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# *Out of equilibrium statistical mechanics of two dimensionnal flows*

One of the most important problem in turbulence is the prediction of the ***large-scale*** of very high Reynolds' flows. The highly turbulent nature of such flows, for instance ocean circulation or atmosphere, renders a probabilistic description desirable, if not necessary. For the two dimensionnal Euler equation, or related geophysical flow models, a statistical mechanics explanation of their self-organization has been proposed by Robert-Sommeria and Miller (RSM). The RSM theory has been successfully applied to the Jupiter's troposphere : cyclones, anticyclones and jets have been quantitatively described by this theory [1]. The RSM theory starts from the conservative dynamics and parametrizes the equilibria by the energy and other dynamical invariants. However, the theory does not predict the long-term effects of the forcing, which is a relevant issue for any application. It is a practical and fundamental problem to understand how the invariants are selected by the presence of a weak forcing and dissipation, what are the associated fluctuations, are all forcings compatible with RSM equilibria ? The relaxation towards equilibrium of 2-D flows has been considered in the past, however the out-of-equilibrium statistical mechanics has never been considered yet. From a statistical mechanics point of view, this problem is a logical continuation of the RSM theory. This is the main goal of this work.

We consider the 2-D Navier-Stokes equation with weak stochastic forcing and dissipation (Euler limit). Because we are interested in large-scale flows, we do not consider the classical inertial self-similar cascade ; rather we consider forcing acting on all scales. In that case, the existence of an invariant measure, and its Euler limit, has been mathematically proved recently, together with mixing and ergodic properties (see for instance [2] and references therein). This problem has however never been considered from a physical point of view. We address the following issues : when is the measure concentrated on RSM equilibria, how are the large scales selected by the forcing, what is the level of the fluctuations ? Because we do not expect universality, we do consider different types of dissipation (bulk diffusion, boundary layer dissipation, linear drag) and different geometries.

This study is based on intensive numerical computations, theoretical predictions, and for simple basic situations, it is based on mathematical results. From the numerical studies, we prove that the large scale stationary flow are actually close to some RSM equilibria. In a situation of phase transition, by tuning the forcing, we can obtain flows with different topologies, as predicted by the RSM theory. We qualitatively understand the link between the Energy and other invariants which parametrize the RSM theory, and the forcing parameters which are our control parameters. A more precise study show that besides a strong qualitative similarity, these out of equilibrium states have some properties which are not compatible with RSM equilibria. We explain the physical mechanism for that discrepancy.

From a mathematical point of view, we have considered the nonlinear stability of the RSM equilibria, for the Euler dynamics. Most of such equilibria do not verify the hypothesis of the celebrated Arnold's theorems. However, they are thought to be stable. We have clarified the nonlinear stability of such equilibria in a large number of situations.

We also address the stochastic stability of RSM equilibria, for the *linearized* Navier Stokes equation with random forcing. We address the existence of a stationnary measure for that problem (stochastic stability). In the stable cases, we numerically compute the second order correlation operator for the flow large scales, and we compare it to the fluctuations around the average flow obtained from the direct numerical simulation of the forced Navier Stokes equation, discussed in the previous paragraphs.

# References

[1] BOUCHET F. and SOMMERIA J., 2002 Emergence of intense jets and Jupiter's Great Red Spot as maximum entropy structures J. Fluid Mech.464, 165-207.

[2] KUKSIN S. B., 2004, The Eulerian limit for 2D statistical hydrodynamics, J. Stat. Physics 115:1/2, 469–492.