Is there life between MAK and NAM?

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The Euler-Poisson system

$$
\begin{aligned}
\partial_\tau v + (v \cdot \nabla) v &= -\frac{3}{2\tau}(v + \nabla\phi) \\
\partial_\tau \rho + \nabla \cdot (\rho v) &= 0 \\
\nabla^2 \phi &= \frac{\rho - 1}{\tau}
\end{aligned}
$$

Notation of Brenier et al (MNRAS, 2003):

- coordinates comoving with expansion
- τ the growth factor

The variational principle

$$\frac{1}{2}\int_0^T d\tau \int d^3x\, \tau^{3/2} \left(\rho\, |v|^2 + \frac{3}{2}\, |\nabla\phi|^2 \right) \to \min$$

$$\rho(x, \tau = 0) = 1, \quad \rho(x, \tau = T) = \rho_T(x)$$

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- MAK: "$\frac{3}{2} \to 0$"

“Kicking”

$$\frac{1}{2}\int_0^{\tau_k} d\tau \int d^3x\, \tau^{3/2} \rho\, |v|^2$$

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“Kicking”

$$\frac{1}{2}\int_0^{\tau_k} d\tau \int d^3x\, \tau^{3/2} \rho\, |v|^2 + \frac{1}{2}\tau_k^{3/2} \int d^3x\, \frac{3}{2}\, |\nabla \phi|^2 + \frac{1}{2}\int_{\tau_k}^{T} d\tau \int d^3x\, \tau^{3/2} \rho\, |v|^2$$

The minimization strategy

The principal unknown is the map

$$x|_{\tau=\tau_k} \mapsto X_T(x)|_{\tau=T}$$

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ρ, ϕ at τ_k follow easily:

$$\begin{aligned} \rho(x,\tau_k) &= \det(\partial_{x_i}\partial_{x_j} X_T)\rho_T(X_T(x)) \\ \nabla^2\phi(x,\tau_k) &= \frac{\rho - 1}{\tau_k} \end{aligned}$$

The minimization strategy

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Kinetic integral between τ_k and T:

$$\min \frac{1}{2} \int_{\tau_k}^{T} d\tau \int d^3x \, \tau^{3/2} \rho \, |v|^2$$
$$= \frac{1}{2} \int d^3x \, \rho(\tau_k, x) \, |X_T(x) - x|^2$$

The minimization strategy

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Kinetic integral between 0 and τ_k vanishes after minimization!

$$\min \int_0^{\tau_k} d\tau \int d^3x \, \underline{\tau^{3/2}} \rho \, |v|^2 = 0$$