# Driven Cavity Flows in Soap Bubbles: An Illustration of the Spherical Dirichlet Problem

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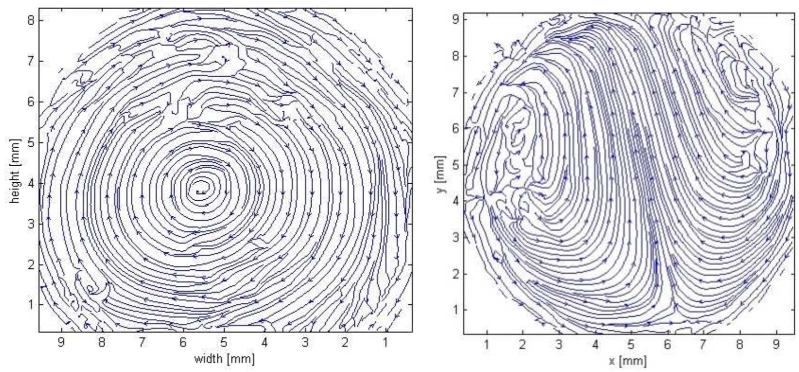
There are few physical objects as beautiful and fascinating as soap bubbles. They exhibit a perfection of geometrical form and an appealing simplicity. Soap bubbles are characterized by the structure of their thin liquid shell of low surface tension. It is well known that the dynamics of the surface of soap bubbles are the resultant of both 1) liquid film thinning under the influence of gravity and 2) the presence of local surface tension gradients which create complex flows (Marangoni flows). In turn, the net motion of the thin liquid shell induces the displacement of gas inside the cavity due to the no-slip boundary condition which holds at the liquid-gas interface. Both flows on the liquid shell and within the cavity are incompressible, $\nabla \cdot \underline{u} = 0$. Moreover, for small soap bubble sizes, O($< 10^{-2}$ m), flows are viscosity dominated and characterized by low-Reynolds numbers (typically Re=O($< 1$)).

While soap bubbles exhibit intrinsic internal flows [1], thermally-induced Marangoni flows in the liquid film may lead to steady-state forced recirculation inside the cavity. Experimentally, this may be achieved by applying a thermal gradient in the vicinity of the liquid shell, which produces a fixed shear stress at the liquid surface given by $\tau = \nabla\sigma = (\partial\sigma/\partial T)\nabla T$, where $\sigma$ is the surface tension, and $T$ the temperature.

In the present discussion, we will show experimental flow visualizations of the boundary-driven cavity flows inside soap bubbles (Fig. 1), which may be controlled by such thermal forcing schemes. Variations in the number and location of heat sources applied near the liquid shell effectively yield different internal flow topologies.




(a) sideview (b) topview

Figure 1: Reconstructed internal streamlines obtained from particle image velocimetry (PIV) for a soap bubble shell heated on one side.

Such incompressible, steady-state, low-Reynolds number flows on the surface of the shell and within the soap cavity may be mathematically described by the simplified vorticity equation:

$$\triangle \underline{\omega} = 0, \tag{1}$$

where $\underline{\omega}$ corresponds to the vorticity field on the boundary and within the cavity. In particular, we will attempt to demonstrate that driven cavity flows in soap bubbles illustrate here the well-known Dirichlet problem for a sphere. Indeed, by prescribing a function $f|_{\partial B}$ on the boundary of the sphere modelling the forced motion of the liquid shell, the flow topology within the cavity may be reconstructed by implementing the Poisson integral for a sphere which yields a harmonic function, as described by Eq. (1).

# References

[1] Sznitman J, Rösgen T. *PIV investigation of internal recirculating flows in thin liquid shells*, Proc. $12^{th}$ Inter. Symp. Flow Visualization, 2006.