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List of abstracts

Dissipative soliton resonances

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Dissipative soliton resonance is a phenomenon where the energy of a soliton in a dissipative system becomes infinitely high at certain values of the system parameters. Specifically, the systems that can be modeled using the cubic-quintic complex Ginzburg-Landau equation admit a region of parameters with stable solitons whose energy goes to infinity at the boundary of that region. The edge of this region in five-dimensional parameter space is a co-dimension one surface. Thus, the energy of the dissipative solitons remains infinite even when the parameters are changed in a continuous way along that surface.This phenomenon can be useful in designing optical oscillators generating pulses with exceptionally high energies.

Modeling of ultrashort optical pulses

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Optical waves are usually described under the slowly varying envelope approximation (SVEA). The latter yields a reduced model equation for the complex wave amplitude, e.g., nonlinear Schrödinger equation or complex Ginzburg Landau equation. A notable progress in generation of ultrashort and few-cycle optical pulses for which the SVEA does not apply awakened interest in new non-envelope models. With this respect we discuss the so called short pulse equation and its modifications accounting for the complex medium dispersion within an optical transparency window. We also present the recently found solitary solutions.

The transition from solitonic to dissipative behavior of solitary waves

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The solitary waves are spatially localized structures that propagates into media without deformation. If the considered media conserves energy, these waves survive after a frontal collision: this is a signature of soliton behavior. In dissipative media, the balance between the energy injection and the dissipation can lead to the propagation of localized structures. In such medias, the frontal collision results in the disappearance of the colliding waves. In this talk, we will present simple insights into the transition from solitonic to dissipative behavior of solitary waves.

Directly and parametrically driven wobbling kinks in the ϕ^4 theory

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In the first part of this paper, we use the method of multiple scales to construct the wobbling-kink solution of the “free” (i.e. undamped undriven) ϕ^4 equation,

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0.$$

The amplitude of the free wobbling decays due to the emission of the second-harmonic radiation. However, we show that the amplitude decays only as $t^{-1/2}$ and hence the wobbler is an extremely long-lived object.

In the second part, we study the compensation of these radiation losses (as well as dissipative losses in case those are present in the system) by the resonant driving of the kink. We consider both the direct

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = -\gamma\phi_t + h\cos(n\Omega t)$$

and parametric driving,

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = -\gamma\phi_t + h\cos(n\Omega t)\phi,$$

at a range of resonance driving frequencies. In each case, we derive equations for the adiabatic evolution of the amplitude of the kink's wobbling coupled to the velocity of its translational motion. These equations predict multistability and hysteretic transitions for some driving frequencies — the conclusion verified in the numerical simulations of the full partial differential equation. We show that the strongest parametric resonance occurs when the driving frequency equals the natural wobbling frequency and not double that value, and provide a qualitative explanation for this atypical behaviour. For direct driving, we confirm the existence of the superharmonic resonance at half the natural frequency, and demonstrate that a weaker resonance at the natural frequency itself does exist, contrary to the claim made in literature. We show that this resonance can only be sustained if the kink performs translational motion.

Femtosecond filaments for pulse compression: Simulating a complete experiment

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In the mid-1990's, first experiments on the meter-range propagation of femtosecond (fs) laser pulsed beams were performed. Infrared laser pulses with a duration of about 100 fs produced narrow filaments of several meters, localizing more than 10% of the energy in the near-axis area. Similar results, but on smaller length scales, are known for propagation in dense media like silica or water. This so-called "filamentation" is attributed to the initial self-focusing of laser radiation, which originates from the Kerr response of the medium and leads to an increase of the light intensity. This growth is then saturated by the defocusing action of the electron plasma created by photoionization of the ambient atoms. Understanding the complex dynamics of these filaments is crucial for potential applications such as supercontinuum generation, pulse compression, generation of high-order harmonics, remote sensing or even material processing.

Pulse compression by femtosecond filaments formed in pressurized cells filled with noble gases is one of the most challenging topics in nonlinear optics. In this talk, I will focus on recent progresses in this field. It is generally possible to use femtosecond filaments to compress ultrashort pulses down to the single cycle limit, potentially without any further post-compression techniques. Fully space-time resolved simulations give insight into the details of the compression mechanism, which are not accessible in experiments otherwise. Numerical simulations can even be performed through all stages undergone by the pulse in a realistic experiment, describing successively a compression stage in a gas cell, the propagation of the pulse through the cell exit window, and finally the residual propagation in air, before the pulse reaches the diagnostics. We show that the temporal pulse profiles are severely broadened during propagation in the thin glass window. However, the compressed states previously formed in the noble gas can be "self-restored" in air.

Inverse Scattering in 2+1 dimensions via the extended resolvent"

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The Inverse Scattering theory is extended to 2+1 dimensions including the case in which the solution describes N wave like-soliton solutions on a generic background. In order to achieve this result a new mathematical tool, called extended resolvent, is introduced. The theory is applied to the Kadomtsev-Petviashvili equation of type I and II.

Parabolic optical pulses under the action of the third-order dispersion

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Recent developments in nonlinear optics reveal an interesting class of pulses with a parabolic intensity profile in the energy-containing core and a linear frequency chirp that can propagate in a fiber with normal group-velocity dispersion. Parabolic pulses propagate in a stable self-similar manner, holding certain relations (scaling) between pulse power, width, and chirp parameter. In the additional presence of linear amplification, they enjoy the remarkable property of representing a common asymptotic state (or attractor) for arbitrary initial conditions. Analytically, self-similar (SS) parabolic pulses can be found as asymptotic, approximate solutions of the nonlinear Schrödinger equation (NLSE) with gain in the semi-classical (large-amplitude/small-dispersion) limit. By analogy with the well-known stable dynamics of solitary waves - solitons, these SS parabolic pulses have come to be known as similaritons. In practical fiber systems, inherent third-order dispersion (TOD) in the fiber always introduces a certain degree of asymmetry in the structure of the propagating pulse, eventually leading to pulse break-up. To date, there is no analytic theory of parabolic pulses under the action of TOD. Here, we develop a WKB perturbation analysis that describes the effect of weak TOD on the parabolic pulse solution of the NLSE in a fiber gain medium. The induced perturbation in phase and amplitude can be found to any order. The theoretical model predicts with sufficient accuracy the pulse structural changes induced by TOD, which are observed through direct numerical NLSE simulations.

Soliton methods in Geometry

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It is well known that that many special submanifolds of interest to geometers are related to soliton theory through their local integrability conditions. It is also well known that methods of soliton theory can be expressed in a particularly simple and natural way in the framework of loop groups. I will try to indicate how this framework, moreover, leads naturally and directly to the special submanifolds mentioned above, and illustrate this with an example from recent work.

Exact solutions for some partial differential equations

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A direct method with a computerized symbolic computation is used to derive exact travelling wave solutions expressed by Jacobi elliptical functions for a 4th order nonlinear evolution equation with constant coefficients.

The validity and reliability of the method is tested by its applications for a $K(m,n)$ equation, among others. Solutions with physical interest as travelling waves are obtained: compactons, solitons, kinks and antikinks.

By using the G'/G-expansion method, three new types of travelling wave solutions for a $K(1,1)$ equation are derived.

Analytical and numerical investigation of a damped sine-Gordon equation and novel effects in the optical dynamic holography

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We consider the four-wave mixing model that appears in optical dynamic holography to describe the process of self-diffraction of waves from a dynamical Bragg grating. The system consists of four equations of coupled waves, where the dynamical grating amplitude comes as a coupling coefficient, and one evolution equation for the grating amplitude, which includes both an amplification of the grating caused by wave interference pattern and a dielectric relaxation of the grating. Transformation of a homogeneous grating amplitude distribution into a spatial localized structure (as well as formation of a localized profile for the interference pattern) is observed in the case of nonlocal response, i.e. there exist a phase delay during interference between a forward wave and a wave diffracted by the grating. For exact nonlocal response the phase delay is equal either to +Pi or to -Pi, and the initial system is reduced to one damped sine-Gordon (SG) equation. We consider the reduction of this damped SG equation to a generalized nonlinear Schrodinger (NLS) equation by using a perturbation method. Several experimental techniques to achieve a nonlocal response in a Kerr-like media are described. Possible observed effects (e.g. all-optical logic elements, optical switching, pulse narrowing, beam shaping control) that are arisen due to manipulation of laser pulses during the four-wave mixing in a medium with a nonlocal response are considered by numerical simulation of the damped SG equation.

KdV-Type Flows on Star-Shaped Curves

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We discuss the connection between the projective and centro-affine geometry of curves, and the relation between invariant curve evolutions in both geometries and associated geometric Hamiltonian structures.

For star-shaped planar curves and parametrized maps into the projective line, we show that projectivization induces a map between differential invariants and a bi-Poisson map between Hamiltonian structure. Our main example in this setting is Pinkall's flow, a Hamiltonian evolution equation for closed star-shaped planar curves closely related to the KdV equation, and whose projectivization is the Schwarzian KdV equation.

Using algebro-geometric methods and the relation of group-based moving frames to the AKNS system for the KdV equation, we construct examples of closed solutions of Pinkall's flow associated with periodic finite-gap KdV potentials.

This is joint work with Tom Ivey (College of Charleston) and Gloria Mari'-Beffa (University of Wisconsin-Madison)

Cavity with an embedded polarized film: an adapted spectral approach

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Joint work with E. V. Kazantseva, L. Loukitch and A.I. Maimistov .

We consider the modes of the electric field of a cavity where there is an embedded polarized dielectric film. The model consists in the Maxwell equations coupled to a Duffing oscillator for the film which we assume infinitely thin. We derive the normal modes of the system and show that they are orthogonal with a special scalar product which we introduce. These modes are well suited to describe the system even for a film of finite thickness. By acting on the film we demonstrate switching from one cavity mode to another. Since the system is linear, little energy is needed for this conversion. Moreover the amplitude equations describe very well this complex system under different perturbations (damping, forcing and nonlinearity) with very few modes. These results are very general and can be applied to different situations like for an atom in a cavity or a Josephson junction in a capacitor and this could be very useful for many nano-physics applications.

Reference:
J. G. Caputo, L. Loukitch, E. Kazantseva and A. Maimistov, *Cavity with an embedded polarized film: an adapted spectral approach*, arXiv: 0807.1287 [physics.gen-ph]

Interaction soliton – envelope soliton: A model for tsunamis predictions

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The source of tsunamis (earthquake, landslide) is often situated in deep sea. Long waves generated in such environnement having a small amplitude, they are hardly observable directly. However, these long waves interact with the ambient short waves and modify their characteristics. Thus, understanding the interaction between long and short waves may lead to criteria for the early predictions of tsunamis.

The classical simple model describing long and short waves (i.e. the KdV and NLS equations) have disjointed domains of validity. Thus, none of these models can be used to describe, e.g., the interaction between a soliton and an envelope soliton.

This problem is investigated via a generalized perturbation scheme based on nonlinear WKB expansions, allowing to describe both long (shallow water) and short (Stokes) waves at the same time. This schemes leads to explicit analytic approximations describing phase shifts and Doppler effects.

Analytic patterns of the four-wave mixing model

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The four-wave mixing model is a ten-dimensional nonlinear evolution system which describes the dynamical self-diffraction of waves. We first build a five-dimensional closed subsystem which removes the useless freedom and unveils the only relevant intrinsic physical variables: the dynamical grating, the intensity pattern, the total net gain. We then isolate by the Painlevé test all the cases when singlevalued solutions may exist. Finally, with one minor exception, we provide closed form expressions for all the singlevalued solutions, for any kind of response, whether local or nonlocal. The stability of the patterns defined by these solutions is determined numerically.

1. R. Conte and S. Bugaychuk, Analytic structure of the four-wave mixing model in photorefractive materials, 177–186, *Waves and stability in continuous media*, eds. N. Manganaro, R. Monaco and S. Rionero (World scientific, Singapore, 2008). http://arXiv.org/abs/0806.1183

2. R. Conte and M. Musette, *The Painlevé handbook*, xxiv+256 pages (Springer, Berlin, 2008).

An analytical model for synthetic- and bio- polymers based on continuous elastic rods: general framework and circular helix solutions

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The study of elastic deformations in thin rods has recently seen renewed interest due to the close connection between these systems and coarse-grained models of widespread application in life- and material-sciences. Until now, the analysis has been restricted to the solution of equilibrium equations for continuous models characterized by constant bending and twisting elastic moduli and/or by isotropic rod section. However, more realistic models often require more general conditions: indeed this is the case whenever microscopic information issuing from atomistic simulations is to be transferred to analytic or semi-analytic coarse-grained or macroscopic models. In this paper we will show that integrable, indeed solvable, equations are obtained under quite general conditions and that regular (e.g. circular helical) solutions merge from reasonable choices of elastic stiffnesses.

Nonlinear aspects of tsunami waves

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The giant tsunami occurred in the Indian Ocean on 26th December 2004 draws attention to this natural phenomenon. The given presentation deals with nonlinear physics of tsunami wave propagation on all stages from the source to the coast. Characteristic scales of tsunami source of various origins (earthquakes, landslides, volcanos, asteroids) are discussed in terms of nonlinearity and dispersion. It is shown that shallow-water theory is an appropriate model for describing tsunami of seismic origin; meanwhile modeling of tsunami caused by a landslide or explosion should take into account dispersive effects. Applicability of existing nonlinear dispersive theories for tsunami wave propagation, refraction, transformation and climbing the coast is demonstrated. The role of solitons is discussed especially. The main attention is paid to nonlinear transformation and runup of tsunami waves in the cases, when analytical solutions can be found. These cases are used for tsunami risk evaluation in different areas of the World Ocean.

KP-equation in Riemann geometry

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Notion of the Riemann extensions of affinely connected spaces for construction of examples of the eight-and six-dimensional the Ricci-flat Riemannian spaces related with the Kadomtsev-Petviashvili equation are used. Their properties and a possible applications are discussed.

Disordered lattice solitons

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We study the properties of a nonlinear Schrödinger equation in the presence of a disordered potential in one and two dimensions. We find that, for both signs of the nonlinearity, there is a large number of solitary wave families each one possessing different quantitative properties. However, all these families can be categorized to only a few classes with the same qualitative properties. Highly confined solitons exist in each waveguide of the lattice. In addition, nonlinear modes originate from each Anderson mode. Bifurcations between a solitary wave and an Anderson mode can take place, leading to broadening of the soliton profile.

Integration of the Vector Nonlinear Schrödinger Equation

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A comprehensive algebro-geometric integration of the two-component Nonlinear Vector Schrödinger Equation will be described. The allied spectral variety is a trigonal Riemann surface, which will be described explicitly, and the solutions of the equation are given in terms of theta-functions of the surface. In particular, it will be shown that the only solution for the genus-2 case is the separable solution.

Gravity-like force supports formation of unusual quasi-solitonic bound states in optical fibres

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Short pulses of light propagating in optical fibres experience dispersion, leading to the pulse spreading. Typically in bulk materials longer wavelength components travel faster - this is known as normal group velocity dispersion (GVD). In fibers GVD can be tailored to become anomalous in certain frequency windows, in which case shorter wavelength components travel faster. The great importance of anomalous GVD is that it can be balanced by self-phase modulation due to Kerr nonlinearity, leading to formation of non-dispersing nonlinear wavepackets - solitons. We report a novel mechanism of localization of light pulses with spectra in the normal GVD range of optical fibres, due to the inertial gravity-like force acting on pulses from the accelerating solitons. Together with solitons, such localized pulses form a very specific type of bound states in which two components are slightly delayed with respect to each other and co-propagate with acceleration. We also pr esent the corresponding bright-bright coupled quasi-soliton numerical solutions of the underlying coupled nonlinear Schrödinger-type equations with opposite signs of dispersion.

Transcritical flow over an obstacle; the forced Korteweg-de Vries equation

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The interaction of aflow with topography can generate large-amplitude, horizontally propagating solitary waves. Often these waves appear as a wave-train, or undular bore. In this talk we will focus on the situation when theflow is critical, that is, the flow speed is close to that of a linear long wave mode. In the weakly nonlinear regime, this is modeled by the forced Korteweg de Vries equation. The solution consists of upstream and downstream propagating wavetrains, whose structure is related to the polarity of the obstacle vis-a-vis that of the polarity of free solitary waves. We will be especially concerned with the case of an obstacle ofnegative polarity, when the wavetrains interact in a complicated manner over the obstacle

Spatial solitons and instabilities of light beams in a three-level atomic medium with a standing-wave control field

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We consider propagation of a light beam propagating in a resonant three-level atomic system in the presence of electromagnetically induced transparency. In such a system stable two-dimensional spatial solitons and as well as several types of instabilities can be observed. Different dynamical regimes can be implemented by simple control of the lattice parameters, achievable by change of either the geometry of the control field or its intensity.

We employ a unified theory to solve the respective nonlinear system with different geometries of the control fields inducing an optical lattice. The formation of spatial solitons and instabilities are studied both analytically and numerically. Different from conventional passive optical media, the solitons in such resonant system can be created by light at a very weak intensity.

Fast-Light Solitons in Active-Raman-Gain Media

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In recent years, much attention has been paid to the study of slow light and fast light in resonant multi-level atomic systems [1,2]. A stable optical pulse propagation in multi-level atomic systems has not only fundamental theoretical interest but also many important applications in optical information processing and transmission. This talk reports our recent research results on fast (or called superluminal) light (i. e. faster than light in vacuum) optical solitons in multi-level atomic systems based on an active-Raman-gain (ARG) mechanism. This mechanism is fundamentally different from that via electromagnetically induced transparency, and is capable of eliminating all attenuation and distortion of signal field. We show that a giant Kerr nonlinearity can be obtained and fast-light solitons can be generated and propagate stably in the system at very low probe-field intensity [3-5].

References:
[1] R.W. Boyd and D. J. Gauthier, Progress in Optics (Elsevier Science, Amsterdam, 2002), Vol. 43, Chap. 6, p. 275.
[2] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. 77, 633 (2005), and references therein.
[3] G. Huang, C. Hang, and L. Deng, Phys. Rev. A 77, 011803(R) (2008).
[4] H. Li, C. Hang, G. Huang, and L. Deng, Phys. Rev. A 78, 023822 (2008).
[5] C. Hang and G. Huang, to be published.

Water wave theory as an application of local reduction methods

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The introduction of spatial dynamics by K.Kirchg ssner in the eighties allowed big progresses in the mathematical theory of water waves. Several new forms of localized waves were discovered, as well in 2D as in 3D. The talk will give elements of the reduction methods used in spatial dynamics (Center manifold reduction and normal forms for infinite dimensional reversible systems) and examples of results for water vave theory. We shall indicate the limitations of the method in physical limiting cases, and open problems.

Reference
F.Dias, G.Iooss. Water-waves as a spatial dynamical system. Handbook of math. fluid dynamics, Chap. 10, p.443 -499. S.Friedlander, D.Serre Eds., Elsevier 2003.

Time-periodic oscillations in weakly inhomogeneous nonlinear lattices”

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(collaboration with Bernardo Sanchez-Rey and Jesus Cuevas).

This work concerns time-periodic solutions (in particular discrete breathers, i.e. spatially localized oscillations) in a chain of coupled nonlinear oscillators, where coupling constants, particle masses and on-site potentials can have small variations along the chain. We analyze the bifurcation of small amplitude solutions using the ideas of discrete spatial dynamics and center manifold reduction. We show that small amplitude oscillations are determined by finite-dimensional nonautonomous mappings (denoted as "reduced maps"), whose dimension depends on the frequency range under consideration. We analyze in detail the parameter regime corresponding to a two-dimensional map. For an homogeneous chain, the reduced map is autonomous and reversible, and bifurcations of reversible homoclinics or heteroclinic solutions are found for appropriate parameter values. Homoclinic orbits to 0 correspond to discrete breather solutions of the infinite lattice. For an inhomogeneous chain, the nonautonomous reduced map is studied for a finite number of defects. At leading order, using the assumption of weak inhomogeneity, we show that homoclinics to 0 exist when the image of the unstable manifold under a linear transformation (depending on the sequence) intersects the stable manifold. As a consequence, tangent bifurcations of discrete breathers occur when defects are varied in such a way that the action of the linear transformation modifies the structure of the intersection. This provides a new geometrical understanding of breather bifurcations in the presence of impurities.

Ocean freak Waves in one and two dimensions.

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Wave forecasting is about forecasting the mean sea state, as reflected by the ocean wave spectrum, and for quite some time it was not realized that it was possible to make statements about extreme events. Recently, however, it has been shown how to relate deviations from the mean sea state to the wave spectrum. Therefore, when the wave spectrum is known, the probability distribution function (pdf) of the sea surface elevation can be determined. The tails of the pdf give vital information on the occurrence of extreme events such as freak waves. Starting from the Hamiltonian description of surface gravity waves it is shown that the short-term dynamics of ocean waves is governed by the Zakharov equation. I have used its one-dimensional version to study the statistical properties of the generation of extreme events using a Monte Carlo simulations. Indeed, deviations from the Normal distribution are shown to be related to the mean sea state. Good agreement with an approximate statistical theory is found, which at the same time describes the evolution in time of the wave spectrum owing to quasi-resonant four wave interactions. In order to better understand the formation of extreme events we study the properties of the narrow-band version of the Zakharov equation, which turns out to be the well-known Nonlinear Schr dinger (NLS) equation. For one-dimensional propagation, the NLS equation may be solved by means of the Inverse Scattering approach and for large times an initial disturbance evolves towards a train of envelope solitary waves, explaining the formation of extreme events. In fact, if the ocean would be truly one-dimensional, shipping would be a hazardous enterprise. In the case of two-dimensional propagation however, envelope solitons are unstable to transverse pertubations and therefore in that event the formation of freak waves is less frequent.

Covering Sets of Squared Eigenfunctions

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Integrable equations in one dimension have Lax pairs and Lax pairs consists of a spatial eigenvalue problem, which has a spectral parameter, and another eigenvalue problem which is the “spectral evolution equation”. A spatial eigenvalue problem usually defines an heirarchy of integrable equations. It also is usually at the top of a series of possible reductions, wherein various components of the potential matrix become identitfy. Each of these reductions then gives rise to a set of symmetries among the Jost functions and among the coefficients of the scattering matrix. For any given spatial eigenvalue problem, there is also a most “general eigenvalue problem” wherein all nonzero components of the potential matrix are taken to be uniquely different. We shall take as an example, the AKNS eigenvalue problem, which is the general eigenvalue problem for the 2×2 Dirac case, since the potential matrix has only two nonzero components, q and r, each of which are taken to be uniquely different form the other. An example of a reduction for this general eigenvalue problem would be to take $r = q$, which would then give rise to a series of symmetries among the Jost functions and the scattering coefficients. When one considers perturbations of these potentials in a general eigenvalue problem, one finds that the pertubations of the potentials can be given as an expansion over the perturbations of the scattering data. The coefficients in this expansion are called "squared eigenfunctions" and are products of a Jost function and an adjoint Jost function. On the other hand, it is known for some systems that the “squared eigenfunctions” are a SUM of different products of a Jost function and an adjoint Jost function. In this talk, we shall explore this phenomenon wherein the perturbation theory for some eigenvalue problems becomes divergent in this manner from other eigenvalue problems What we shall finally demonstrate is that the re is a set of squared eigenfunctions which can be taken to be a “covering set”.

Nonlinear effects in clouds of cold and confined atoms

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In the last ten years or so, there has been a remarkable activity on the field of nonlinear effects in clouds of Bose-Einstein condensed atoms, both on the theory side, as well as experimentally. These systems are described by a nonlinear Schrödinger equation, with an extra term that results from the external potential that confines the atoms. This term introduces novel effects and makes these problems very appealing.

In this talk I will give a very brief review of the experimental observations. I will also mention a few theoretical results which have been motivated by the above experiments.

Solitary waves and generalized solitary waves in layered elastic structures.

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In this talk I will discuss our current theoretical and experimental studies on this topic, including the scattering of the waves by inhomogeneities modelling delamination. In particular, I will consider the analytical solution for scattering of a solitary wave in a delaminated bar (K.R. Khusnutdinova, A.M. Samsonov, PRE 77 (2008) 066603). The developed approach is based on matching two asymptotic multiple-scales expansions and results from the theory of integrable systems, due to a Lax pair formalism. Experiments have been performed in the Ioffe Institute of the RAS by the group headed by Prof. A.M. Samsonov. This is a joint work with them and Dr. A.V. Zakharov from Loughborough.

Ernst equation and hyperelliptic solutions

C. Klein

The Ernst equation appears in many branches of mathematics and physics: it corresponds to a completely integrable sigma model and is equivalent to the stationary axisymmetric Einstein equations in vacuum. It also appears in the construction of multimonopole solutions to the Einstein-Yang-Mills-Higgs equations and in the description of Bianchi surfaces. The Ernst equation has a non-autonomous Lax pair. Aa a consequence algebro-geometric solutions to the equation are given on families of hyperelliptic Riemann surfaces, where some of the branch points depend on the physical coordinates. The corresponding solutions are discussed and compared to almost periodic solution to the Korteweg-de Vries equation.

On operator representations of integrable equations"

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Operator representations of soliton equations from the celebrated Lax pair, Manakov triad to Yano algebroids are discussed. Various examples and associated algebraic and geometrical structures are considered.

Dynamics of matter solitons in linear and nonlinear lattices

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A survey of dynamical phenomena, observable in quasi one-dimensional Bose-Einstein condensates loaded in linear and nonlinear optical lattices will be presented. The system is described by the nonlinear Schrödinger equation with linear and nonlinear potentials periodically varying in space. We concentrate on long-lived Bloch oscillations of solitons, dynamical localization, Rabi oscillations of solitons and macroscopic tunneling, and on the delocalizing transition.

Spatially localized structures in finite domain with general boundary conditions (P)

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We analyze asymptotically the localized solutions of the quadratic-cubic Swift-Hohenberg equation in a large but finite domain. In the vicinity of a modulation instability, these solutions are constructed as combinations of slow spatial fronts between the homogeneous and periodic solutions. The effect of the distant boundaries balances with the exponentially small pinning forces between the fronts and the spatial oscillations, the latter being determined by a beyond-all-orders calculation. This procedure allows us to investigate the incidence of general boundary conditions on the dynamics. We find that the snaking bifurcation diagram is strongly modified with respect to the usual picture obtained for an infinite domain. Moreover, the locations available to localized structures now form a discrete set which develops from a cascade of bifurcations as their size is varied.

Optical solitons in nonlocal nonlinear media

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In nonlinear media with a nonlocal nonlinear response the nonlinearity in a particular point is not determined solely by the strength of the wave in that very point (as it is the case for a local medium), but also in its neighborhood. It appears that nonlocality is a generic feature of various nonlinear systems ranging from optics to matter waves. It may result from certain transport processes such as atom diffusion, heat transfer, drift of electric charges etc., or the long range of the inter-particle interaction. It turns out that nonlocality may dramatically affect propagation of waves and their stability. In this talk I will we discuss the impact of nonlocality on the modulational instability of plane waves, the collapse of finite-size beams, and the formation, stability and interaction of spatial bright and dark solitons.

Bifurcations of solitons and their universality

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This lecture provides a brief review of the recent results devoted to bifurcations of solitary waves. The main attention is paid to the universality of soliton behavior and stability of solitons while approaching supercritical bifurcations. Near the transition point from supercritical to subcritical bifurcations, the stability of two families of solitons is studiedin the framework of the generalized nonlinear Schrödinger equation. It is shown that one-dimensional solitons corresponding to the family of supercritical bifurcations are stable in the Lyapunov sense. The solitons from the subcritical bifurcation branch are unstable. The development of this instability results in the collapse of solitons. Near the time of collapse, the pulse amplitude and its width exhibit a self-similar behavior with a small asymmetry in the pulse tails due to self-steepening.

Asymptotic models for water waves

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This talk will be a survey of the different asymptotic equations used to model water waves. I will show how they can be derived from the Euler equations and comment on the range of validity of these models.

Modelling Pulses in Molecular Chains and Conservative Time-Discrete Hamiltonian Systems

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A new approach is described for generating time discretizations for a large class of Hamiltonian systems which exactly conserve the energy and other quadratic conserved quantities of the corresponding differential equations.

An essential feature is a procedure for constructing discrete approximations of partial derivatives in a way that mimics essential properties of derivatives, in particular for the quadratic forms of most "momenta".

The approach is applied to a class of systems which includes models of energetic pulse propagation in protein due to Davydov, Scott et al. These models have the integrable nonlinear Schrödinger equation as a continuum limit, with sech pulse solutions. Extensions will be shown to systems like coupled systems of discrete nonlinear Schrödinger equations and oscillators. The discrete models have self-focusing effects not seen in the 1D cubic NLS.

The resulting time-discrete systems serve as numerical methods for Hamiltonian DE's, but are also of possible intrinsic interest as fully discrete dynamical system models, with solutions respecting symmetries and invariants of the Hamiltonian in the spirit of Noether's Theorem.

Nonlinear supratransmission and bistability in two-level media

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The Maxwell-Bloch system describes a quantum two-level medium interacting with a classical electromagnetic field by mediation of the variations in the density of states. This variation is a purely quantum effect which is actually at the very origin of nonlinear coupling. Such a nonlinearity possesses particularly interesting consequences at the resonance (when the frequency of the electromagnetic field is close to the ransition frequency of the two-level medium) as e.g. slow-light gap solitons that result from the nonlinear instability of the evanescent wave at the boundary (nonlinear supratransmission). For short-length media this instability generates a bistable behavior of the stationary cavity waves, allowing to conceive ultra-sensitive detectors. Moreover, as nonlinearity couples the different polarizations of the electromagnetic field, the dynamics of the slow-light gap soliton can be driven by applied laser beams or by external electrostatic potential.

Q-ball Ansatz in the scalar nonlinear electrodynamics - the case of the V-shaped matter potential

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Encouraged by the simplicity of the Q-ball solution in the complex scalar field theory with a V-shaped potential (and a global U(1) symmetry) we have undertaken the investigation of the case with the local symmetry. Basing on numerical data we have described the asymptotical behavior (both for small and large values of electric charge) of the physically most relevant family of solutions. The family interpolates between conventional Q-balls and solutions that resemble rather shells than balls. In between of these two kinds of solutions the charge-energy graph has a surprising, phase-transition-like, structure.

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A long wave model for the electrodynamics of point Josephson junctions in a cavity (P)

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We review a new long wave model describing the electrodynamics of point Josephson junctions in a superconducting cavity. It consists in the wave equation with Dirac delta function sine nonlinearities. This model allows an detailed and integrated description that was unavaible up to now. In the static case, it gives an excellent agreement with experiments.

The dynamical behavior can be solved by in the case of a single junction. It is remarquable that sweeping in current generates all of the cavity modes of the device. For multi-junction devices, we estimate the interaction between junctions throught the resonances. If the capacities are unmatched, we develop adapted modes on which to project the dynamics. We investigate too a dissipative kink which is an exact solution of the problem.

Parametric instability of coherent states of a nonlinear scalar field. Can gravity stabilize pulsations of soliton stars?

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We investigate stability of both localized time–periodic coherent states (pulsons) and uniformly distributed coherent states (oscillating condensate) of a real scalar field satisfying the Klein–Gordon equation with a logarithmic nonlinearity. Such singular nonlinearities appear, in particular, in cosmology and in some supersymmetric extensions of the Standard Model.

The analysis of time–dependent parts of the pulson's perturbation leads to the Hill equation with a singular coefficient. To evaluate the characteristic exponent we extend the Lindemann–Stieltjes method, usually applied to the Mathieu and Lamé equations, to the case that the periodic coefficient in the general Hill equation is an unbounded function of time. As a result, we derive a formula for the characteristic exponent and plot the stability–instability chart. We verify these calculations by the direct numerical integration of the Hill equation. The consideration of the space–dependent parts of the perturbation leads to the Schrödinger equation having a discrete spectrum of localized solutions. In particular, for the nodeless pulsons these solutions are expressed in terms of the Hermite polynomials. Using these results we show that the pulsons of any amplitudes, remaining well–localized objects, lose their coherence with time. This means that, strictly speaking, all pulsons of the model considered are unstable. Nevertheless, for the nodeless pulsons the rate of the coherence breaking in narrow ranges of amplitudes is found to be very small, so that such pulsons can be considered as long–lived.

Further, we use the obtaned stability–instability chart to examine the Affleck-Dine type condensate oscillating around the minimum of the logarithmic potential. We estimate the wavenumber and the oscillation amplitude for which the characteristic exponent is maximal and discuss the fragmentation processes. We conclude that, due to the parametric instability, the oscillating condensate can decay into an ensemble of the nodeless pulsons.

At the end we consider the Einstein–Klein–Gordon system and discuss the effect of gravity on the above instabilities in the context of the soliton star and dark matter dynamics.

Optical solitons and similaritons in fiber based systems

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Optical fiber systems are well-known to provide convenient platforms with which to investigate a host of fundamental fascinating non-linear phenomena. One of the best illustrations is the observation of optical solitons by Mollenauer et al. more than 20 years ago and their practical application for ultra long-haul telecommunications. Since then, a wide range of configurations has been experimentally studied and has outlined the extensive richness of optical fibers, especially in the context of optical supercontinuum generation where various soliton effects have been reported : compression, fission, Raman self-frequency shift, as well as generation of dispersive waves or formation of Raman bound pairs. But the interest of optical fibers is not restricted to the anomalous dispersion regime and the propagation of ultrashort pulses in normally dispersive fiber amplifiers has recently generated considerable interest and has revealed a self-similar asymptotic dynamics : any initial pulse progressively reshapes during its amplification into a pulse having a parabolic intensity profile combined with a linear chirp, often referred to as a similariton pulse.

We will first review the various solitonic effects which impact the evolution of a femtosecond pulse towards a supercontinuum and in a second part of the talk, we will describe the features and somme applications of optical similaritons.

Optical rogue waves and giant solitons

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It has been recently reported that the probability of observing a giant wave in the middle of the ocean is stronger that what is predicted by classical statistical models. These rare events are not yet well understood, namely the conditions which favour the emergence of these waves. Very recently, it has been demonstrated that the governing equations as well as the statistical properties of an optical pulse propagating inside an optical fiber [1] are very closed than those of these gigantic surface waves [2]. As a consequence, a better understanding of the reason why strong and rare optical waves can be generated in optical fiber could help to understand what happens in the seas. To our knowledge, all reported studies in the field of optics has been realised under pulsed pumping conditions [1,3]. However, the parallel with these input pulses and any state of the sea seems difficult to estabilish. The aim of this work is twofold. Firstly we numerically and experimentally d emonstrate that the generation of these rare and strong optical pulses can be achieved with a continuous wave pump (CW). We believe that the correspondence between a calm sea and this CW initial condition is easier to achieve. Secondly, we analytically demonstrated that the generalized Shrdinger (GNLSE) equation, which governs the soliton propagation in fibers leads to convective instabilities [4]. The convective nature of the optical rogue waves is fundamental in the understanding of their origin. This reveals the extreme sensitivity of these rare events to noisy initial conditions. As a result input noise is dynamically connected to convective modulationnal instabilities producing eventually optical rogue waves in form of rare giantlike propagating solitons.

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Modelling resonances in fragmented solids

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Fragmented solids are solids separated into fragments or blocks with no mechanical bonding. The blocks are hold together either due to attractive forces of a sort or by the geometric constraints imposed by the neighbouring blocks as in topologically interlocking structures. (The topological interlocking structure being constrained at the periphery maintains its integrity without the help of any binder or connectors.) The main feature of interlocking structures is the ability of separate blocks to move (and rotate) independently within the kinematic constraints imposed by the neighbours. This affects the way the oscillations are induced and propagate within the structure and results in considerable vibration damping. It is hypothesised that the main features of vibrations in an interlocking structure can be captured by representing it as a system of coupled bilinear oscillators. Bilinear oscillator is the oscillator in which the spring has different stiffness in compression and tension. In a single bilinear oscillator this is known to lead to the emergence of multi-harmonic and sub-harmonic resonances. We investigated a system consisted of two coupled bilinear oscillators and proposed a method of determination resonance frequencies. The two resonance frequencies correspond to the oscillations which, in an appropriate coordinate frame, are equivalent to oscillations of two independent bilinear oscillators. Numerical simulations of both, single and double oscillators show that the shape of oscillations is stable for all types of deterministic and stochastic driving force if the ratio of partial frequencies (the frequencies corresponding to stiffness in tension and compression) is large. The resonances are also accompanied by a phase shift and, in the case of sub- harmonic resonances, by a reduction in frequency.

Lax pairs, sine-Gordon and elliptic sine-Gordon

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This talk will survey the role of Lax pairs when solving boundary value problems for integrable PDEs in one space dimension, using the case of sine-Gordon and elliptic sine-Gordon as illustrative examples.

Controlling events at the atomic and molecular scales through Hamiltonian manipulation

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Since the development of the laser some 40 years ago, a long standing dream has been to utilize this special source of radiation to manipulate dynamical events at the atomic and molecular scales. Hints that this goal may become a reality began to emerge in the 1990's, due to a confluence of concepts and technologies involving (a) control theory, (b) ultrafast laser sources, (c) laser pulse shaping techniques, and (d) fast pattern recognition algorithms. These concepts and tools have resulted in a high speed instrument configuration capable of adaptively changing the driving laser pulse shapes, approaching the performance of thousands of independent experiments in a matter of minutes. Each particular shaped laser pulse acts as a Photonic Reagent much as an ordinary reagent would at the molecular scale. Although a Photonic Reagent has a fleeting existence, it can leave a permanent impact. Current demonstrations have ranged from manipulating simple systems (atoms) out to the highly complex (biomolecules), and applications to quantum information sciences are being pursued. In all cases, the fundamental concept is one of adaptively manipulating quantum systems. The principles involved will be discussed, along with the presentation of the state of the field.

Oscillons in water waves

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We show experimental evidences of stable solitary standing water waves of large amplitudes. These waves resemble the oscillons first evidenced by Umbanhovar et al. (1996) in vibrated granular materials.

Nonlinear Waves in Mathematical Models of Traffic Flow

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I will describe a few qualitative aspects of mathematical models of traffic flow. The tools used here are mainly hyperbolic systems of conservation laws, the corresponding nonlinear waves: shocks when braking, rarefaction waves when accelerating, contact discontinuities between cars with different Lagrangian properties (e.g. cars/trucks), and how these waves are modified when one adds a relaxation term towards an equilibrium velocity, e.g. rarefactions are replaced with traveling waves (TW).

One of the main issues is to obtain a system with enough staibility, in order to avoid crashes, or on the contrary negative velocities (!), but not too much, in order to preserve a rich dynamical behaviour.

Indeed, in real traffic flow, the existence of oscillating solutions (stop and go waves) is a fact of life, that people are trying to describe in particular on the prototype of a ring road, with competing approaches.

An interesting example of system with relaxation is when there is just L^∞ stability (bounded invariant regions) but no BV stability, in particular the celebrated Whitham stability condition is not satisfied. In this case, the first issue is the existence of periodic solutions, typically a TW followed by a shock with the same speed. I will discuss some of these issues. Part of the material is in collaboration with J. Greenberg (Pittsburgh).

Deriving the fine structure constant from quantum transitions between nonlinear eigenstates

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A numerical coincidence in the quasi-exact nonlinear Schrödinger-Poisson description of a parabolically trapped opposite-spin electron pair is pointed out, where the probability of coupling between either electron and the trap harmonic quantum approaches the value 1/137 of the fine-structure constant within 1It can be explained by quantum transitions between the non-orthogonal stationary eigenstates of the corresponding non-relativistic and nonlinear differential system.

Conservation properties of multisymplectic integrators

On Nonlinearity, Dissipation and Decoherence in Quantum Mechanics

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There exist different modifications of the linear Schrödinger equation introducing nonlinearities for different purposes, e.g., for the stabilization of the shape of wave packets or for consideration of dissipation in quantum mechanics. Gaussian wave packet-type solutions can be found in all these cases but different effects on the coherence and, thus, the wave packet shape, are obtained which are sometimes very contradictory. So, some models predict stabilization and reduction of decoherence and wave packet spreading due to dissipation, whereas others arrive at exactly the opposite conclusion. Equations of motion for the wave packet width, and hence decoherence, can be obtained from quantum mechanical model systems with exact analytic solutions. This leads to nonlinear complex Riccati equations or nonlinear real Ermakov equations with corresponding invariants. The changes in these equations due to the consideration of nonlinearities and dissipation in the Schrödinger equation will be discussed and, in particular, a physically-motivated Schrödinger equation with a logarithmic nonlinearity that combines relaxation and decoherence effects will be presented. Similar Riccati or Ermakov equations can also be found when considering the cubic nonlinear Schrödinger equation (Gross-Pitajewski) and a nonlinear formulation of time-independent quantum mechanics.

Explosive instability due to 3-wave or 4-wave mixing

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Forty years ago, Coppi, Rosenbluth & Sudan (1969) discovered the "explosive instability" in a model of plasma physics. They showed that under the right conditions, three nonlinearly interacting wave modes can all gain energy from a background source, and all three modes can blow up together in finite time. Explosive instabilities are now known to occur in a variety of physical systems that admit three-wave interactions (also called resonant triads), when these systems also admit "negative energy modes".

More recently, Safdi & Segur (2007) showed that explosive instability can occur even in systems that admit no three-wave interactions. Instead, four nonlinearly interacting waves can all gain energy from a background source, and all four modes can blow up together in finite time. This singular behaviour has no relation to the well-known self-focussing singularity, which can also occur in systems with four-wave interactions. Both singularities can grow out of smooth, bounded initial data in finite time, but they are entirely different.

This talk describes explosive instability, due to either three-wave or four-wave interactions. As time permits, we can also discuss the effect of dissipation on an explosive instability, and/or the meaning of "negative energy modes" for systems that blow up due to four-wave interactions.

Microcavity soliton-polaritons

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Existence and dynamical properties of spatially localized exciton-polaritons (or cavity solitons) in semiconductor microcavities operating in strong coupling regime will be presented and discussed. These localized structures is a promising alternative for applications in all-optical signal processing because their excitation time and required pump powers are few orders of magnitude less than those of their better known weakly coupled counterparts. Exciton-polariton condensate in microcavities can be manipulated by the various potentials (including periodic and guiding potentials), which offers a multitude of methods to complement nonlinear localisation. This system also paves the way to a practical realisation of a new type of 'compacton'-like waves.

Azimuthons in nonlocal nonlinear media

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Spatial trapping of light in nonlinear media is a result of a balance between diffraction and the self-induced nonlinear index change. However, local nonlinearity cannot stabilize complex localized structures such as vortex beams, but nonlocal nonlinearty can. In such media nonlinear response in a particular spatial location is typically determined by the wave intensity in a certain neighborhood of this location. Nonlocality often results from certain transport processes such as atomic diffusion or heat transfer. It can be also a signature of a long-range interparticle interaction such as in nematic liquid crystals. Recently, it has been shown that nonlocal media can also support stable propagation of the rotating solitons, the so called azimuthons [1]. Azimuthons are azimuthally modulated beams with nontrivial phase structures exhibiting steady angular rotation upon propagation [2]. In this work, we will show that both, rotational frequency and intensity profile of the azimuthons can be uniquely determined by analyzing eigenmodes of the linearized version of the corresponding nonlocal problem. We link families of azimuthons to internal modes of classical non-rotating solitons, like vortices or multipoles. This offers a straightforward and exhaustive method to identify rotating soliton solutions in a given nonlinear medium.

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The effect of noise and coupling on beta cell excitation dynamics

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Bursting electrical behavior is commonly observed in a variety of nerve and endocrine cells, including that in electrically coupled beta-cells located in intact pancreatic islets. However, individual beta-cells usually display either spiking or very fast bursting behavior, and the difference between isolated and coupled cells has been suggested to be due to stochastic fluctuations of the plasma membrane ion channels, which are supposed to have a stronger effect on single cells than on cells situated in clusters (the channel sharing hypothesis). This effect of noise has previously been studied using numerical simulations. We show here how the application of two recent methods allows an analytic treatment of the stochastic effects on the location of the saddle-node and homoclinic bifurcations, which determine the burst period. Thus, the stochastic system can be analyzed similarly to the deterministic system, but with a quantitative description of the effect of noise. This approach supports previous investigations of the channel sharing hypothesis.

For beta cells coupled via gap junctions we briefly discuss the effects of the ATP driven potassium ion gate on reaction diffusion type waves. It is shown how these effects lead to wave block phenomena in glucose gradients across an islet of Langerhans.

Interactions of shallow water solitons and their applications

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Interactions between solitons are not only the corner stone of the concept of solitons but also one of the most fascinating features of soliton phenomena. A selection of phenomena occurring at and practical applications of soliton interactions is discussed from the viewpoint of low-dimensional (line) solitons in shallow water.
Already the Russell's observations in 1834 implicitly involved certain effects created through soliton interactions. The straight crest of the Russell's soliton is not intrinsically one-dimensional structure; instead, it is formed owing to a specific mechanism of soliton interactions from precursor solitons with curved crests occurring at the sidewalls (Pedersen 1988). For a certain range of incidence angles, the crests of the approaching and the reflected wave fuse together to an analogue of the Mach stem near the wall. The resulting structure is equivalent to half of the pattern created by two interacting Kadomtsev-Petviashvili solitons.
The prominent feature of both the phenomena is that the height of the common crest can be up to four times as high as the incoming waves and the slope of water surface may eight times exceed the water slope in counterparts. Unexpectedly large elevations, extreme slopes, or changes in orientation of wave crests owing to the (Mach) reflection or oblique interactions of solitonic waves frequently cause an acute danger and can be interpreted as a potential mechanism of freak waves.
An increasing source of solitonic waves is the fast ship traffic in relatively shallow areas. The low decay rates of the solitonic ship wakes has led to a significant impact on the safety of people, property and craft, and to a considerable remote impact of the ship traffic in shallow areas. Such a wake has probably caused a fatal accident as far as about 10 km from the sailing line already in 1912. The interactions of ship-induced solitons may lead to dangerous waves in the vicinity of coastal fairways and harbour entrances in otherwise sheltered areas because ship wakes often approach seawalls or breakwaters from other directions than

wind waves do.
An intriguing use of soliton interactions has been made for reducing the wave resistance in channels and for catamaran design: the bow wave (optionally reflected from a side wall) may be used to cancel the stern wave. The use of this effect probably is the first intentional use of the features occurring during soliton interactions in the design of a certain technology (Chen and Sharma 1994, 1997). Yet already J.S. Russell may have been aware of this possibility. He describes efforts of a spirited horse which, pulling a boat in a canal had drawn the boat up into its own wave leading to a significant reduction in resistance. This led to high-speed service on some canals in the 1820's and 1830's (Russell 1837).

Solitons on an undeformable free drop generated by the Marangoni effect (P)

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The Marangoni effect (sometimes also called the Gibbs-Marangoni effect) represents the mass transfer at the surface of a liquid due to surface tension differences. Marangoni flow might be generated by gradients in either temperature or in chemical concentration (especially in the presence of surfactants) at the interface. Fluids react differently to stress in microgravity when compared to the same stress on Earth. Under Earth conditions the effect of gravity, inducing density driven convection in a system with a surface tension gradient along a fluid/fluid interface, is usually much stronger than the Marangoni effect. Therefore, many experiments have been conducted under microgravity conditions to observe the Marangoni effect without the influence of gravity. Thus, we have studied experimentally the effect of the adsorption of a surfactant on the surface of an undeformable free liquid drop immersed in an unbounded liquid. In this respect, we have simulated the microgravity by taking equal densities for the drop and bulk liquid. Our experimental results demonstrate that the interfacial tension gradient, caused by the surfactant adsorption at the drop surface, generates a real surface flow (Marangoni flow) in front with a solitary wave, a soliton. Further, we have proposed a hydrodynamic model to explain the generation of this soliton and we have calculated the soliton velocity which is in good agreement with the experimental one.

Linear stability of stationary wave solutions for second harmonic generation and degenerate two-photon propagation (P)

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Both second harmonic generation (SHG) and degenerate two–photon propagation (DTPP) are known to be S–integrable systems. They permit solutions where there is no exchange of energy a) between ground and harmonic waves for SHG or b) between the optical field and the two–level atoms for DTPP. These solutions prove to be linearly stable. With small distortions of the considered solutions for both systems we arrive at one and the same linear third–order differential equation, $b_{1,\chi\tau\tau}+4b_{1,\chi}+8b_{1,\tau}=0$, and the ansatz $b_1=\cos[K\chi-W\tau]$ leads to the dispersion relation $K=2W/(1-W^2)$, $W=(-1\pm\sqrt{1+K^2})/K$. I.e., for any real $W\neq\pm1$ there is a real K, and for any real K there is a pair of real values of W.
From the point of physical intuition the linear stability is surprising. An investigation of nonlinear stability has not been done yet.

Solitonic solutions of the kinetic derivative nonlinear Schrödinger equation for dispersive Alfvén waves (P)

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The kinetic derivative nonlinear Schrödinger (KDNLS) equation extends the regular DNLS equation for parallel Alfvn waves in a magnetized plasma, by retaining kinetic effects (such as Landau damping). It can be directly derived from the Vlasov-Maxwell equations, using a long-wavelength reductive perturbative expansion. Landau damping contributes by a non local nonlinear dissipative term involving a Hilbert transform, that breaks the integrability of the DNLS equation and plays a significant role even when relatively small compared with the usual DNLS nonlinearity. Numerical simulations of the KDNLS equation indicate that this equation admits solutions in the form of solitonic waves whose characteristics (width, amplitude and velocity) can strongly vary in time, possibly displaying strong acceleration and quasi-collapse, together with abrupt changes in the direction of propagation

Dissipative localized structures and self-organization

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Out of equilibrium systems support a large variety of dissipative structures, which can be either periodic or localized in space. The latter are often called dissipative solitons. They have been investigated theoretically and observed experimentally in various natural systems [1, 2]. In this communication, an overview on the formation of localized structures will be given with a special emphasis on:
(i) small scale self-organisation in diffractive cavity nonlinear optics
(ii) large scale self-organisation in plant ecology, generically called vegetation patterns. These are typical of (semi-)arid regions where the potential evapotranspiration substantially exceeds the mean annual precipitation. This hydric deficit impedes the development of individual plants and, at the community level, promotes clustering behaviors, via a modulational instability, even if the topography is isotropic.

References:

[1] M. Tlidi, T. Kolokolnikov, and M. Taki, Focus Issue "Dissipative Localized Structures in Extended Systems" Chaos, 17, Issue 3 september, 2007.
[2] N. Akhmediev and A. Ankiewicz "Dissipative Solitons: From Optics to Biology and Medicine" (Springer-Verlag, Berlin, Heidelberg, 2008).

Singular solutions of a modified two-component shallow-water equation

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The Camassa-Holm equation (CH) is a well known integrable Hamiltonian equation describing the velocity dynamics of shallow water waves. In its dispersionless limit, this equation exhibits spontaneous emergence of singular solutions (peakons) from smooth initial conditions. The CH equation has been recently extended to a two-component integrable system (CH2), which includes both velocity and density variables in the dynamics. Although possessing peakon solutions in the velocity, the CH2 system does not admit singular solutions in its density profile.

We modify the CH2 system to allow dependence on average density as well as on pointwise density. The modified CH2 system (MCH2) now admits peakon solutions in both velocity and average density, although it may no longer be integrable. We analytically identify the steepening mechanism that summons the emergent singular solutions from smooth spatially-confined initial data.

Numerical results for MCH2 are given and compared with the pure CH2 case. These numerics show that the modification in MCH2 to introduce average density has little short-time effect on the emergent dynamical properties. However, an analytical and numerical study of pairwise peakon interactions for MCH2 shows a new asymptotic feature. Namely, besides the expected soliton scattering behavior seen in both overtaking and head-on peakon collisions, MCH2 also allows the phase shift of the peakon collision to diverge in certain parameter regimes.

Enhancement of interaction of dissipative solitons above self-pulsing instability treshold

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Short optical pulses have numerous technological applications in high bit rate communications, optical tomography, spectroscopic measurements, material processing, frequency standards, etc. In relation to their applications for all-optical transmission, storage, and processing of information, the problem of pulse interaction is of particular importance. In this presentation, the interaction of weakly overlapping dissipative solitons below and above self-pulsing threshold in active (mode-locked laser) and passive (driven cavity) optical devices of an individual soliton solution is studied analytically and numerically. Being separated from each other, solitons in these devices interact via their exponentially decaying tails. Using an asymptotic approach we derive a set of ordinary differential equations governing the slow time evolution of the positions and oscillation phases of the interacting pulses. Being independent of specific details of the model, the form of these interaction equations is determined mainly by the asymptotic behavior of the pulse tails and the symmetries of the model equations. Therefore, they have a universal nature and can be used to study interaction of temporal and spatial localized structures not only in optical, but also in hydrodynamic, plasma, and even biological systems. We present numerical and analytical evidence that an Andronov-Hopf bifurcation leading to a periodic modulation of characteristics of individual solitons can drastically change the character of their interaction. In particular, we demonstrate a very strong enhancement of the soliton interaction just above the self-pulsing instability threshold. This enhancement results in the formation of various new types of soliton bound states.