Lagrangian modeling and alignment trends of vorticity with pressure-Hessian eigendirections in turbulence

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The local statistical and geometric structure of three-dimensional turbulent flow can be described by properties of the velocity gradient tensor $A_{ij} = \partial u_i/\partial x_j$. Recently, based on prior works, a stochastic model has been developed for the Lagrangian time evolution of this tensor, in which the exact nonlinear self-stretching term $-\mathbf{A}^2$ accounts for the development of well-known non-Gaussian statistics and geometric alignment trends [1]. The pressure Hessian $\mathbf{P}$ and viscous term $\nu\Delta\mathbf{A}$ are accounted for by a closure that models the material deformation history of fluid elements. The resulting model can be written in the following way:

$$d\mathbf{A} = \left(-\mathbf{A}^2 - \mathbf{P} + \nu\Delta\mathbf{A}\right) dt + d\mathbf{W} \ , \qquad (1)$$

where $\mathbf{P} \sim \mathbf{C}_\Gamma^{-1}$ is modeled using the recent material deformation as described by the (inverse) recent Cauchy-Green tensor $\mathbf{C}_\Gamma$. The viscous term is modeled as a friction term with a non-linear coefficient (see Ref. [1] for details). The forcing term $\mathbf{W}$ entering in Eq. (1) is chosen Gaussian and delta-correlated in time and describes the joint action of large-scale forcing and neigboring eddies at moderate Reynolds number. A parameter $\Gamma = \tau/T$, defined as the ratio between the Kolmogorov and integral time scale, has to be given. The resulting stochastic system reproduces many statistical and geometric

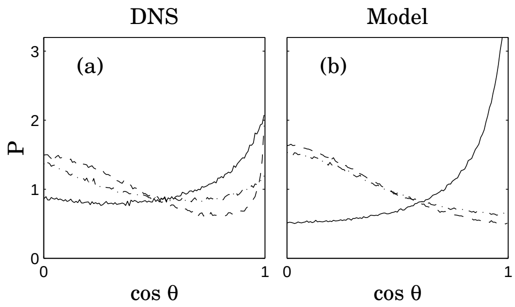

Figure 1: Comparison of the PDFs obtained from DNS ($\mathcal{R}_\lambda = 150$) and the model ($\tau_K/T = 0.1$) of the cosine between vorticity and the eigendirections of the pressure Hessian associated to the smallest (dashed), intermediate (solid) and biggest (dot-dashed) eigenvalues.

trends observed in numerical and experimental 3D turbulent flows, including anomalous relative scaling and the preferential alignment of vorticity [2].

We focus here on the precise topology and eigensystem of the resulting modeled pressure Hessian. Prior work for the inviscid limit described by the Euler equations has shown that vorticity tends to be an eigenvector of the rate of strain tensor $\mathbf{S}$ as well as of the pressure Hessian $\mathbf{P}$. We show that this property is exactly reproduced by the present model [2]. Then, Direct Numerical Simulations (DNS) are used to test alignment trends between vorticity and pressure Hessian eigendirections, and results are compared to predictions from the Lagrangian model. Results are displayed in Fig. 1. We see that the model reproduces the alignment with the eigendirection associated with the intermediate eigenvalue very well. But the model describes only partially the orientation of vorticity with the eigendirection of the smallest eigenvalue, for which in DNS very strong alignment is found. The analysis results provide, overall, additional support for the accuracy of the Lagrangian model proposed in [1] but highlight areas where improvements are required.

References

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