Effect of a turbulent magnetic field on spectral lines polarization

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Motivation : effect of a random magnetic field with a **finite correlation length**

Zeeman effect (LTE)

⇒ **Mean Zeeman propagation matrix** for various magnetic field vector distributions

⇒ **Mean Stokes parameters and rms fluctuations**

Hanle effect

⇒ **Mean Stokes parameters**

Magnetic field model : stepwise constant Markov process
micro and macroturbulent limits when the correlation length goes to zero and to infinity

Turbulent velocity field : model with a finite correlation length (similar to the magnetic field model) or simply a combination of micro and macroturbulent fields

Mean Zeeman propagation matrix

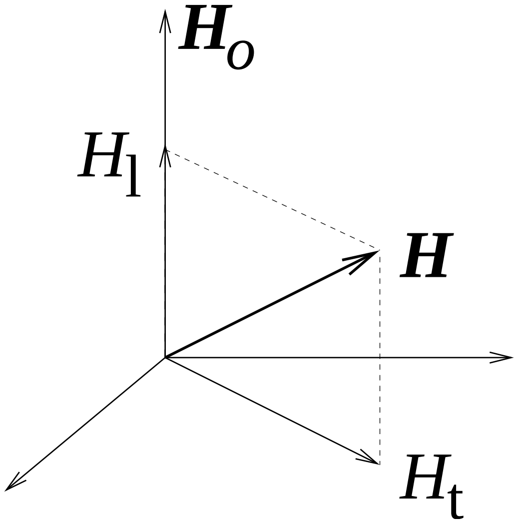
- Previous work : Dolginov & Pavlov 1972; Domke & Pavlov 1979
- Random magnetic field effects :

  $\Rightarrow$ broadening and shifts of the σ components only (intensity variations)

  $\Rightarrow$ averaging over the angular dependence of all the components (σ, π)
- Results

  $\Rightarrow$ General expressions for the mean coefficients $\langle\varphi_{\rm I}\rangle$, $\langle\varphi_{\rm Q}\rangle$, $\langle\varphi_{\rm V}\rangle$, . . . for random fields $\boldsymbol{H}$ rot. invariant around a mean field $\boldsymbol{H}_o$, i.e. $\boldsymbol{H} = \boldsymbol{H}_{\rm l} + \boldsymbol{H}_{\rm t}$

  $\Rightarrow$ Applications to anisotropic and isotropic Gaussian distributions



$$P_{\rm l}(\boldsymbol{H}) \propto e^{-(H_{\rm l}-H_o)^2/2\sigma_{\rm l}^2} \quad \text{longitudinal 1D}$$

$$P_{\rm t}(\boldsymbol{H}) \propto e^{-H_{\rm t}^2/2\sigma_{\rm t}^2} \quad \boldsymbol{H} = \boldsymbol{H}_o + \boldsymbol{H}_{\rm t} \quad \text{transverse 2D}$$

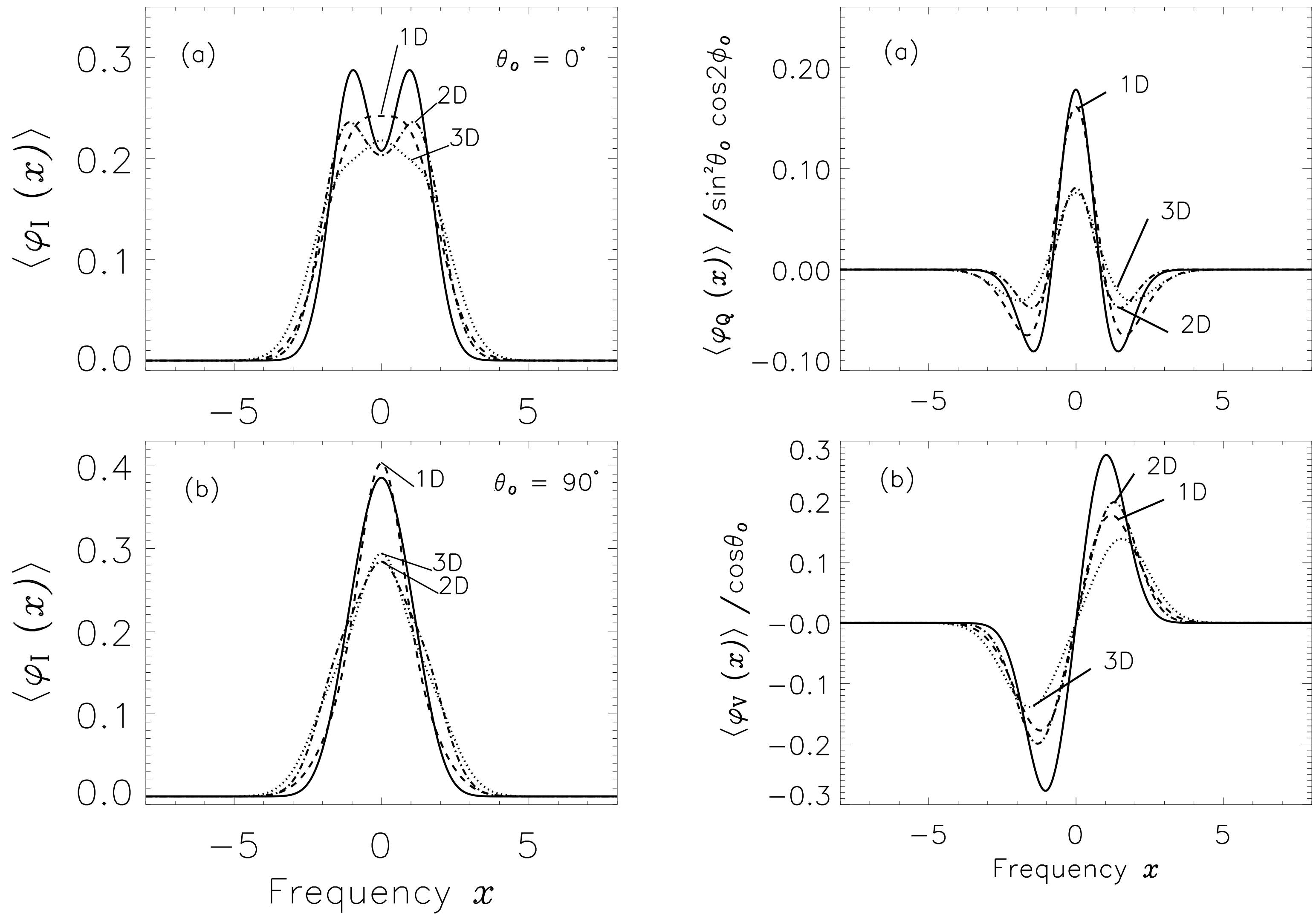
$$P_{\rm i}(\boldsymbol{H}) \propto e^{-(\boldsymbol{H}-\boldsymbol{H}_o)^2/2\sigma^2}; \quad \sigma = \sigma_{\rm l} = \sigma_{\rm t} \quad \text{isotropic 3D}$$

Parameter : $y_o = \frac{H_o}{\sigma_{\rm t,l}}$; $\quad y_o > 1,\ y_o < 1,\ y_o \simeq 1$

weak, strong, moderate fluctuations

second parameter : H_o or $\sigma_{\rm t,l}$

Dependence on the magnetic field distribution



x: in Doppler width units $\Delta_\mathrm{D} = \frac{\nu_o}{c}\sqrt{v_\mathrm{th}^2 + v_\mathrm{turb}^2}$; mean field $\boldsymbol{H}_o(H_o, \theta_o, \phi_o)$;

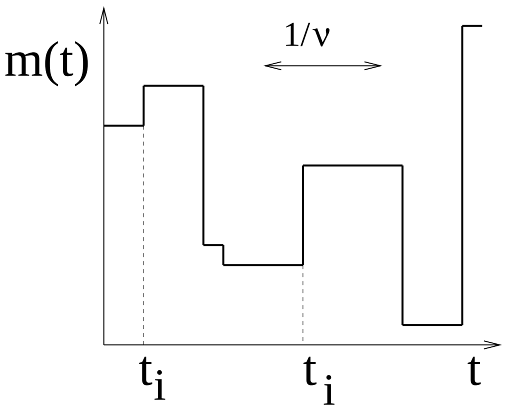
Zeeman shifts : $\Delta H_o = 1.,\ \Delta\sigma \simeq 0.7$; $\Delta = \frac{ge}{4\pi mc}\frac{1}{\Delta_\mathrm{D}}$ moderate fluctuations

left panels : $\langle \varphi_\mathrm{I} \rangle$ for $\theta_o = 0^\circ$ and $\theta_o = 90^\circ$; right panels $\langle \varphi_\mathrm{Q} \rangle$ and $\langle \varphi_\mathrm{V} \rangle$

Zeeman line transfer in a random magnetic field with a finite correlation length

Vector magnetic field model : Kubo-Anderson process

Properties : Markovian; stationary; piece-wise constant; characterized by a correlation length and a probability density.



jumping points t_i : Poisson distribution with density ν

probability density of m : $P(m)$

limit $\nu \rightarrow 0$: macroturbulence

limit $\nu \rightarrow \infty$: microturbulence

autocorrelation function is exponential :

$\langle m(t)m(t')\rangle = \langle m^2\rangle e^{-\nu|t-t'|}$ if $\langle m\rangle = 0$

Remark : model used by E. Landi Degl'innocenti (1994) (see also Landi Degl'Innocenti & Landolfi 2004, book) for random magnetic fields; for the broadening by random velocity fields (1973), stochastic Stark effect (1971), nuclear magnetic resonance (1954)

Velocity field : also a KAP with density ν (same jumping points); probability densities : $P_{\rm H}(\boldsymbol{H})$ and $P_{\rm v}(\boldsymbol{v})$ if $\boldsymbol{H}$ and $\boldsymbol{v}$ are uncorrelated and $P(\boldsymbol{H}, \boldsymbol{v})$ if they are correlated.

Mean Stokes parameters

- Transfer equation for the Stokes parameters $\boldsymbol{I}(I, Q, U, V)$

$$d\boldsymbol{I}/d\tau_c = (\hat{E} + \beta\hat{\Phi})[\boldsymbol{I} - \boldsymbol{S}]$$

$\hat{E}$: 4×4 unit matrix, $\hat{\Phi}$: line Zeeman propagation matrix, τ_c : continuum optical depth

- Milne-Eddington atmosphere :
  $\beta = \kappa_{\text{line}}/\kappa_{\text{cont}}$ =constant
  source vector $\boldsymbol{S}$ linear : $\boldsymbol{S} = (B_o + B_1\tau_c)\boldsymbol{U}$, [*calculations for* $\boldsymbol{U} = (1, 0, 0, 0)$]

- Residual emergent Stokes parameters (at $\tau_c = 0$)

$$\boldsymbol{r}(0) = \frac{1}{B_1}[\boldsymbol{I}_c(0) - \boldsymbol{I}(0)]$$

$I_c(0)$: continuum intensity

- Result : explicit expression for the mean value $\langle \boldsymbol{r}(0) \rangle$ and dispersion around $\langle \boldsymbol{r}(0) \rangle$

random magnetic field is piece-wise constant

- Method : convolution equation for the mean propagation operator; differs somewhat from Landi Degl'Innocenti (1994,2004); for the dispersion around the mean values : resummation method.

Mean residual emergent Stokes parameters (I)

- Explicit expression for the mean value

$$\langle \boldsymbol{r}(0) \rangle_{\rm kap} = (1+\nu)\hat{R}_{\rm macro}\left(\frac{\beta}{1+\nu}\hat{\Phi}\right)\left[\hat{E} + \nu\hat{R}_{\rm macro}\left(\frac{\beta}{1+\nu}\hat{\Phi}\right)\right]^{-1}\boldsymbol{U}$$

with

$$\hat{R}_{\rm macro}\left(\frac{\beta}{1+\nu}\hat{\Phi}\right) = \left\langle \frac{\beta}{1+\nu}\hat{\Phi}\left[\hat{E} + \frac{\beta}{1+\nu}\hat{\Phi}\right]^{-1}\right\rangle_{P(\boldsymbol{H},\boldsymbol{v})}$$

β/ν : correlation length in units of the optical depth at line center

- Micro and macroturbulent limits : Unno-Rachovsky solution

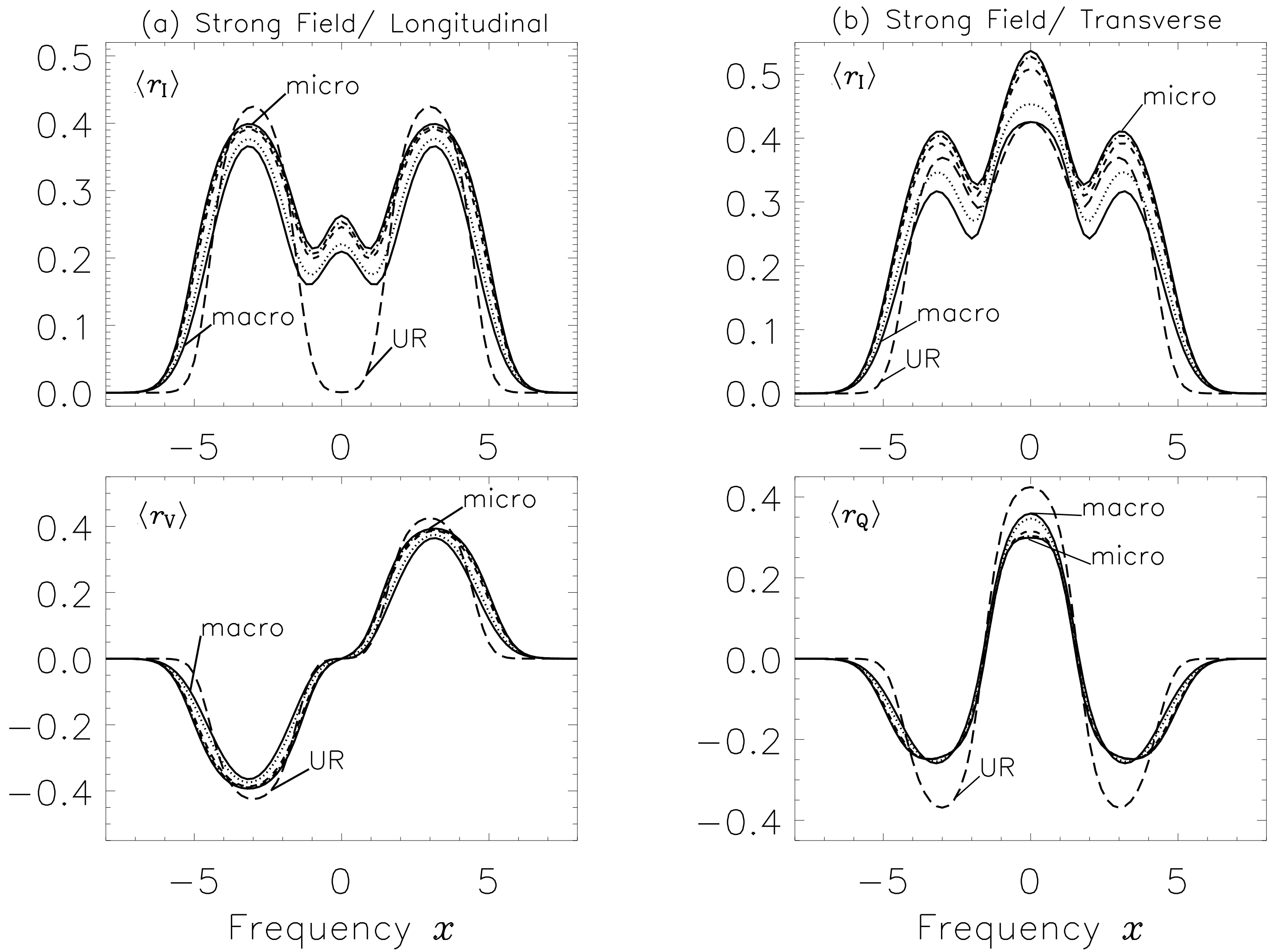
$$\langle \boldsymbol{r}(0) \rangle_{\rm micro} = \beta\langle\hat{\Phi}\rangle[\hat{E} + \beta\langle\hat{\Phi}\rangle]^{-1}\boldsymbol{U}$$
$$\langle \boldsymbol{r}(0) \rangle_{\rm macro} = \langle\beta\hat{\Phi}[\hat{E} + \beta\hat{\Phi}]^{-1}\rangle\boldsymbol{U}$$

- Remarks :

$\Rightarrow$ differences between micro and macro limits significant for $\beta \simeq 10 - 100$

$\Rightarrow$ microturbulent limit for $\nu \geq \beta$ (correlation length smaller than one in $\tau_{\rm line}$ unit)

Mean residual emergent Stokes parameters (II)



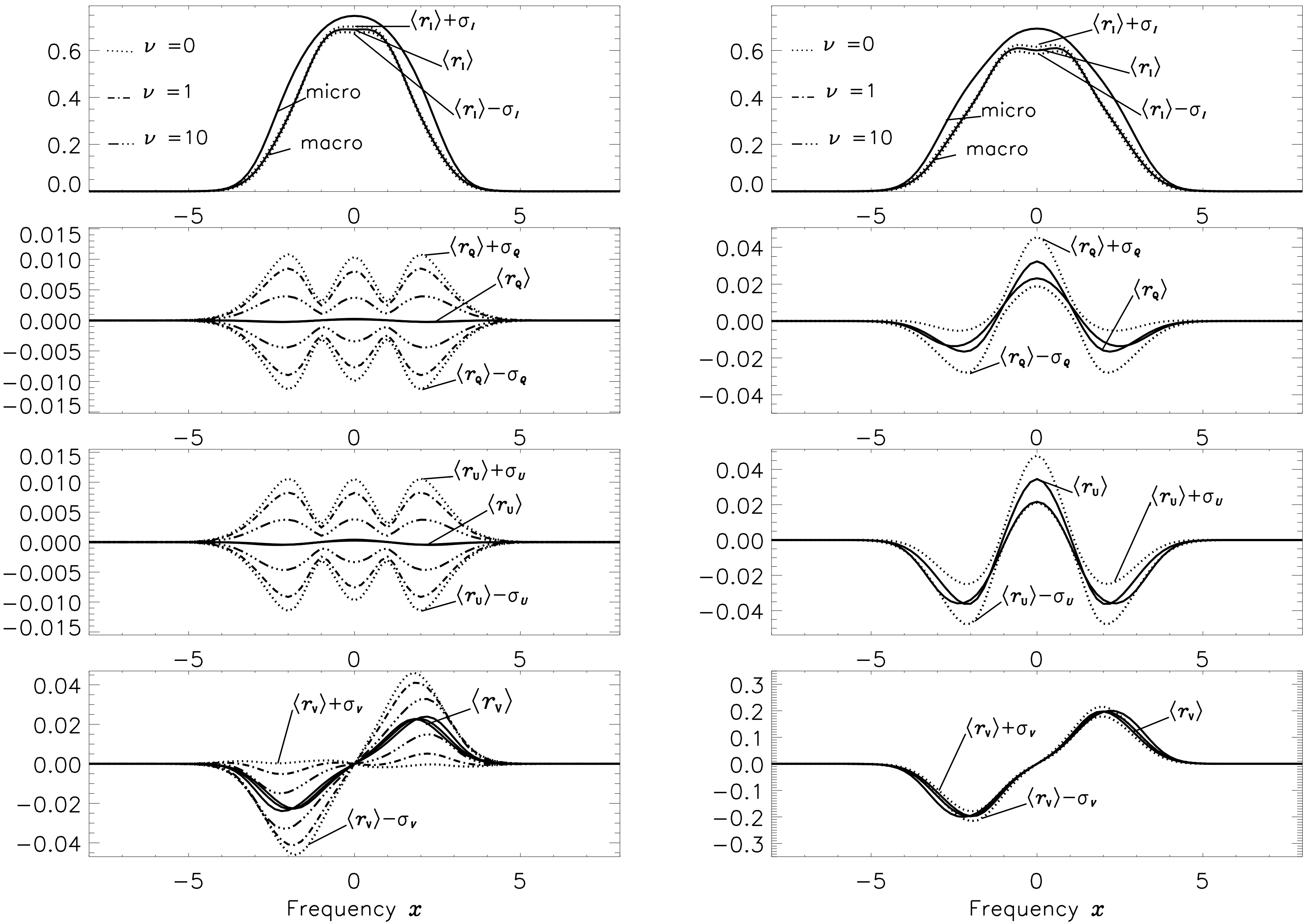
x: in Doppler width units $\Delta_D = \frac{\nu_o}{c}\sqrt{v_{th}^2 + v_{turb}^2}$; $\beta = \kappa_{line}/\kappa_{cont} = 10$

vector magnetic field distribution $P(\boldsymbol{H})$: isotropic Gaussian

Zeeman shifts : $\Delta H_o = 3.$; $\Delta\sigma \simeq 0.7$ weak fluctuations

left panels : $\langle r_I \rangle$ and $\langle r_V \rangle$ for $\theta_o = 0°$; right panels $\langle r_I \rangle$ and $\langle r_Q \rangle$ for $\theta_o = 90°$

Dispersion : $\sigma_X^2 = \langle X^2 \rangle_{\rm kap} - \langle X \rangle_{\rm kap}^2, \quad X = r_{\rm I,Q,U,V}(0)$



$\beta = 10$; $\theta_o = 45^\circ$; $\phi_o = 30^\circ$; isotropic ; $\sigma_X^2 \rightarrow 0$ in the microturbulent limit

$\Delta\sigma \simeq 0.7$; left $\Delta H_o = 0.1$ strong fluctuations; right $\Delta H_o = 1$ moderate fluctuations

Hanle effect in a random magnetic field with finite correlation length

Hanle effect : non-LTE line formation (multiple scattering)

Method

- previous work : turbulent velocity with finite correlation length in the seventies (Nice, Heidelberg, Paris)
- magnetic field and velocity field : KAP along the photon trajectories
- time-dependent transfer equation ; stationary solution as time goes to infinity

Simplified Hanle problem

- Polarization is weak : Stokes I independent of the magnetic field
- Coupling between Stokes Q and Stokes U is neglected
- Only a few number of scatterings suffice to build up the polarization

Explicit expressions for the mean Stokes parameters

Mean Stokes parameters (I)

Formulation of the problem

two-level atom ; no polarization of the lower level; plane-parallel atmosphere ;

scattering phase matrix $\hat{R}(x, \boldsymbol{\Omega}, x', \boldsymbol{\Omega}') = \varphi(x)\varphi(x')\hat{P}_{\rm H}(\boldsymbol{\Omega}, \boldsymbol{\Omega}', \boldsymbol{H})$;

x and $\boldsymbol{\Omega}(\theta, \phi)$: frequency and direction of incident beam ; x' and $\boldsymbol{\Omega}'(\theta', \phi')$ of scattered beam

Stokes I : scalar problem ; Stokes I affected by the random velocity field only

Stokes Q :

$$\langle Q(\tau, x, \boldsymbol{\Omega})\rangle_{\rm kap} \simeq \frac{3}{2\sqrt{2}} W_2(1-\mu^2)\bar{I}_2(\tau, x, \mu) \qquad \mu = \cos\theta$$

At the surface $\tau = 0$, in the outward direction, *microturbulent velocity*

$$\bar{I}_2(0, x, \mu) = \int_0^\infty e^{-\varphi(x)/\mu}\bar{S}_2(\tau)\varphi(x)\,\frac{d\tau}{\mu}$$

with $\bar{S}_2 = \langle \mathcal{S}_2(\tau|\boldsymbol{H})\rangle_{P(\boldsymbol{H})}$. The mean conditional source function, $\mathcal{S}_2(\tau|\boldsymbol{H})$ satisfies an integral equation. Solution by a Neumann series expansion.

Result : explicit expressions for the average over $P(\boldsymbol{H})$ of the two first terms in the expansion

first term : *single-scattering* $\Rightarrow$ *local average*

second term : *two-scattering* $\Rightarrow$ *depends on correlation length of the random magnetic field*

- Expression of $\bar{S}_2$: $\bar{S}_2(\tau) = \underbrace{\bar{S}_2^{\rm ss}(\tau)}_{single-scattering} + \underbrace{\bar{S}_2^{\rm 2s}(\tau)}_{two-scattering}$

⇒ Single-scattering term

$$\bar{S}_2^{\rm ss}(\tau) = (1-\epsilon_{\rm p})\langle M_{22}\rangle C_{\rm I}(\tau)$$

$\langle M_{22}\rangle$: usually denoted $W_{\rm B}$; $\epsilon_{\rm p}$: rate of destruction by elastic collisions
$C_{\rm I}(\tau)$: depends on I only (*dominant term in the frequency averaged spherical tensor* $J_0^2(\tau)$)

⇒ Two-scattering term

$$\bar{S}_2^{\rm 2s}(\tau) = (1-\epsilon_{\rm p})^2 W_2 \int_0^\infty K_{\rm 2s}(|\tau-\tau'|)C_{\rm I}(\tau')d\tau'$$

with the kernel

$$K_{\rm 2s}(|\tau-\tau'|) = \int_{-\infty}^{+\infty} \frac{1}{2}\int_0^1 \varphi(x)\Psi_{22}(\mu)\Gamma_{22}(|\tau-\tau'|)e^{-(|\tau-\tau'|)\varphi(x)/\mu}\, d\mu\, dx$$

$\Psi_{22}(\mu) = \frac{1}{4}(5-12\mu^2+9\mu^4)$ with $\frac{1}{2}\int_{-1}^{+1}\Psi_{22}(\mu)\,d\mu = 7/10$ *resonance scattering*
and
$\Gamma_{22}(|\tau-\tau'|) = e^{-\nu|\tau-\tau'|/\mu}\langle M_{22}^2\rangle + (1-e^{-\nu|\tau-\tau'|/\mu})\langle M_{22}\rangle^2$ *correlation fct. of* M_{22}
Limits micro $\nu\to\infty$: $\Gamma_{22}^{\rm micro} = \langle M_{22}\rangle^2$; macro $\nu\to 0$: $\Gamma_{22}^{\rm macro} = \langle M_{22}^2\rangle$

⇒ Remark : Explicit expressions also, if the random velocity field is a KAP ; with effective medium approximation

Further work : contact with observations and numerical simulations

Inversion problems : how to retrieve the random magnetic field parameters

Enrich the models : e.g. depth dependent or magnetic field dependent, correlation length

Publications

H. Frisch, M. Sampoorna, & K.N. Nagendra 2005 *Stochastic polarized line formation I. Zeeman propagation matrix in a random magnetic field* (A&A, in press)

H. Frisch, M. Sampoorna, & K.N. Nagendra 2005 *Stochastic polarized line formation II. Zeeman line transfer in a random magnetic field* (preprint)

H. Frisch 2005 *The Hanle effect in a random medium*, (preprint, submitted to A&A)

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