# Global Well-posedness of $3D$ Primitive Equations

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*Abstract submitted to EE250*

Large scale dynamics of oceans and atmosphere is governed by the primitive equations which are derived from the Navier–Stokes equations, with rotation, coupled to thermodynamics and salinity diffusion-transport equations, which account for the buoyancy forces and stratification effects under the Boussinesq approximation which are the $3D$ Navier–Stokes equations (NSE) with rotation couple to the heat (and/or salinity) transport equations.

The global existence and uniqueness of smooth solution to the $3D$ NSE is one of the most challenging mathematical problems in applied analysis. Consequently, the Boussinesq system is equally challenging from this point of view. However, due the geophysical nature of the rotating earth geophysicists derive "simpler" models to be studied analytically and simulated computationally. For instance, by taking advantage of the shallowness of the oceans and the atmosphere they introduce the hydrostatic balance approximation for the vertical motion (see, e.g., [3], [4], [5], [6], [7] and references therein). The Boussinesq system under the hydrostatic assumption is known as the primitive equations.

Here, we will focus on the $3D$ primitive equations in a cylindrical domain

$$\Omega = M \times (-h, 0),$$

where $M$ is a smooth bounded domain in $\mathbb{R}^2$:

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v + w\frac{\partial v}{\partial z} + \nabla p + f\vec{k} \times v = \frac{1}{Re_1}\Delta v - \frac{1}{Re_2}\frac{\partial^2 v}{\partial z^2}, \tag{1}$$

$$\partial_z p + T = 0, \tag{2}$$

$$\nabla \cdot v + \partial_z w = 0, \tag{3}$$

$$\frac{\partial T}{\partial t} + v \cdot \nabla T + w\frac{\partial T}{\partial z} = Q + \frac{1}{Rt_1}\Delta T - \frac{1}{Rt_2}\frac{\partial^2 T}{\partial z^2}, \tag{4}$$

where the horizontal velocity field $v = (v_1, v_2)$, the three-dimensional velocity field $(v_1, v_2, w)$, the temperature $T$ and the pressure $p$ are the unknowns. $f = f_0(\beta + y)$ is the Coriolis parameter, $Q$ is a given heat source, $Re_1, Re_2$ are positive constants representing the horizontal and vertical Reynolds numbers, respectively, and $Rt_1, Rt_2$ are positive constants which stand for the horizontal and vertical heat diffusivity, respectively.

In [3], [4] and [8] the authors set up the mathematical framework to study the viscous primitive equations for the atmosphere and ocean circulation. The short time existence and uniqueness of strong solutions to the viscous primitive equations model was established in [1] and [8]. In [2] the authors proved the global existence and uniqueness of strong solutions to the viscous primitive equations in thin domains for a large set of initial data whose size depends inversely on the thickness of the domain. In this paper we show the global existence, uniqueness and continuous dependence on initial data, i.e. global regularity and well-posedness, of the strong solutions to the $3D$ viscous primitive equations model (1)–(4) in general cylindrical domain, $\Omega$, and for any initial data. It is worth stressing that the ideas developed in this paper can equally apply to the primitive equations subject to other kinds of boundary conditions.

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