal{R}^3(y_1)\frac{\partial \mathcal{R}(y_1)}{\partial p} + \mathcal{R}^3(y_2)\frac{\partial \mathcal{R}(y_2)}{\partial p}) \tag{22}$$

where $\partial\mathcal{R}(y_1)/\partial p$ are deduced from (2) and (18).

Convergence analysis of the proposed algorithm is beyond the scope of this paper. However computer simulations are presented in order to demonstrate that the proposed algorithm works.

5. COMPUTER SIMULATIONS

The performances of the algorithm are associated to an index/measure of performance defined on the global filtering matrix $\{\mathbf{S}\}$ according to

$$\begin{aligned} \mathrm{ind}(\{\mathbf{S}\}) \quad &\stackrel{\triangle}{=} \quad \frac{1}{2}\left[\sum_i \left(\sum_{j,k} \frac{|s_{ij}(k)|^2}{\max\limits_{\ell,m}|s_{i\ell}(m)|^2} - 1\right)\right. \\ &\left. + \sum_j \left(\sum_{i,k} \frac{|s_{ij}(k)|^2}{\max\limits_{\ell,m}|s_{\ell j}(m)|^2} - 1\right)\right] \end{aligned}$$

This positive index is indeed zero if $\{\mathbf{S}\}$ is such that $\mathbf{S}(z)$ satisfies (6) and a small value indicates the proximity to the desired solution. We present simulations in the case of two sources. Three kind of sources are considered: i) two 4-QAM communication sources; ii) two 16-QAM communication sources and iii) two constant modulus sources: $\exp(j\phi)$ where ϕ is a random variable with uniform probability density over $[0, 2\pi[$. The mixing filter is of the form (18) where $\theta_1 = \pi/6$, $\phi_1 = \pi/18$, $\theta_2 = \pi/9$ and $\phi_2 = \pi/36$. The algorithm (22) is tested via Monte Carlo simulations. In Fig.1, 2 and 3 we have plotted the sample average over 500 data realizations of the index as a function of iterations respectively in cases i), ii) and iii). The index decreases monotically and achieve the steady state level of -33dB, -27dB and -28dB respectively in the three cases. In Fig.4 and 5 we have plotted one realization of the performance index, the estimated parameters, the observed signals and the reconstructed signals at channel 1 when steady state is achieved.

6. REFERENCES

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fig1.eps scaled 550

Figure 1: Sample average of the performance index for two 4-QAM sources.

fig2.eps scaled 550

Figure 2: Sample average of the performance index for two 16-QAM sources.

fig3.eps scaled 550

Figure 3: Sample average of the performance index for two constant modulus sources.

figQ4.eps scaled 450

Figure 4: Performance index + parameters + observed signals + reconstructed signals for two 4-QAM sources.

figQ16.eps scaled 450

Figure 5: Performance index + parameters + observed signals + reconstructed signals for two 16-QAM sources.