Detection Bounds for Automatic Target Recognition in Compressed Domain[1]

Pierre Moulin
Univ. of Illinois, Beckman Institute and ECE Dept
405 N. Mathews Ave., Urbana, IL 61801

Michael I. Miller
Washington Univ., Dept of Electrical Engineering
St Louis, MO 63130

Abstract — **We study a problem of target detection given transform–coded image data. We formulate optimal likelihood ratio tests and derive detection bounds by application of Stein's lemma. This study provides a framework in which to study degradation in detection performance due to lossy compression.**

I. Introduction

A variety of civilian and military applications require detection of *targets* using remote sensors such as cameras and radars. Assuming suitable statistical models for both target and sensors, the detection problem may be treated in a Bayesian framework, and bounds on detection performance may be computed. The analysis of such problems is complicated due to the presence of unknown parameters such as target orientation, motion, and reflectance properties [1]. In some applications, the detection problem is further complicated due to the fact that the data available to the detection algorithm are not the original sensor data, but a *compressed version* thereof. In this paper, we seek optimal detection algorithms that operate on compressed data, and compute bounds on detection performance.

We introduce structural constraints on the coder. Without such constraints, the ideal coder would "simply" solve the detection problem and transmit the one bit necessary to describe the target type! We use orthonormal transform-based coders, due to their inherent simplicity, flexibility, and high coding performance over a broad class of images [2]. Coefficients with magnitude lower than some specified threshold λ are set to zero. The threshold provides a tradeoff between rate and detection performance. While we are not concerned with quantization of significant transform coefficients in this paper, our analysis can be easily extended to include quantization effects. See [3] for an example of such analysis.

We consider a known target I with orientation $s \in \mathcal{S}$, seen through a projection map T that produces N sensor data (N–pixel images). Here $\mathcal{S}$ is a K–dimensional Lie group such as $SO(3)$, the group of proper rotations in $\mathcal{R}^3$. We assume an additive white Gaussian noise model for sensor data $I^D \in \mathcal{R}^N$. The detection problems considered here are formulated as binary hypothesis tests H_1 {target present}: $I^D = TsI + n$ *vs* H_0 {target absent} : $I^D = n$, where n is iid $N(0, \sigma^2)$.

II. Known Target with Known Orientation

Likelihood Ratio Test. We set up the detection problem in the transform domain as $H_1 : \tilde{c}^D = Q(c+n)$ *vs* $H_0 : \tilde{c}^D = Q(n)$, where Q denotes componentwise scalar quantization, L is the orthonormal transform, $\tilde{c}^D = Q(LI^D)$ is the compressed data, $c = LTsI$ is the image of the known target, and n is iid $N(0, \sigma^2)$. The log likelihood ratio is additive over n.

Performance of Optimal Detector. Stein's lemma relates the asymptotic probability of a miss P_{miss} to the Kullback distance $D(\tilde{p}_0||\tilde{p}_1)$ (expressed here in nats) between probability distributions $\tilde{p}_1(\tilde{c}^D)$ and $\tilde{p}_0(\tilde{c}^D)$ with and without the target, for a fixed probability of false alarm. We obtain closed–form expressions for $D(\tilde{p}_0||\tilde{p}_1)$ as a function of λ/σ and $\{c(i)/\sigma\}$. $D(\tilde{p}_0||\tilde{p}_1)$ decreases monotonically with λ (grouping data can only reduce information distances [4]) and tends to zero as $\lambda \to \infty$. Also, the rate R of the coder is a decreasing function of λ. Hence $D(\tilde{p}_0||\tilde{p}_1)$ can be used to evaluate the compression efficiency of a particular transform in a detection context. More specifically, P_{miss} viewed as a function of R gives the *operational rate–performance curve* of the detector.

III. Known Target with Unknown Orientation

Consider now the case of a random orientation parameter s, distributed as $\pi(s)$. Under H_1, the distribution of $\tilde{c}^D$ is the mixture $p(\tilde{c}^D|H_1) = \int p(\tilde{c}^D|s)\pi(s)\,ds$, where $p(\tilde{c}^D|s)$ is the same as in Sec. II. We assume that the maximum-likelihood estimator $\hat{s} = argmax_{s\in\mathcal{S}}\, p(\tilde{c}^D|s, H_1)$ is unique.

Likelihood Ratio Test. Here, evaluation of the LRT is computationally difficult due to the nonseparability of the mixture distribution $p(\tilde{c}^D|H_1)$. However, in the asymptotic case $\sigma \to 0$, the unknown orientation parameter s may be estimated with high accuracy [1], and $p(\tilde{c}^D|H_1)$ admits a simple expression in terms of $\hat{s}$, as given by Lemma 1 below.

Lemma 1. Let $h(s) \stackrel{\triangle}{=} -\ln p(\tilde{c}^D|s, H_1)$. Then
$$p(\tilde{c}^D|H_1) \sim p(\tilde{c}^D|\hat{s}, H_1)\,\pi(\hat{s})\,(2\pi)^{K/2}[\det \ddot{h}(\hat{s})]^{-1/2} \quad \text{as } \sigma \to 0.$$

Performance of Optimal Detector. While Lemma 1 makes evaluation of the LRT tractable, evaluation of $D(\tilde{p}_0||\tilde{p}_1)$ is still very difficult due to the dependency of $\hat{s}$ on $\tilde{c}^D$, and the need to integrate the log–likelihood ratio over all $\tilde{c}^D$. So we seek lower and upper bounds on $D(\tilde{p}_0||\tilde{p}_1)$. We define $\tilde{p}_{1|s}(\tilde{c}^D) = p(\tilde{c}^D|s, H_1)$ and obtain $\min_{s\in\mathcal{S}} D(\tilde{p}_0||\tilde{p}_{1|s}) \le D(\tilde{p}_0||\tilde{p}_1) \le \int_{\mathcal{S}} D(\tilde{p}_0||\tilde{p}_{1|s})\,\pi(s)\,ds$, where the lower bound holds in an asymptotic sense, provided that the distributions under H_0 and H_1 become increasingly separated as $\sigma \to 0$. The upper bound follows from the concavity of the log function and holds for all values of σ.

References

[1] U. Grenander, A. Srivastava and M. I. Miller, "Performance Analysis of Bayesian Object Recognition in Computer Vision," *preprint*, 1997.

[2] M. Vetterli and J. Kovačević, *Wavelets and Subband Coding*, Prentice-Hall, Englewood Cliffs, NJ, 1995.

[3] H. V. Poor and J. B. Thomas, "Applications of Ali–Silvey Distance Measures in the Design of Generalized Quantizers for Binary Decision Systems," *IEEE Trans. Comm.*, Vol. 25, No. 9, pp. 893—900, 1977.

[4] S. M. Ali and S. D. Silvey, "A General Class of Coefficients of Divergence of One Distribution from Another," *J. Roy. Stat. Soc. B*, Vol. 28, pp. 131—143, 1966.

[1] This work was upported by ARO under contract numbers ARO DAAH-04-95-1-0494 and ARMY WUHT-011398-S1.