On axisymmetric intrusive gravity current in stratified ambient: the approach to self-similarity solutions of the shallow-water equations

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The axisymmetric intrusion of a fixed volume of fluid, which is released from rest and then propagates horizontally-radially at the neutral buoyancy level in a stratified ambient fluid of height $2H$ is investigated. The typical system configuration is sketched in Fig. 1(a).

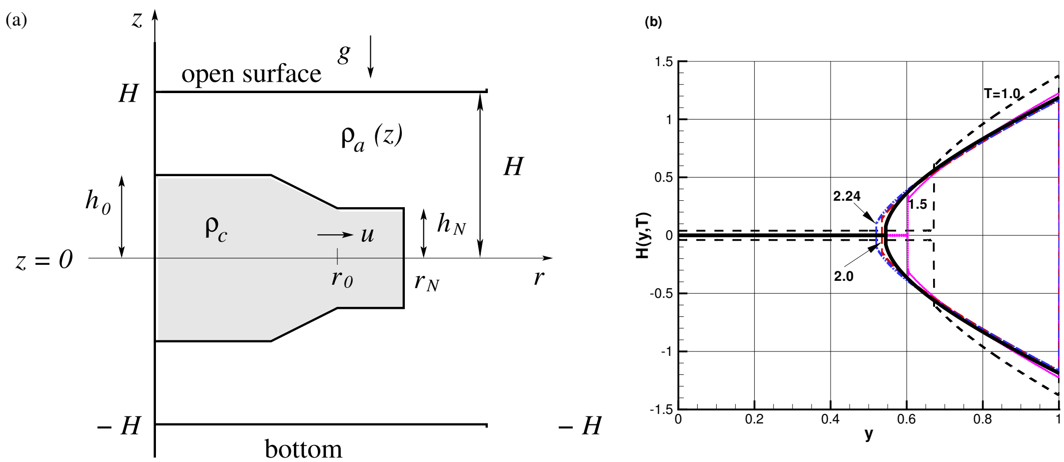

Figure 1: (a)Schematic description of the system: in the dimensionless form, the horizontal lengths are scaled with r_0 and the vertical lengths with h_0. The subscripts denote: N - nose (or front); a - ambient; b - bottom; c - current (intrusion); o - open surface. (b)The comparison between the similarity and numerical solutions of the shallow-water equations for $Fr = 1.19$. Computed height profiles $H(y, T)$. The bold solid curve is the similarity analytical solution.

The extension from the rectangular to the axisymmetric geometry is non-trivial and produces some unexpected results. The predicted velocity field and the the shape of the interface gain some complexities with have no counterpart in the Cartesian case.

Attention is focused on the development of self-similar propagation. It is assumed that the ambient fluid is deep, its density change is linear and the Reynolds number Re of the flow is large. The shallow-water one-layer equations representing the motion are used for the dependent variables: the half-height $h(r,t)$ and the averaged longitudinal velocity $u(r,t)$ of the intrusion, where r is the radius and t is time. The dimensionless variables are scaled by

$$. \{r^*, z^*, h^*, H^*, t^*, u^*\} = \{r_0 r, h_0 z, h_0 h, h_0 H, T_{ref} t, U_{ref} u\} \quad (1)$$

where

$$U_{ref} = \mathcal{N} h_0, \;\; T_{ref} = \frac{r_0}{U_{ref}}, \;\; \mathcal{N} = \left(\frac{\rho_c - \rho_o}{\rho_o}\frac{g}{H}\right)^{1/2}, \quad (2)$$

The appropriate necessary front condition uses Froude number, Fr, in the domain $1 \leq Fr \leq \sqrt{2}$. For the long-time developed motion an analytical similarity solution indicates radial expansion with $t^{1/3}$, but the shape is peculiar: the intruding fluid propagates like a ring with a fixed ratio of inner and outer radii; the inner domain contains clear ambient fluid.

To verify the similarity analytical prediction, it should be compared with the numerical solution of the shallow-water problem for the very long time. However, as noted by Grundy & Rottman [2], for large time after release, the current becomes very thin and numerical finite difference solutions of the original shallow-water equations are prone to round-off error. Therefore, we found it necessary to reformulate the problem in terms of new dependent variables $H(y,T)$ and $U(y,T)$, defined by

$$\begin{cases} r_N(t) = Kt^{1/3}R(T) \\ h(r,t) = \frac{1}{3}Kt^{-2/3}H(y,T); \\ u(r,t) = \frac{1}{3}Kt^{-2/3}U(y,T), \end{cases} \tag{3}$$

where $y = \frac{r}{r_N}$ and $T = \frac{1}{3}\ln t$. K is the positive constant which is determined by volume continuity considerations to be

$$K = 3Fr(\frac{V_0}{3\sqrt{2}})^{1/3}, \tag{4}$$

where V_0 is the constant volume of the intrusion and is equal to 0.5 for the standard initial cylinder and to 1/3 for an initial ellipsoid. The choice of variables keeps the new independent and dependent variables within reasonable ranges. The numerical solution of the problem was obtained by finite-difference McCormack scheme for various values of Fr and two initial geometries of the lock: cylindrical and elliptic. The results are shown in Fig. 1(b): at the initial phase of propagation ($T \le 1$) the shape resembles the behavior of the dam-break problem, however at $T \approx 1$ the similarity "tail-ring" form is already obtained. It is noted that the initial configuration of the lock does not change the similarity solution. The comparison between the similarity solutions obtained for the non-stratified and stratified cases has also been performed. It was found that for the non-stratified case there is a stage of propagation in which the intrusion has a similar "tail-ring" form; however this stage is only a transient to a similarity solution which has a quite different form.

References

[1] T.Benjamin 1968 *Gravity currents and related phenomena.* J.Fluid Mech. **31**, 209-248

[2] R.E.Grundy & J.W.Rottman 1985 *The approach to self-similarity of the solutions of the shallow-water equations representing gravity-current releases.* J.Fluid Mech. **156**, 39-53

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