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DYNAMICS OF EROS

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ABSTRACT

We have investigated the dynamical evolution of asteroid (433) Eros, soon to be explored by the *Near-Earth Asteroid Rendezvous* (*NEAR*) probe, by performing 16 numerical integrations of "dynamical clones" of Eros's chaotic orbit over a timespan of 5 Myr. By analyzing the results of these integrations we have found the following: (1) In six cases a clone becomes an Earth crosser, typically because of eccentricity increases caused by the ν_3 and ν_4 secular resonances; two clones become Venus crossers, and one eventually collides with the Sun. (2) Some of the Earth-crossing clones go back to the Mars-crossing state after some time, and several have their inclination affected by the ν_{13} and ν_{14} nodal resonances. (3) Nine clones have a slow evolution dominated by Mars encounters, and one of them is temporarily trapped into the 25:24 mean motion resonance with Mars, providing effective protection from close encounters over more than 1 Myr. (4) From the number of planetary encounters recorded during our integrations, Eros's lifetime versus a collision with Earth and Mars can be estimated to be about 1.84 and 2.9 Gyr, respectively. (5) On the other hand, it is impossible to estimate even as an order of magnitude the past or future mean impact rate onto Eros's surface.

These findings have the following implications: Eros's dynamical lifetime is probably of the order of 50–100 Myr, and it has ≈5% probability of eventually hitting Earth. Its shape may have been affected by tidal forces during past Earth encounters. Its birth location in the main belt cannot be traced back with certainty, but if Eros comes from a family-forming catastrophic breakup near one of the main resonances, this must have been one of the last such events to occur in the main belt. More likely, Eros's orbit became Mars crossing by slowly diffusing from the high-eccentricity portion of the main belt; then, Mars and Earth encounters led it to its current state. Unfortunately, the forthcoming observations of Eros's cratering record by the *NEAR* probe will not be useful to constrain its age or collisional lifetime.

Key words: celestial mechanics, stellar dynamics — methods: numerical — minor planets, asteroids — solar system: general

1. INTRODUCTION

In the currently known population of ≈400 near-Earth asteroids (NEAs), (433) Eros deserves particular attention for a number of reasons. First, Eros is the largest asteroid that gets close to Earth, with a perihelion distance of $q = 1.13$ AU and a semimajor axis of $a = 1.46$ AU. Its mean diameter of 22 km is about 3 times that of the largest currently Earth-crossing object, though only about one-half that of the largest Amor asteroid, (1036) Ganymed (which, however, orbits farther away from Earth). Although Eros does not currently cross Earth's orbit, it undergoes relatively frequent close encounters with Mars and long-range perturbations by all the planets, which may cause its perihelion to reach 1 AU in the (astronomically) close future, namely, within ≈1 Myr (Milani et al. 1989, Michel et al. 1996b). It follows that in the distant future there is some chance that Eros will hit Earth—and this would be a truly catastrophic event, some 10 times more energetic than the Chicxulub K-T impact that caused the mass extinction of biota 65 Myr ago (Montanari et al. 1998).

Second, spectroscopic observations have recently revealed that Eros's surface is spectrally similar to those of the main-belt asteroids belonging to the Maria family, which is located near the outer edge of the 3/1 Kirkwood gap (Binzel et al. 1996, Zappalà et al. 1997). Such a family, generated by the breakup of a sizable main-belt asteroid, would be a plausible birthplace for a fragment as big as Eros. However, the typical lifetimes of NEAs versus falling into the Sun or ejection by Jupiter are only a few tens of Myr (Gladman et al. 1997), and it is unlikely that the formation of the Maria family (or any other prominent breakup event in the main belt) occurred in such a relatively recent past. Dynamical studies based on numerical integrations cannot reconstruct the past orbit of Eros (or any other NEA) over times much longer than the Lyapunov divergence time (≈100 yr), and therefore it is impossible to trace back with certainty where in the main belt Eros was formed. However, numerical integrations can reveal which are the main mechanisms under which Eros's orbit has evolved and allow us to estimate the timescale of such an evolution.

Third, Eros will soon (1999) be explored in situ by the *Near-Earth Asteroid Rendezvous* (*NEAR*) NASA probe, which will orbit the asteroid for at least one year, carrying out a variety of structural, morphological, and compositional studies (for more details, see Cheng et al. 1997). Therefore, soon we will know relatively well Eros's physical

TABLE 1

NUMERICAL RESULTS ON THE ORBITS OF EROS CLONES

Clone	Earth Crosser?	Resonances (Myr)	Effect
Eros v	No	ν_{13} (0−1.8)	i: 8°−22°
		ν_{14} (1−1.4)	
Eros vi	Yes	ν_{13} ($t > 4.2$)	i: 5°−14°
		ν_{14} (close)	
		ν_3 (3.6−4.4)	e: 0.2−0.38−0.1
		ν_4 (3.8−4.4)	
Eros ve	No	ν_{13} (1.8−4)	i: 8°−22°−8°−18°
		ν_{14} ($t > 2.6$)	
Eros va	Yes	ν_3 (2.9−3.6)	e: 0.2−0.6
		ν_4 (2.9−3.5)	
		5:1 J (3.4-3.5)	e: 0.4−0.6
		ν_{16} (2.7−3.2)	i: 10°−4°
		ν_3 (4.4−4.8)	e: 0.4−0.7
		ν_4 (4.4−4.8)	
Eros va2	No	ν_{13} (0.7−2.1)	i: 10°−20°
		ν_{14} (2.0−5.0)	i: 10°−25°
Eros vM	No	ν_{13} (3.1−3.6)	i: 10°−24°
		ν_{14} (3.3−5.0)	
Eros vom	Yes	ν_3 (4.3−5.0)	e: 0.25−0.43
		ν_4 (4.4−5.0)	
		ν_{16} (4.2−5.0)	i: 10°−2°
Eros vno	No	ν_{13} (0.0−1.9)	i: 9°−25°
		ν_{14} (1.1−5.0)	
Eros h	Yes	ν_{16} (1.8−3.1)	i: 5°−14°
		ν_{16} (4.6−5.0)	i: 9°−16°
Eros hi	No	...	...
Eros he	No	ν_{13} (0.8−2.2)	i: 2°−20°
		ν_{14} (1.4−2.2)	
Eros ha	No	ν_{13} (2.4−4.2)	i: 7°−21°
		ν_{14} (3.5−4.4)	
Eros hM	Yes	ν_{16} (1.0−4.0)	i: 10°−20°
		ν_3 (close)	
		ν_4 (3.2−3.4)	
Eros hom	No	ν_{13} (3.2−4.0)	i: 10°−23°
		ν_{14} (3.5−4.0)	
Eros hno	Yes	ν_{13} ($t > 4.6$)	i: 8°−16°
		ν_{14} ($t > 4.6$)	
Eros ha2	No	ν_{13} (0.6−1.2)	i: 10°−24°
		ν_{14} (1.0−1.4)	

NOTES.—The clone labels correspond to the computer used (v and h) and the modified orbital element (a, e, i, M, om, and no). The table lists the resonant mechanisms that have been found to be effective, either secular resonances or i:j mean motion resonances (labelled according to the initial of the involved planet), with the corresponding time intervals (Myr) indicated in parentheses. The resulting inclination (i) and eccentricity (e) changes are given in the sense they have been found to occur.

and surface properties, and it will be natural to wonder which of these properties are related to its origin and past evolution. For instance, cratering rates are likely to differ by a factor $\approx 10^3$ in the main belt and in near-Earth space (Eros's aphelion distance, about 1.78 AU, is located just near the typical perihelia of inner belt asteroids). Thus the cratering record, which will be observed on the surface, is itself related to Eros's dynamical history.

With these motivations, a few years ago we carried out the first dynamical study of the long-term evolution of Eros's orbit and roughly determined its lifetime with respect to a collision against Earth (Michel et al. 1996a, 1996b). However, the chaotic nature of Eros's orbit implies that such studies must have a statistical character, that is, they must be based on a fair number of independent integrations of initially equivalent "dynamical clones." As a consequence, the corresponding results can be refined by improving the statistics, namely, performing more integrations over longer timespans and analyzing in detail whether small-number statistics may have led to wrong estimates. Therefore, in this paper we have furthered our 1996 studies by using a much larger database of numerical integrations and also analyzing the possibility that Eros may in the future hit Mars instead of Earth.

The remainder of this paper is organized as follows: Section 2 briefly describes the numerical method we have used, which is basically the same as that adopted in our previous work (Michel et al. 1996a, 1996b). Section 3 focuses on the transport of Eros to Earth-crossing region. In § 4, we discuss some results suggesting that Eros's orbit might undergo a relatively slow kind of evolution. Orbital inclination changes are discussed in § 5. Updated results on the collision probabilities with Earth and Mars are presented in § 6. Section 7 is devoted to estimating the mean collision probability and corresponding impact velocities with main-belt asteroids. Implications for the analysis of the cra-

TABLE 2

DATA

Body	$a_{\min}$ (AU)	$a_{\max}$ (AU)	$e_{\min}$	$e_{\max}$	$i_{\min}$ (deg)	$i_{\max}$ (deg)	Min. Dist. (AU)	f_1 (%)	f_2 (%)
Eros v	1.404	1.459	0.131	0.295	8.363	24.669	0.349 (V) 7.821×10^{-2} (E) 1.227×10^{-3} (M)	0	100
Eros vi	1.388	1.560	0.085	0.378	4.079	14.266	0.262 (V) 7.945×10^{-4} (E) 3.499×10^{-4} (M)	4.259	82.8
Eros ve	1.415	1.473	0.114	0.290	7.945	22.477	0.357 (V) 8.355×10^{-2} (E) 1.122×10^{-4} (M)	0	65.83
Eros va	1.289	2.156	0.150	0.751	0.159	13.816	4.539×10^{-4} (V) 7.071×10^{-4} (E) 2.618×10^{-4} (M)	40.93	97.64
Eros va2	1.456	1.491	0.153	0.289	8.523	24.855	0.357 (V) 8.499×10^{-2} (E) 8.270×10^{-4} (M)	0	99.78
Eros vM	1.439	1.476	0.135	0.291	6.821	24.234	0.354 (V) 8.267×10^{-2} (E) 1.235×10^{-3} (M)	0	89.10
Eros vom	1.422	1.712	0.167	0.431	2.514	16.702	0.254 (V) 1.158×10^{-3} (E) 1.189×10^{-3} (M)	4.92	93.5
Eros vno	1.434	1.475	0.142	0.287	8.399	25.531	0.358 (V) 8.153×10^{-2} (E) 4.332×10^{-4} (M)	0	95.76
Eros h	0.746	1.493	0.085	0.366	3.121	16.39	4.245×10^{-4} (V) 5.553×10^{-4} (E) 9.642×10^{-4} (M)	50.90	47.40
Eros hi	1.454	1.493	0.159	0.288	8.641	17.492	0.365 (V) 7.794×10^{-2} (E) 4.841×10^{-4} (M)	0	100
Eros he	1.385	1.471	0.089	0.282	1.946	21.643	0.321 (V) 4.135×10^{-2} (E) 4.433×10^{-4} (M)	0	11.55
Eros ha	1.431	1.492	0.098	0.296	6.124	21.400	0.351 (V) 8.095×10^{-2} (E) 1.042×10^{-3} (M)	0	68.31
Eros hM	1.415	1.650	0.204	0.414	5.744	20.117	0.260 (V) 1.694×10^{-3} (E) 1.751×10^{-3} (M)	20	100
Eros hom	1.442	1.491	0.165	0.288	8.534	23.472	0.360 (V) 9.156×10^{-2} (E) 5.855×10^{-4} (M)	0	99.52
Eros hno	1.410	1.459	0.192	0.301	8.267	18.693	0.319 (V) 3.666×10^{-2} (E) 1.178×10^{-3} (M)	2	94.34
Eros ha2	1.447	1.464	0.103	0.282	8.192	24.301	0.355 (V) 8.684×10^{-2} (E) 3.526×10^{-4} (M)	0	54.09

NOTES.—The columns list minimum and maximum values of the orbital elements (a, e, i); minimum distances to the planets Venus (V), Earth (E), and Mars (M); fractions of time spent in the Earth-crossing region, with perihelion distance less than 1.017 AU (f_1) and with aphelion located in the main belt, beyond 1.7 AU (f_2).

tering records soon to be observed by the *NEAR* probe are also discussed. Finally, in § 8 we summarize the main results and discuss some implications of our work.

2. NUMERICAL INTEGRATION METHOD AND GENERAL RESULTS

We refer to Michel et al. (1996b) for a detailed description of the integration method and the choice of initial conditions. Briefly, we used a Bulirsch-Stoer variable step size integrator, optimized for dealing accurately with close approaches to planets. Because of the chaotic nature of planet-crossing orbits, only a statistical study of their long-term evolution can be done. Therefore, our initial conditions were selected as follows: Besides the nominal orbital elements of Eros at a given starting epoch (see Michel et al. 1996b for the corresponding data), we defined a set of *clone* orbits obtained by slightly changing the initial orbital parameters one at a time. To improve the statistics with respect to our previous study, we performed the integration on a longer timespan (5 Myr instead of 2 Myr) and varied not only the initial inclination, eccentricity, and semimajor axis (for which we actually selected two close values), but also the perihelion argument, nodal longitude, and mean anomaly, leading to a total of eight clones. The relative changes in the orbital elements were of about 10^{-3}, small enough to avoid the fact that the clones may start in a qualitatively different region of the phase space with respect to the nominal orbit. Finally, we performed all integrations on two different computers having different round-off errors in order to finally get a total of 16 possible evolutions (for a total of 80 Myr).

Tables 1 and 2 summarize the results for the 16 clones. These results are discussed separately in the following sections. Here we just give a brief summary. In our 16 integrations over 5 Myr we have found that

> Six clones become Earth crossing (that is, approach the Sun at distances smaller than Earth's aphelion distance) within the 5 Myr timespan;
>
> Eleven clones are affected by the ν_{13} and ν_{14} secular resonances, with inclinations ranging between 2° and 25°;
>
> Four clones are affected by the ν_3 and ν_4 resonances, with eccentricities in the range $0.2 < e < 0.7$;
>
> Four clones are affected by the ν_{16} resonances ($2° < i < 16°$);
>
> One clone is affected by the 5:1 mean motion resonance with Jupiter.

Most of these results are statistically similar to those obtained in our previous study (Michel et al. 1996b) with a smaller number of integrations. However, as we shall see, the enlarged data set has also resulted in new findings.

3. TRANSPORT TO THE EARTH-CROSSING ZONE

Over 16 clones, six reach the Earth-crossing region within the 5 Myr integration timespan. Five of them are transported by the same mechanism, namely, an increase of the eccentricity due to crossing of the ν_3 and ν_4 secular resonances. The remaining clone becomes an Earth crosser at the end of the integration time, while its eccentricity is at its maximum on its short-period oscillation cycle, and only as a result of close approaches with Mars, which decrease slightly its semimajor axis and consequently its perihelion distance.

Two clones (Eros va and Eros h) are even transported into the Venus-crossing zone, and one of them "dies" by a collision with the Sun. For this object, the transport mechanism to the Venus region is as follows: As a consequence of close approaches with Earth, its semimajor axis is increased up to 1.8 AU where the 5:1 mean motion resonance with Jupiter is present; this resonance increases the eccentricity so that the perihelion distance becomes lower than the orbital distance of Venus and the orbit is then strongly affected by close approaches to both Earth and Venus until the end of the 5 Myr. Figure 1 shows this evolution in a semimajor axis a versus eccentricity e plot. At $t = 4.4$ Myr, a deep close approach to Venus transports the semimajor axis to values corresponding to the main belt. The object even crosses the powerful 4:1 mean motion resonance with Jupiter (at $a \approx 2.06$ AU) without being captured.

This is probably due to the fact that the inclination is very small (see Fig. 2) so that frequent and deep close encounters with Earth and Venus occur. Consequently the semimajor axis suffers big jumps across the resonant value. Then, continuing the integration on a longer timespan, we have found that a close approach to Venus injects the orbit into the 7:2 mean motion resonance with Jupiter (at $a \approx 2.27$ AU). As an effect of this resonance, the orbital eccentricity is raised up to almost 1, so that the asteroid finally collides with the Sun, a typical end state of NEAs according to Farinella et al. (1994) and Gladman et al. (1997). Actually, looking at this evolution in the reverse way, i.e., going from $t = 5$ Myr to $t = 0$, one can see a possible (but not necessarily likely) mechanism that may have lead Eros in its current state in which it could then have remained for a long time, according to the quasi stable evolutions observed for some of the other clones (see § 4).

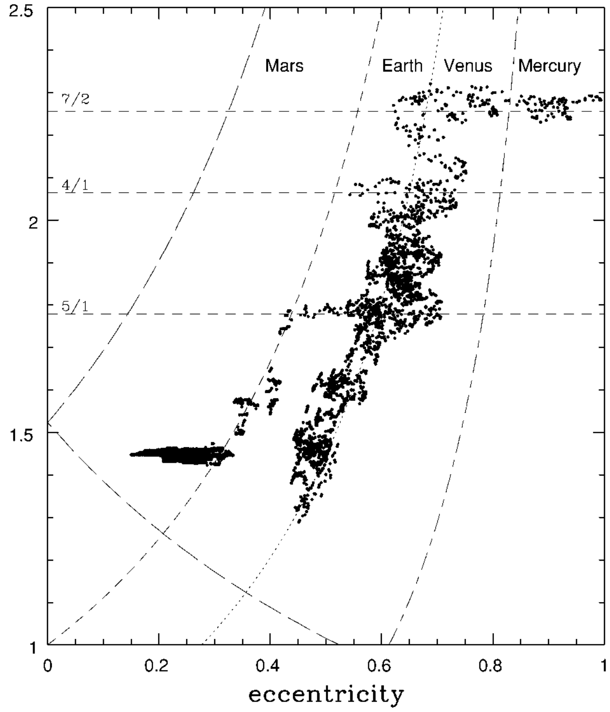

FIG. 1.—The evolution of clone Eros va in the semimajor axis (a, in AU) vs. eccentricity (e) plane. The horizontal lines correspond to mean motion resonances with Jupiter. The planet-crossing lines (perihelion/aphelion distance equal to planetary semimajor axes) are also shown in the diagram with the names of the corresponding planets.

Four of the clones (Eros va, Eros vom, Eros h, and Eros hM) also undergo inclination changes related to the occurrence of the ν_{16} secular resonance. For two of them (Eros va and Eros vom), a secular inclination decrease is observed. As it has been previously noted (Michel et al. 1996b), this explains the fairly high frequency of the close encounters with Earth and their efficiency in causing strong variations of semimajor axis.

It has to be stressed that two of the Earth crossers remain only temporarily in the Earth-crossing zone and are ejected again from it by the ν_3 and ν_4 resonances, which in these cases decrease the eccentricities. This shows that Eros might already have evolved into the Earth-crossing region and that its current orbit might be a result of the ejection from this region due either to close approaches to Earth or to secular resonances. Some interesting implications can be derived from this scenario. First, it would be consistent with the recent results by Gladman et al. (1997): their numerical integrations of ≈ 1500 test particles injected into the main-belt resonances have shown that, due to its small mass, Mars is not effective in extracting bodies from resonant "fast tracks." Therefore, nearly all the "evolved" orbits (with semimajor axes $a < 1.7$ AU), including those that are not currently Earth crossing, such as Eros, must have been extracted by past encounters with Earth (or Venus). This has been confirmed by recent work of Migliorini et al. (1998) on the orbital evolution of Mars-crossing asteroids. This complex orbital history, involving both Earth encounters and secular resonances, probably requires a significantly longer duration of the transport phase from the main

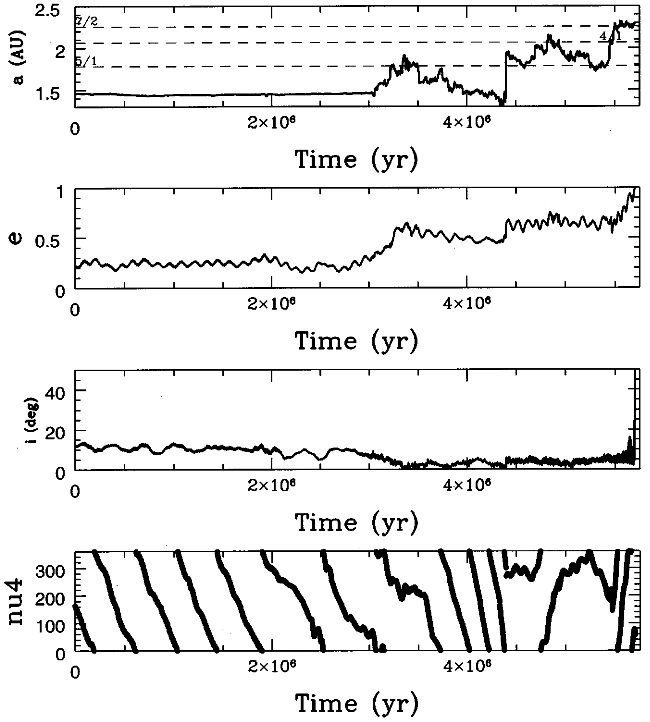

FIG. 2.—Evolution of the orbital elements of clone Eros va. Besides semimajor axis, eccentricity, and inclination, the figure also shows the critical argument ($\varpi - g_4 t - \alpha_4$) of the ν_4 secular resonance (ϖ is the perihelion longitude of the clone, $g_4 = 17''.85\ \mathrm{yr}^{-1}$ is the fundamental frequency corresponding to the resonance, and α_4 is the phase at time $t = 0$).

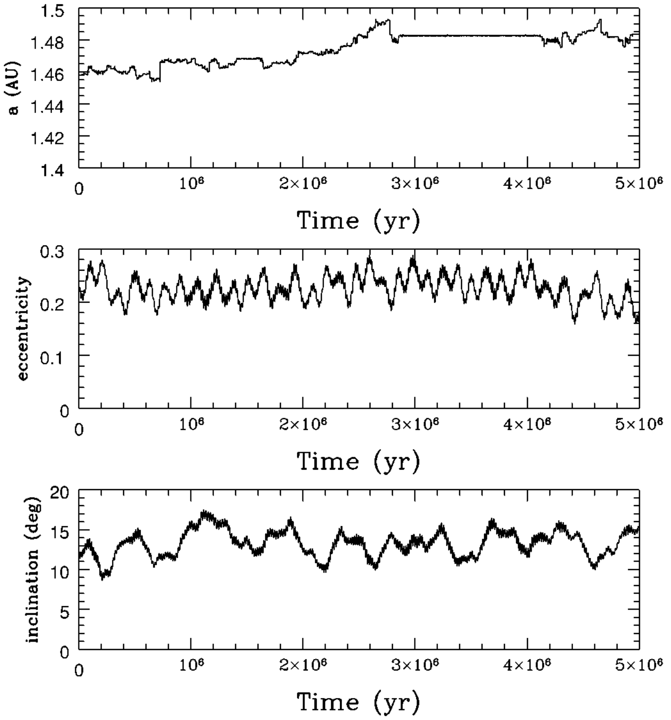

FIG. 3.—Evolution of the semimajor axis, eccentricity, and inclination of clone Eros hi.

belt to the current orbit than it would be expected otherwise. Second, Eros might have been reshaped by tidal effects during previous close approaches to Earth (Bottke et al. 1998). We will further discuss this possibility in § 5.

4. SLOWLY EVOLVING CLONES

Nine of our 16 Eros clones showed, over the full 5 Myr of the integration timespan, a "slow" kind of orbital evolution dominated by Mars encounters. In these cases the semimajor axis evolves in a random-walk mode over a range not exceeding a few hundredths of AU, with the eccentricities and the inclinations undergoing quasi regular, moderate-amplitude oscillations (see Fig. 3).

Basically, this behavior is typical of the "Eros class" asteroids as defined by Milani et al. (1989). However, there is an interesting complication: for part of the time, these bodies get trapped into high-order mean motion resonances with Mars, the equivalent of the "Toro class" of Milani et al., which were affected by Earth resonances (note that Milani et al. also found this kind of behavior for asteroid 1951 SX, which was locked inside the 15:11 resonance with Mars over the entire 0.2 Myr timespan of their integration). For instance, between about 3 and 4 Myr since the beginning of the integration, clone Eros hi (Fig. 3) is trapped inside the 25:24 Mars resonance, and therefore in this interval Mars encounters are avoided since the body is phase protected. Consequently, its semimajor axis remains almost constant. We have verified that the dynamical mechanism at work in this case is exactly the same that has been discussed recently for the Toro asteroids by Milani & Baccili (1998): as shown in Figure 4, the critical argument of the resonance alternates between libration and circulation, depending on whether the asteroid's orbit is close or not to a node crossing. Note that when a Mars resonance is involved, as in the current case, its stabilizing effect can be effective for times much longer than with Earth, thus increasing the possibility of long lifetimes in the Mars-crossing state.

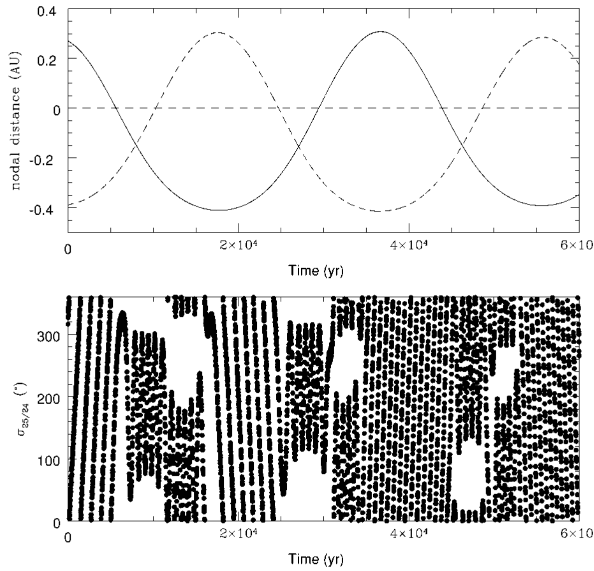

FIG. 4.—The top plot shows the evolution of the nodal distances to Mars for the ascending and descending nodes of clone Eros hi, while the bottom plot shows the evolution of the critical argument $\sigma_{25/24} = 25\lambda - 24\lambda_M$ (here λ and λ_M are the mean longitudes of the asteroid and Mars, respectively) of the 25:24 mean motion resonance with Mars. The 6×10^4 yr timespan has been chosen within the time interval from about 3 to 4 Myr from the beginning of the integration, when the semimajor axis stays almost constant (see Fig. 3). These plots show that when a node crossing occurs, $\sigma_{25:24}$ librates, thus providing an effective protection mechanism against close encounters with Mars, whereas when the asteroid is far from the node-crossing points, it cannot undergo close approaches although $\sigma_{25/24}$ is circulating.

How long can a slowly evolving Eros clone remain in a slowly evolving state? Although much longer integrations would be needed to answer this question in a quantitative way, our results, as well as those of Gladman et al. (1997) and Migliorini et al. (1998), suggest that typically the duration of these quasi stable evolutions does not exceed several tens of Myr. The reasons for this (tentative) conclusion are the following: (1) for some of our clones, we indeed observe a transition between a slowly and a fast-evolving orbit within 5 Myr; (2) as shown in Figure 5, even in the slowly evolving cases the random walk "diffusion" over 5 Myr spans a region in *a*-*e* space of size comparable to the remaining "distance" from the Earth-crossing line; (3) at the semimajor axis of Eros but at slightly lower inclinations (6°–8°, see Fig. 2 in Michel & Froeschlé 1997,), the ν_3 and ν_4 secular resonances are present, which, as we have discussed in § 3, can easily pump the eccentricity up to Earth-crossing values (Michel 1997); (4) the median dynamical lifetimes of the observed NEA orbits, according to the extensive numerical integrations of Gladman et al. (1997, Fig. 2) is about 10 Myr, and very few such orbits survive for ≈ 100 Myr.

If this conclusion is correct, it raises a problem concerning the origin of "giant" NEAs such as Eros and (1036) Ganymed, as discussed already by Morbidelli et al. (1995) and more recently by Zappalà et al. (1997). Main-belt collisions disrupting targets large enough and close enough to a resonance to produce and inject into a resonant channel fragments tens of kilometers across are probably very rare (no more than one per Gyr, according to Farinella & Davis 1992 and Farinella et al. 1996) and should generate detectable asteroid families. According to Zappalà et al., the Maria family at the outer edge of the 3/1 Kirkwood gap is the most plausible candidate, as it may have injected into the gap ≈ 10 Eros-sized objects. However, this may not be enough to yield one long-lasting Eros-like asteroid (or two of them, if one includes Ganymed, which is about twice as large as Eros), since according to Gladman et al. (1997) only 4.5% of the Maria fragments are extracted from the 3:1 resonance by a Mars encounter before ending up into the Sun or close to Jupiter, and anyway 90% of them "die" within 11 Myr. Thus, the results of long-term dynamical studies would suggest that the age of the Maria family (or anyway of the breakup event forming Eros and possibly Ganymed) is less than 100 Myr, and this is consistent with current collision rates in the belt only if this event was the last or one of the few last ones involving targets about 100 km in diameter (Farinella et al. 1996). This would be a somewhat surprising conclusion, given the peculiar location in orbital element space required to get a high-injection efficiency into a strong resonance (like in the case of the Maria family). In other words, this scenario would mean that we are living near the end of a "peculiar" period, lasting several tens of Myr and characterized by an intense shower of NEAs, up to several tens of kilometers in size, coming from a catastrophic breakup near a resonance in the main belt (Zappalà et al. 1998).

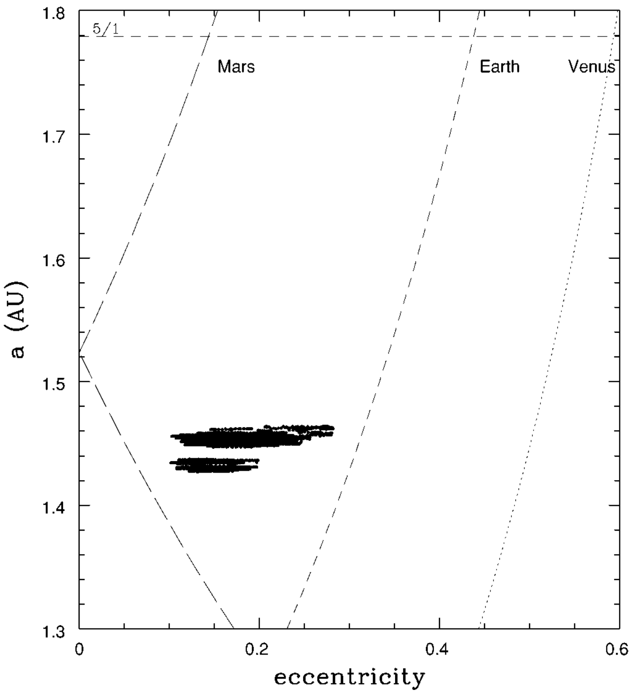

FIG. 5.—Same as in Fig. 1, but for the clone Eros ha2

An alternative, more likely possibility is that Eros was formed in the main belt long before its eventual insertion into the current Mars-crossing orbit. This is consistent with the existence in the main belt of "sticky" regions of the orbital element space where very slow diffusion of the eccentricity takes place, eventually leading into a strong resonance or to Mars crossing. Evidence for such behavior has been found near the mean motion resonances (Milani & Farinella 1995; Milani et al. 1997; Gladman et al. 1997; Morbidelli & Gladman 1998) and, more recently, in the high-eccentricity region of the main belt (Migliorini et al. 1998; Nesvorný & Morbidelli 1998). In particular, Migliorini et al. 1998 found that starting from the current population of sizable Mars crossers it is possible to maintain about five objects larger than 5 km in diameter in Eros-like orbits over several tens of Myr; assuming that the population of Mars crossers is itself in steady state thanks to the slow diffusion of asteroids from the main belt, this would provide a plausible scenario for the origin of Eros and its siblings.

5. ORBITAL INCLINATION CHANGES

Eros currently evolves on an orbit whose parameters are very close to the values at which the secular resonances ν_{13} and ν_{14} occur (Michel & Froeschlé 1997). This is the reason why 11 out of 16 clones pass through these resonances, suggesting that it is highly probable that the real object will cross or has already crossed them. These resonances can produce variations in the orbital inclination of about 15° (Michel 1997); since the current inclination of Eros is close to the lowest value along the resonant cycle of the secular angle, the effect would be an increase to $i \approx 24°$. If the complete cycle were to be completed, i.e., if close approaches to Mars did not eject the body from the region where these resonances work before the end of their cycle, the inclination would decrease again to the minimum value and its evolution could be described as a large-amplitude oscillation ($8° < i < 24°$). This is the case for five of our clones: Eros ve, Eros va2, Eros vm, Eros vno, and Eros ha (see Fig. 6). For the other three clones, only half a cycle is completed and the resonances just push the inclination to a higher value ($i \approx 14°$, $i \approx 22°$, and $i \approx 20°$, respectively, for Eros vi, Eros v, and Eros he).

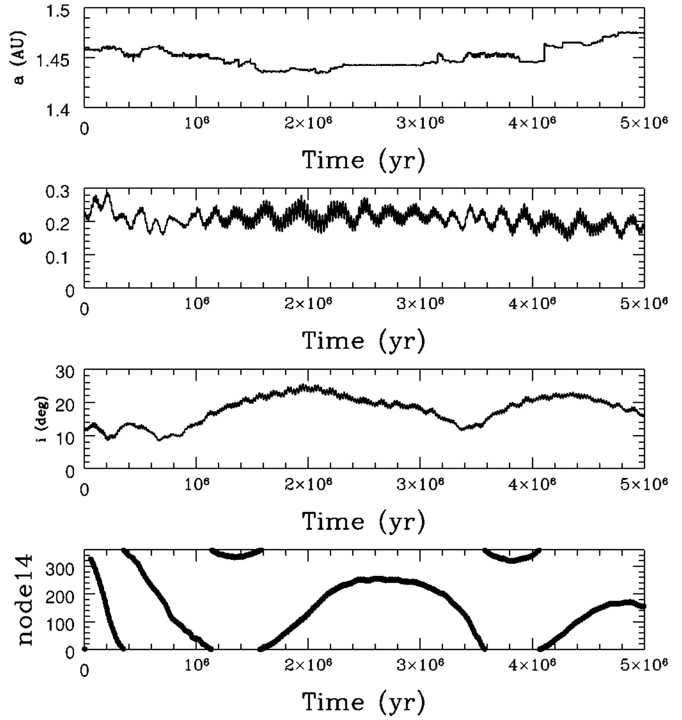

FIG. 6.—Evolution of the orbital elements of clone Eros vno. Besides semimajor axis, eccentricity, and inclination, the figure also shows the critical argument ($\Omega - s_4 t - \beta_4$) of the ν_{14} secular resonance (Ω is the nodal longitude of the clone, $s_4 = -17''.8\ \mathrm{yr}^{-1}$ is the fundamental frequency corresponding to the resonance, and β_4 is the phase at $t = 0$).

The consequences of these inclination changes can be important since they modify the geometry of close encounters with the planets. Mars being fairly small, the efficiency of close approaches in changing the orbital elements is strongly dependent on their geometry and is much higher at small inclinations. Therefore, were Eros led to higher inclinations by secular resonances, it could remain in the Mars-crossing region for a longer time. However, if its inclination were decreased again as a result of the completion of the resonant cycle, close approaches would become more effective again and would result in more frequent kicks in semimajor axis. Then, depending on the region where Eros would be transported, other dynamical mechanisms could play a dominant role.

Another consequence of the inclination changes is related to the shape of Eros. As shown in a recent paper (Bottke et al. 1998), planetary tidal forces can change the shape of an asteroid during a close approach. But to be effective, these forces require that the close approach occurs at a low relative velocity (Bottke & Melosh 1996). Again, this depends on the value of the inclination of the asteroid, which must be small. It is well known that Eros has already a highly elongated, bean-like shape (e.g., Mitchell et al. 1998) and this could be the result of a past evolution under the effect of tidal forces during a sequence of close encounters with a planet. Moreover, if tidal forces will act again in the future, Eros could eventually break into two or more pieces and/or become a binary asteroid (Chauvineau et al. 1995). In our integrations, the most frequent effect of the ν_{13} and ν_{14} resonances is to increase the inclination. If this will occur in reality, Eros will be less likely to undergo deformations by tidal forces in the close future.

Note finally that an increase of inclination would lead this orbital parameter closer to the values where the ν_2 and ν_5 resonances are effective (Michel & Froeschlé 1997). For some of our clones, the critical angles of these resonances circulate slowly and related oscillations of the eccentricity can be detected. However, no clone has its inclination reaching the value where the resonances actually occur and no substantial consequence is observed. Nevertheless, the possibility that Eros will encounter these resonances cannot be ruled out.

6. PROBABILITY OF COLLISION WITH EARTH AND MARS

Following the procedure described in Michel et al. (1996a, 1996b), we have estimated the encounter rate of the Eros clones with Earth and Mars. During the integrations we recorded all the encounters corresponding to impact parameters $d \leq 0.01$ AU and then grouped all the 16 clones together, corresponding to a total timespan of 80 Myr (in other words, we have assumed that the 16 evolutions are independent, which is correct on a timescale much longer than the Lyapunov time). Up to $d = 3 \times 10^{-3}$ AU the relationship between the cumulative number of encounters N and d is well fitted by a quadratic relationship, as would be expected for an isotropic distribution of relative velocity vectors. Since we found $N = 83$ for $d = 3 \times 10^{-3}$ AU and since for Earth a collision would occur whenever $d < 10,300$ km (corresponding to a gravitational focusing factor $1 + (V_{\mathrm{esc}}/V_{\infty})^2 = 2.61$, derived from the encounter data themselves), it is easy to estimate that the averaged collision rate is $5.435 \times 10^{-4}\ \mathrm{Myr}^{-1}$, that is, the collisional lifetime of Eros with Earth is 1.84 Gyr. Of course this is just a statistical estimate based on a fairly limited data set. For instance, if we had considered only the six Earth clones, the number of encounters would have remained the same, but the total timespan would be only 30 Myr, corresponding to a collision rate of $1.449 \times 10^{-3}\ \mathrm{Myr}^{-1}$ and a lifetime of 690 Myr. We recall that in one of the 2 Myr integrations reported in Michel et al. (1996b) we detected an actual Earth impact, although it is clear that it was just a fluke, fairly unlikely a priori.

We used the same methodology to estimate the collision probability with Mars. In this case the N versus d relationship is well fitted by a parabola up to $d = 2 \times 10^{-3}$ AU and the maximum value of d corresponding to an impact is 4140 km (using this time an estimated gravitational focusing factor of 1.57). Extrapolating the observed encounter rate yields a collision rate of $3.452 \times 10^{-4}\ \mathrm{Myr}^{-1}$, that is, a collisional lifetime of about 2.90 Gyr.

This shows that although the current Eros is a Mars crosser but not an Earth crosser it is about 2 times more likely to impact Earth than Mars. Of course, both events are probably quite unlikely compared with the most frequent fates for NEAs, which are a collision with the Sun or an ejection onto hyperbolic orbit following a Jovian encounter (Farinella et al. 1994; Gladman et al. 1997). We cannot directly estimate the timescales for these end states in the specific case of Eros from our integrations, which are too short for them to show up frequently enough to draw statistical inferences. However, according to Gladman et al., for the observed NEA population the median lifetime is about 10 Myr, and taking into account that Eros is probably a member of the relatively long-lived tail of the population, we can guess that its lifetime is probably of the order of 50–100 Myr (this is consistent with our finding that one out of 16 clones hits the Sun, and also with recent results by Migliorini et al. 1998). Comparing this estimate with the previous results for a collision with Earth or Mars, we conclude that there is a chance of ≈5% and a few percent that Eros will end its life with a collision against Earth and Mars, respectively. On the other hand, it is worth noting that the existence of Eros and Ganymed in orbits which in the future may well become Earth crossing, shows that unless we are living in a peculiar transient epoch following a big main–belt breakup (as discussed in § 4), a catastrophic collision with a NEA a few tens of km in size should be expected to occur on Earth about once per gigayear.

7. COLLISIONAL PROBABILITY WITH MAIN–BELT ASTEROIDS

Can our results on the dynamical evolution of the Eros clones be used to find out about the past cratering rate on its surface? As it has already been done in the cases of (951) Gaspra (Belton et al. 1992; Greenberg et al. 1994), Ida (Greenberg et al. 1996) and Mathilde (Davis 1998), the abundance and size distribution of craters on an asteroid surface can be used to infer constraints on the age of the surface itself, provided one can estimate the flux of impactors as a function of size. In the case of Eros (and other NEAs as well), a complicating factor is the fraction of its lifetime that the asteroid has spent in crossing the main asteroid belt (say, beyond 1.7 AU) instead of the near-Earth region (inside 1.7 AU), since there is about a factor of 10^3 between the fluxes typical of these two regions (e.g., a few thousands of NEAs vs. a few millions of main-belt objects larger than 1 km; see Rabinowitz et al. 1994; Farinella & Davis 1994).

Unfortunately, our integrations do not solve this problem because (as we already noted in Michel et al. 1996a) the different clones behave in very different ways from this point of view. As shown by Table 2, the fraction of time spent with the aphelion distance exceeding 1.7 AU ranges from 11.5% to 100%. To obtain a more quantitative estimate, we have proceeded as follows: At time intervals of 10^3 yr, we used the orbital parameters of the Eros clones as an input for the code of Farinella & Davis (1992), which calculates the average intrinsic collision probability $\langle P_i \rangle$ (as defined by Wetherill 1967) and impact velocity $\langle V_i \rangle$ of a given body with all the 682 main-belt asteroids of diameter greater than 50 km. Assuming that small main-belt asteroids have an orbital distribution that resembles that of the (nearly complete) sample of asteroids larger than 50 km, the results should provide reliable values for the collision rates and velocities prevailing when an Eros clone enters into the main belt. Of course it is possible that the orbital distribution in the belt of small (subkilometer) impactors does not mimic that of larger asteroids, for a variety of reasons including the relevance of nongravitational removal mechanisms (Farinella et al. 1998), but this assumption is just intended to provide a reasonable first guess.

With this caveat, the results shown in Table 3 indicate that the average impact velocity of the Eros clones with main-belt bodies ranges from about 10 to 16 km s^{-1}, namely, 2–3 times the average impact speeds for main-belt asteroids with themselves. Of course, this is due to the average higher eccentricity of the Eros clones and to the fact that they cross the main-belt near aphelion. As for the intrinsic collision probabilities P_i, they range from about 5×10^{-21} km^{-2} yr^{-1} to 4×10^{-19} km^{-2} yr^{-1}, compared with an average value of 2.85×10^{-18} km^{-2} yr^{-1} for main-belt asteroids. Note that taking into account the higher impact speed, the maximum value of P_i corresponds to a flux of impact energy quite comparable to that hitting a typical main-belt asteroid, whereas for other clones the collision rate is at least a factor 100 lower than for main-belt asteroids. This means that unfortunately it will not be possible to estimate Eros's age from the density of craters observed on its surface. Nor will it be possible to estimate its future collisional lifetime, for the same reason. In either case, the chaotic nature of the orbit prevents us from knowing the collisional environment where the real Eros has spent and will spend most of its lifetime.

TABLE 3

MEAN INTRINSIC COLLISION PROBABILITY ($\langle P_i \rangle$) AND IMPACT VELOCITY ($\langle V_i \rangle$) OF THE EROS CLONES

Body	$\langle P_i \rangle$ (yr^{-1} km^{-2})	$\langle V_i \rangle$ (km s^{-1})
Eros v	6.648×10^{-21}	16.339
Eros vi.........	2.196×10^{-20}	13.782
Eros ve	7.609×10^{-21}	15.369
Eros va	2.172×10^{-18}	12.253
Eros va2.......	1.018×10^{-20}	15.023
Eros vM	6.811×10^{-21}	15.514
Eros vom......	1.418×10^{-19}	13.624
Eros vno	6.844×10^{-21}	15.909
Eros h	2.407×10^{-20}	12.696
Eros hi.........	1.782×10^{-20}	13.514
Eros he	5.009×10^{-21}	15.227
Eros ha	7.168×10^{-21}	15.291
Eros ha2.......	5.807×10^{-21}	16.446
Eros hM.......	4.361×10^{-19}	9.767
Eros hom......	1.155×10^{-20}	14.719
Eros hno	8.331×10^{-21}	15.133

NOTES.—Probabilty and velocity with the main-belt asteroids of diameter greater than 50 km over the integration timespan of 5 Myr.

8. CONCLUSIONS

The main conclusions of this paper can be summarized as follows:

From our numerical integrations of dynamical clones of (433) Eros, we estimate that there is a significant probability (6/16) that this sizable NEA will become an Earth crosser within the next 5 Myr. The mechanism leading to this is in most cases related to the ν_3 and ν_4 secular resonances, which are effective enough in pumping up the orbital eccentricity. Two clones are found to become Venus crossing, and one hits the Sun.

Two of the Earth-crossing clones go back to the Mars-crossing state after some time, and several have their inclination affected by the ν_{13} and ν_{14} nodal resonances. A small inclination while in the Earth-crossing state leads to very frequent close approaches to Earth. If this has been the case for the real Eros, Earth tidal forces may have affected its elongated shape.

Nine clones have a slow, random-walking evolution dominated by Mars encounters. One of them is temporarily trapped into a high-order mean motion resonance with Mars, providing a protection mechanism from close encounters over more than 1 Myr. We estimate that these slow evolutionary tracks may allow Eros to survive for 50–100 Myr, that is, longer than the median dynamical lifetime of NEAs but probably not enough to trace back Eros's origin to the formation of one of the (presumably older) prominent dynamical families in the main belt, such as Maria. It seems more likely that Eros comes from the main belt through slow diffusion to the Mars-crossing region as recently suggested by Migliorini et al. 1998.

From the record of planetary encounters occurred during our integrations, we estimate that Eros's lifetime with respect to a collision with Earth and Mars is about 1.4 and 2.9 Gyr, respectively. Since the dynamical lifetime of Eros versus a solar collision or ejection by Jupiter is only 50–100 Myr, there is a chance of about 5% that the asteroid will hit Earth some time in the future. Such catastrophic impacts may occur onto our planet about once per gigayear.

The chaotic nature of Eros's orbit and the very different collisional fluxes in and out of the main asteroid belt make it impossible to estimate even as on order of magnitude the past or future mean impact rate onto its surface. Therefore the forthcoming observations of Eros's cratering record by the *NEAR* probe will not constrain the age or collisional lifetime of the asteroid.

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