

$$E_g = \int dV = 16\pi^2 G \int_0^a \rho(R) R \left( \int_0^R \rho(u) u^2 du \right) dR$$