Multi-Dimensional Radiative Transfer with Polarization

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  - Equivalent width of Li I 670.6nm and abundance determination
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- Conclusions

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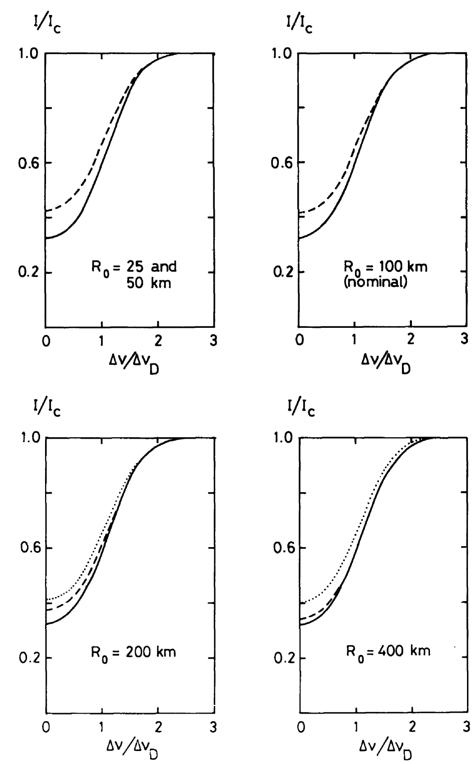
- The optical extent of a structure with size L is: $L\kappa(z) = (L/H)\tau(z)$
- Horizontal transport becomes energetically important when the lateral photon escape probability is equal or larger that the probability of vertical escape: $(L/H)\tau(z) \leq \tau(z)$, or simply when $L \leq H$.

Multi-dimensional Radiative Transfer in Fluxtubes

- Stenholm & Stenflo (1977), A&A 38, 273
- Magnetic fluxtubes represented in cylindrical geometry with Wilson depression
- Multi-dimensional Non-LTE radiative transfer, two-level atom, core saturation method
- "Our calculations demonstrate how plane-parallel models may be completely inadequate and non-physical representations of global averages. The averages may be significantly influenced by three-dimensional geometry of small-scale structures."



Important Papers

- Mihalas, Auer & Mihalas (1978), ApJ 220, 1001: *Two-dimensional Radiative Transfer. I Planar geometry.* "In actual fact, we have not found a single instance of a significant difference between 1.5D and 2D solutions of any periodic cases that we have considered."

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- Auer, Fabiani Bendicho & Trujillo Bueno (1994), A&A, 292, 599: *Multi-level radiative transfer with multi-level atoms: I ALI method with preconditioning of the rate equations*. "These illustrative multi-level calculations in schematic inhomogeneous atmospheres demonstrate the importance of properly including the effects of horizontal radiative transfer and realistic atomic models."

Basic Radiative Transfer: Local Changes

Source function: Transport along a ray:

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu - \alpha_\nu I_\nu$$

$$\frac{\mathrm{d}I_\nu}{\alpha_\nu \mathrm{d}s} = S_\nu - I_\nu$$

$$S_\nu = j_\nu / \alpha_\nu$$

$$S_\nu^{\mathrm{tot}} = \sum j_\nu / \sum \alpha_\nu$$

$$S_\nu^{\mathrm{tot}} = \frac{j_\nu^c + j_\nu^l}{\alpha_\nu^c + \alpha_\nu^l} = \frac{S_\nu^c + \eta_\nu S_\nu^l}{1 + \eta_\nu}, \quad \eta_\nu \equiv \alpha_\nu^l / \alpha_\nu^c$$

Equation of Polarized Radiative Transfer

Transfer Equation:

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}s} = -\mathbf{K}\mathbf{I} + \mathbf{j}$$

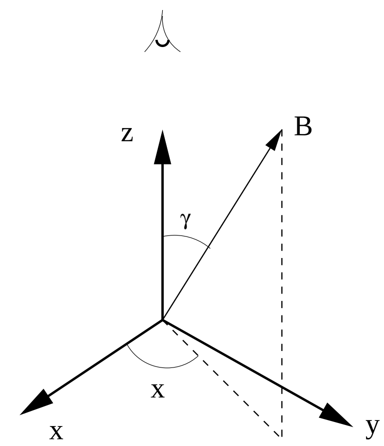
$$\mathbf{I} = (I, Q, U, V)^{\dagger}, \quad (\text{Stokesvector})$$

$$\mathbf{j} = (j_c + j_l\mathbf{\Phi})\mathbf{e}_0, \quad \mathbf{e}_0 = (1, 0, 0, 0)^{\dagger}$$

$$\mathbf{K} = \alpha_c\mathbf{1} + \alpha_c\mathbf{\Phi}, \quad (\text{Absorptionmatrix})$$

Line Absorption Matrix

$$\mathbf{\Phi} = \begin{pmatrix} \phi_I & \phi_Q & \phi_U & \phi_V \\ \phi_Q & \phi_I & \psi_V & -\psi_U \\ \phi_U & -\psi_V & \phi_I & \psi_Q \\ \phi_V & \psi_U & -\psi_Q & \phi_I \end{pmatrix}$$



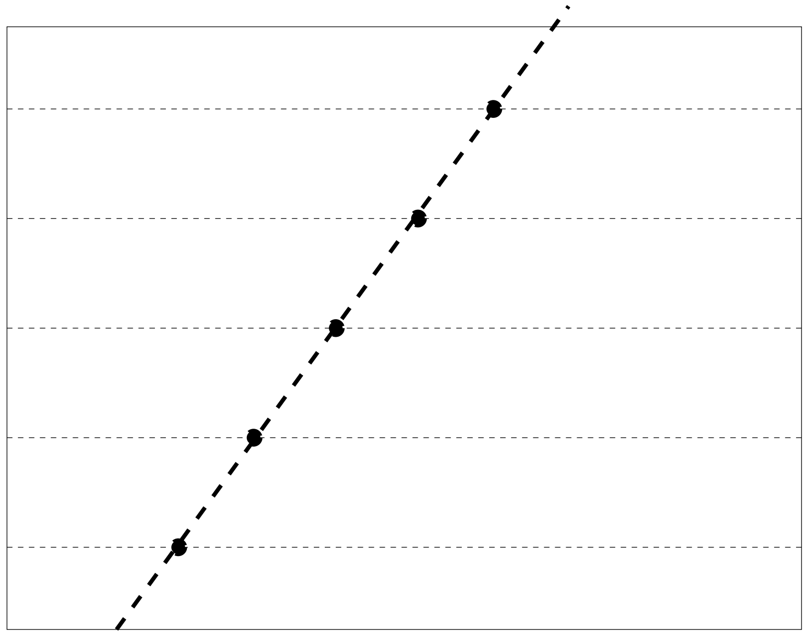
$$\phi_I = \phi_\Delta \sin^2\gamma + \tfrac{1}{2}(\phi_+ + \phi_-), \quad \phi_\Delta = \tfrac{1}{2}\left[\phi_0 - \tfrac{1}{2}(\phi_+ + \phi_-)\right]$$

$$\phi_Q = \phi_\Delta \sin^2\gamma \cos 2\chi$$

$$\phi_U = \phi_\Delta \sin^2\gamma \sin 2\chi$$

$$\phi_V = \tfrac{1}{2}(\phi_+ - \phi_-) \cos\gamma$$

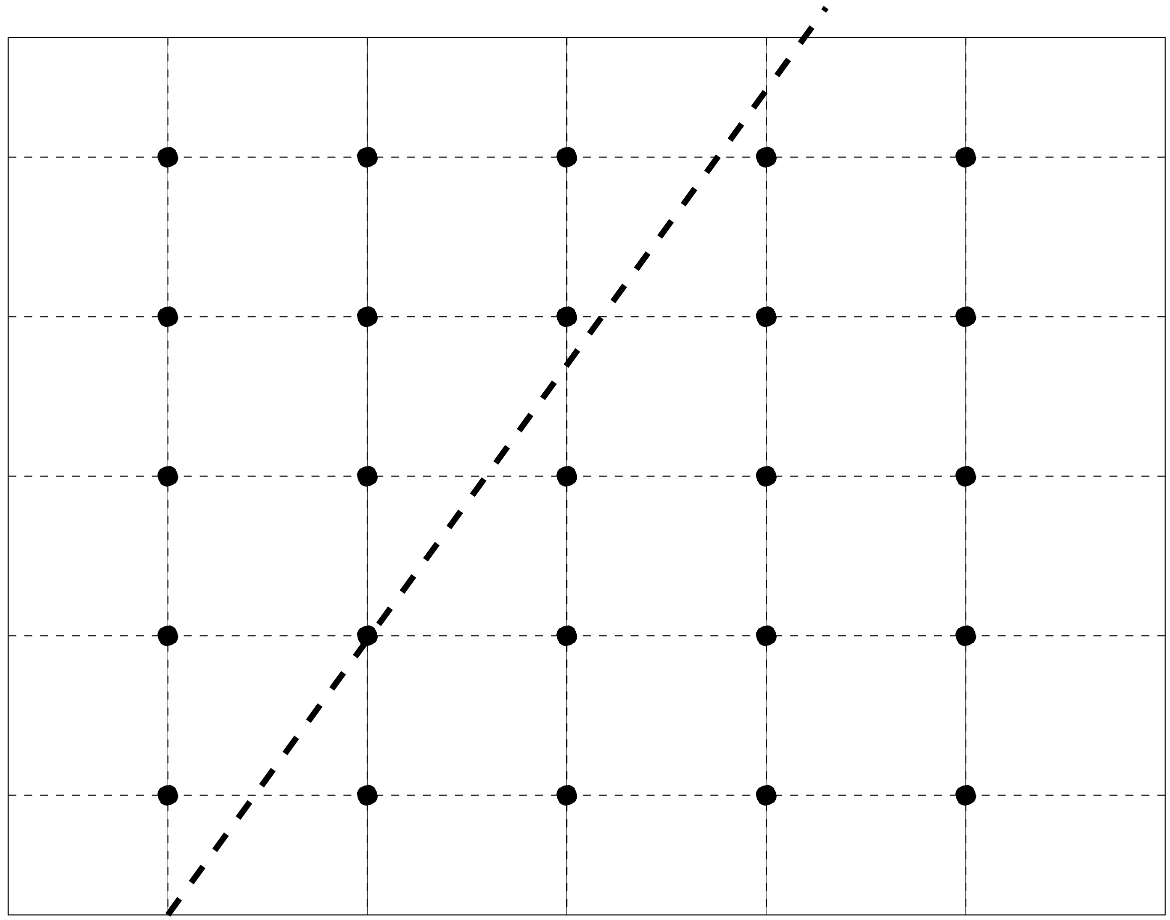
Formal solution in 1-D



Solution to transfer equation:

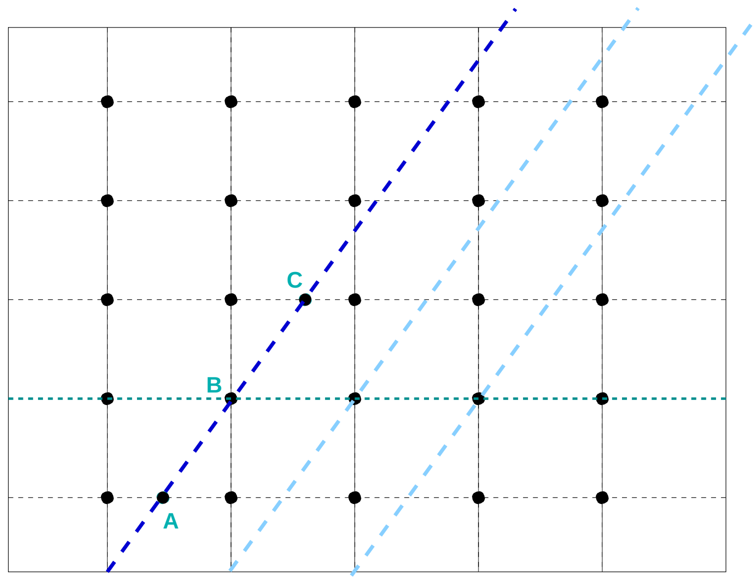
$$I_\nu(\tau_\nu) = \int_0^\infty S_\nu(t) e^{-t} \mathrm{d}t$$

Formal solution in 2- and 3-D



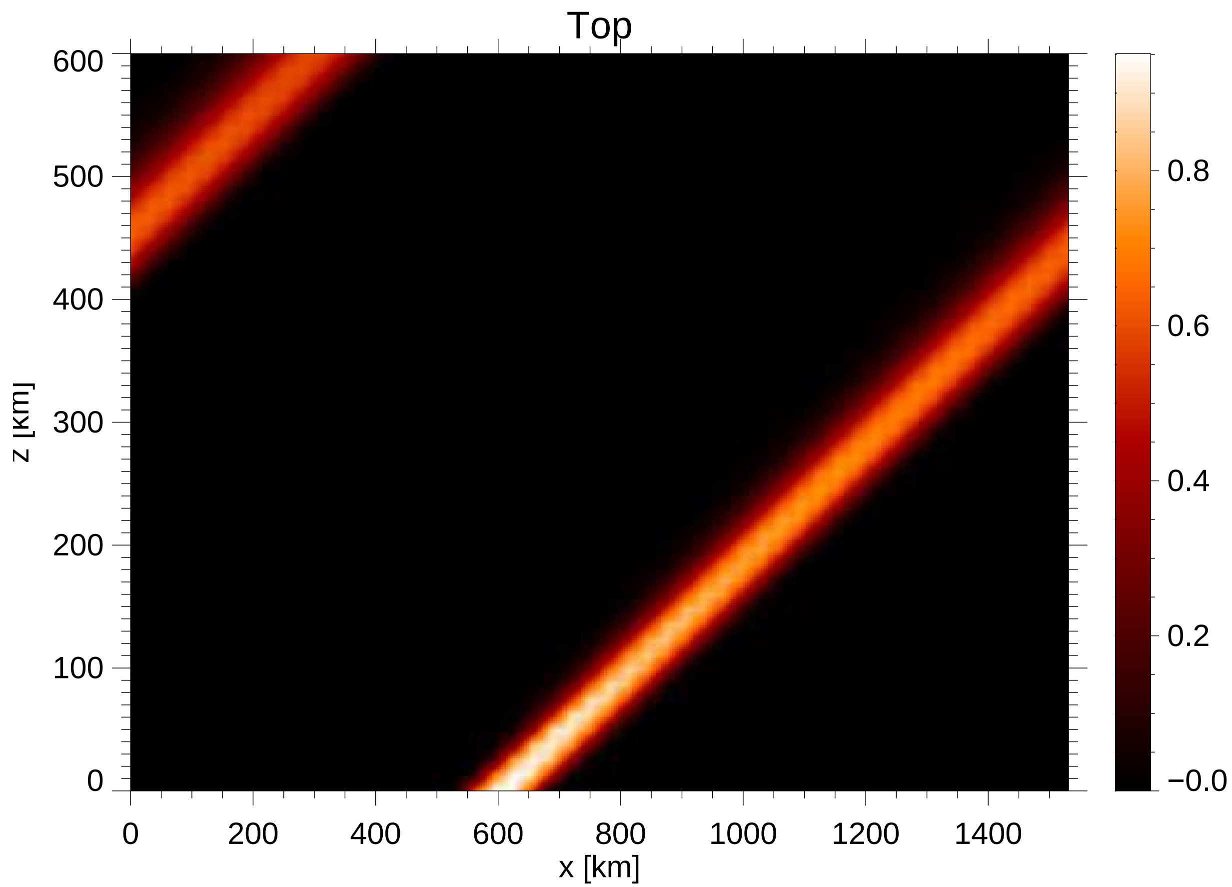
Short-Characteristics in Multi-Dimensional Geometry

Kunasz & Auer (1988), J. Quant. Spectrosc. Radiat. Transfer, 39, 67



$$I_B = I_A e^{-\tau_{AB}} + \int_{\tau_A}^{\tau_B} S(\tau) e^{-(\tau - \tau_{AB})} \mathrm{d}\,\tau$$

Search light problem in two dimensions



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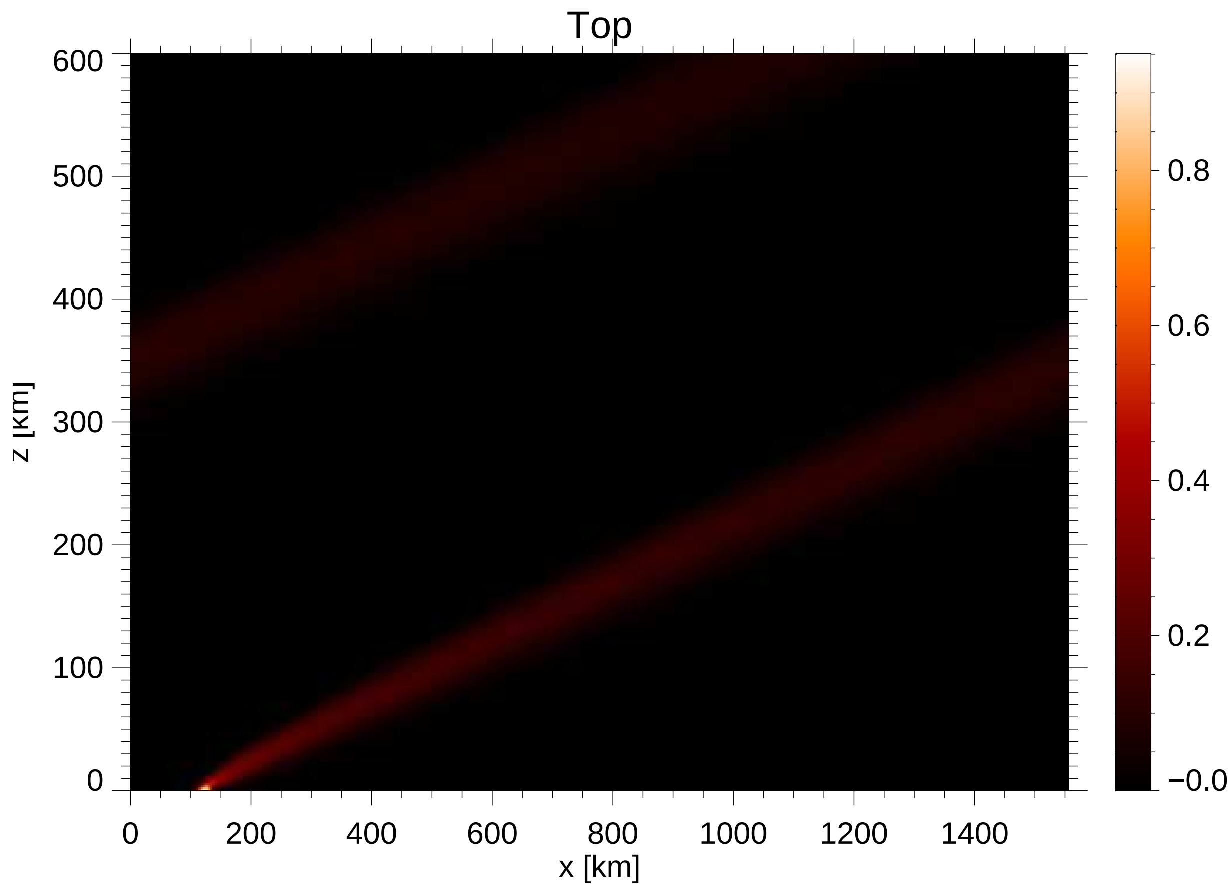
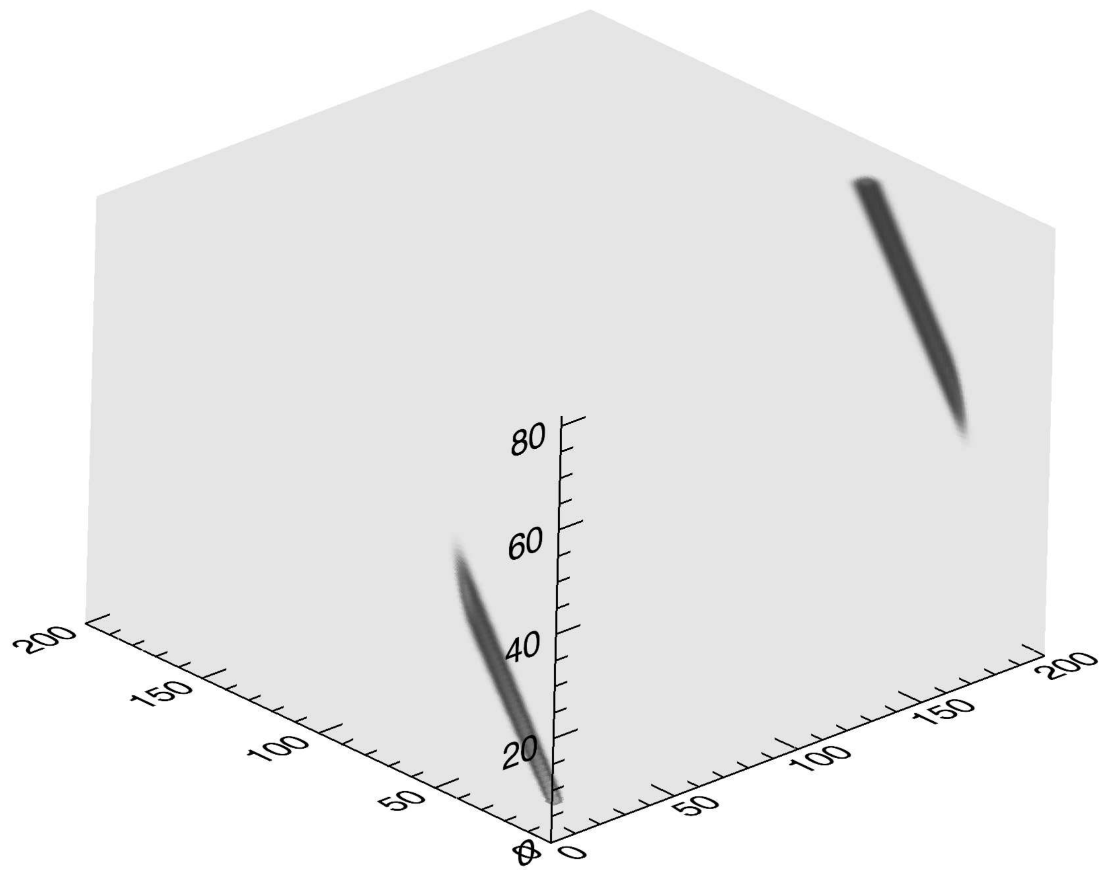
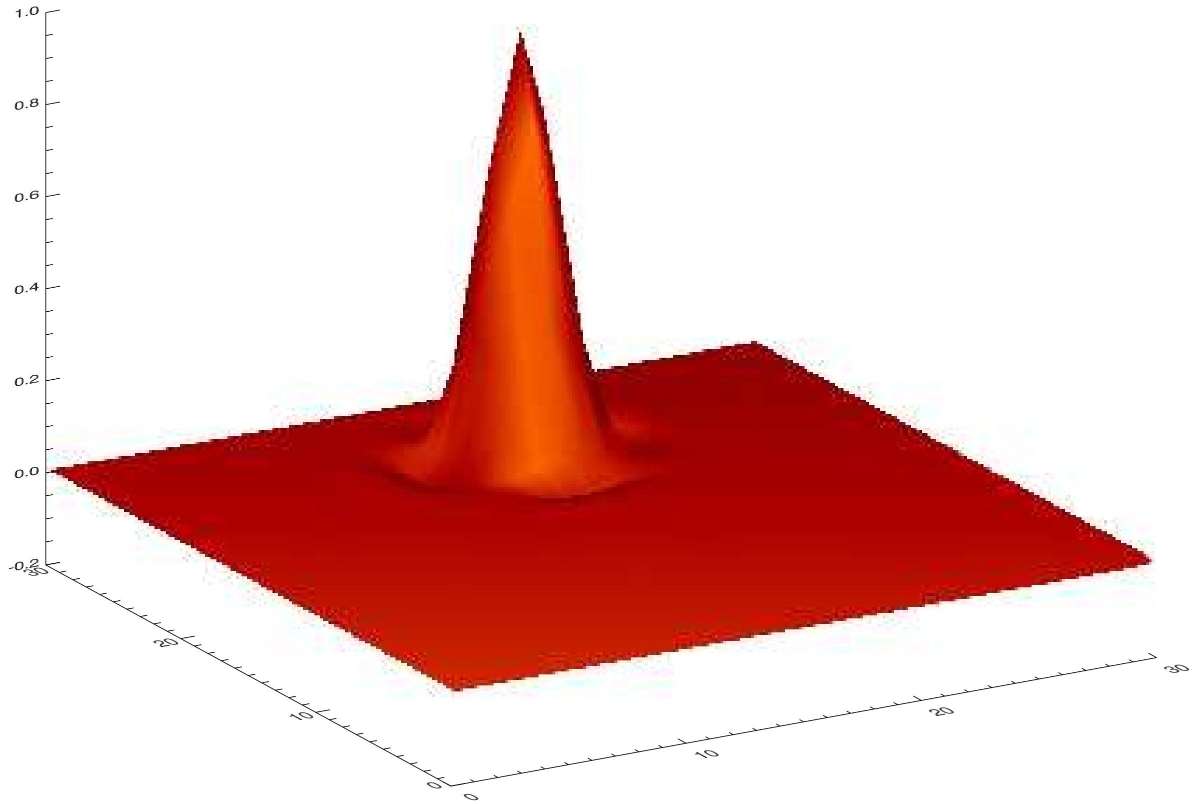


Search light problem in three dimensions



Interpolation in three dimensions: Monotonicity



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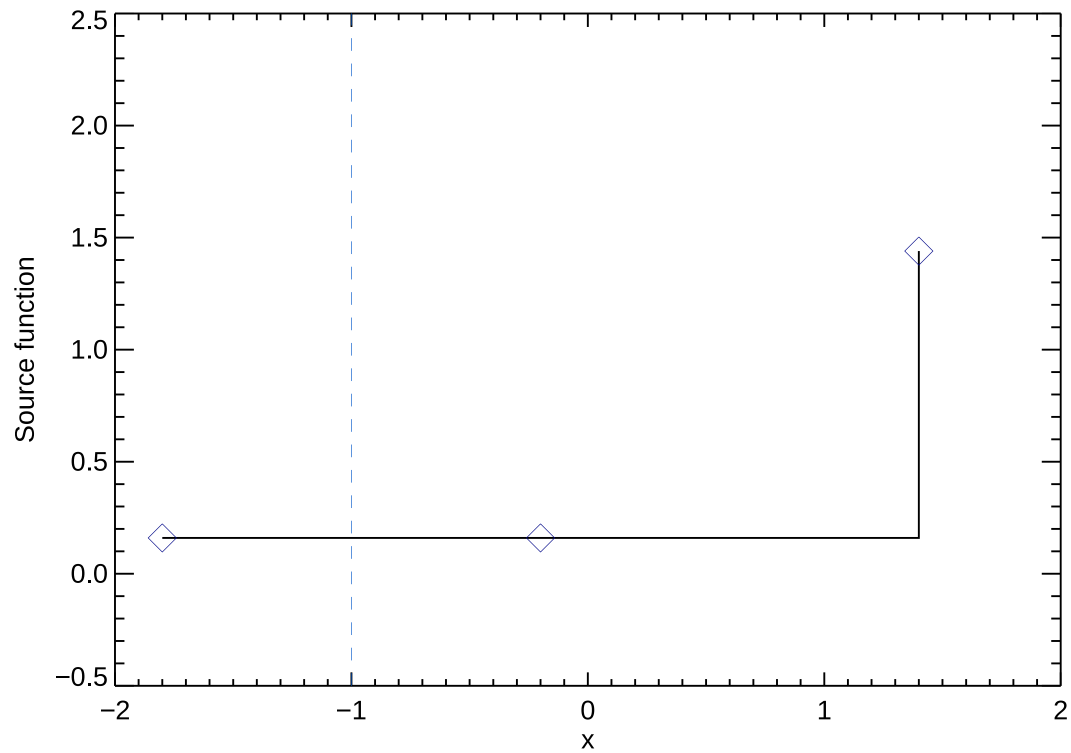

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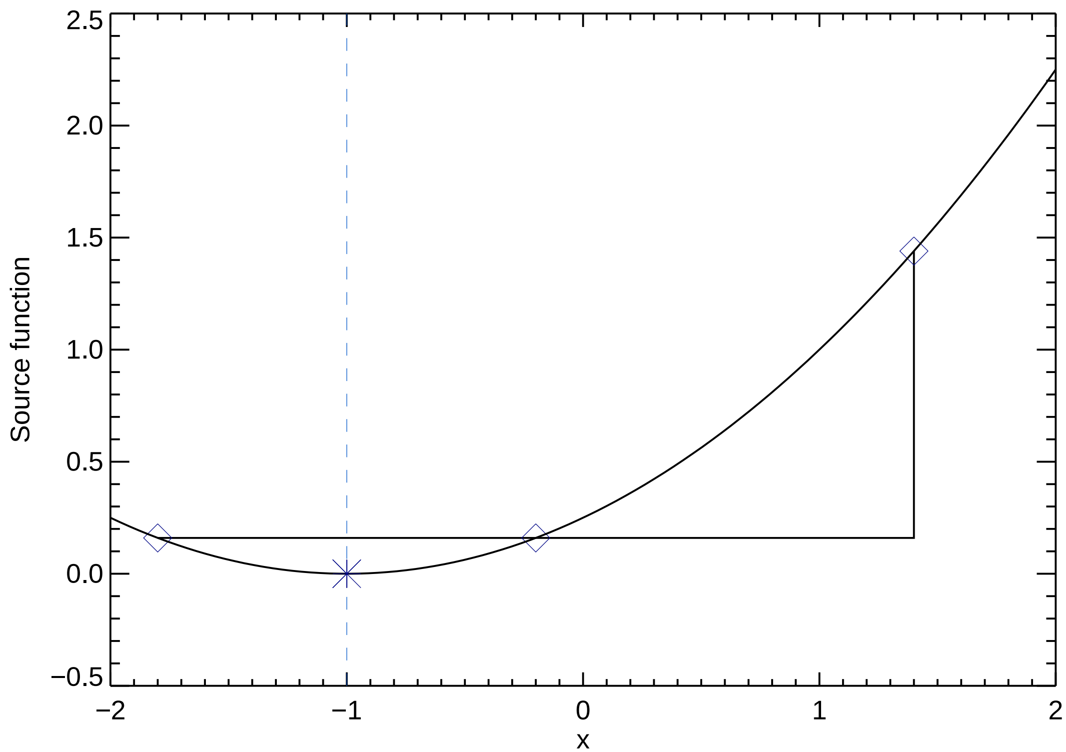

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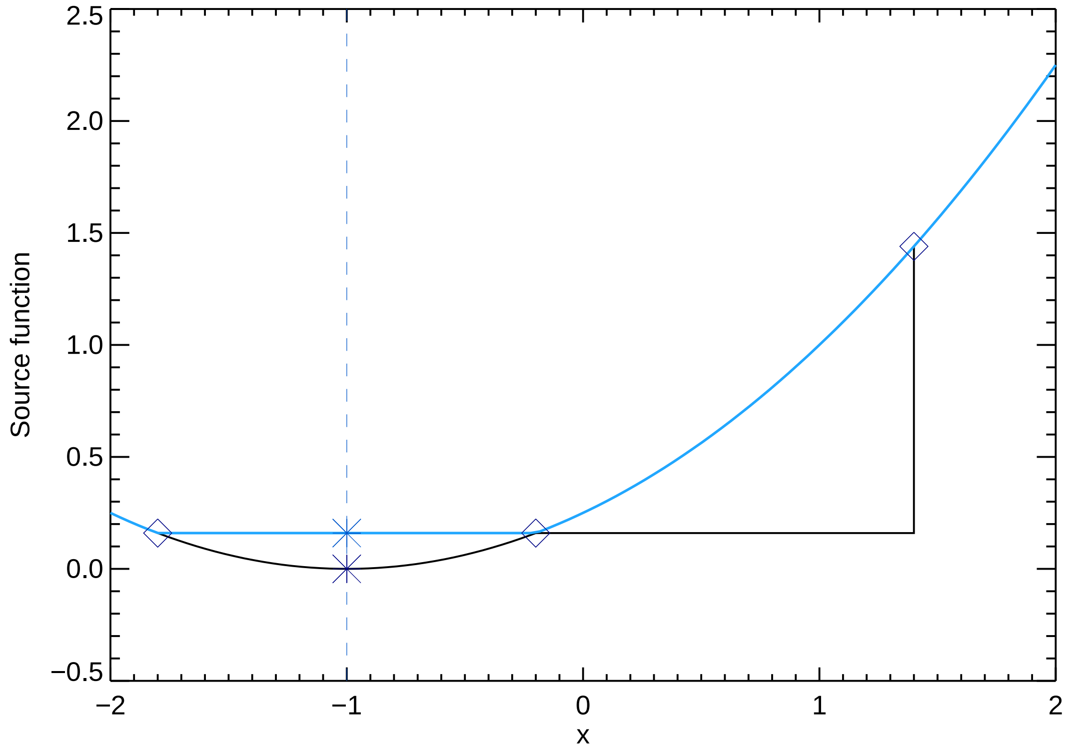

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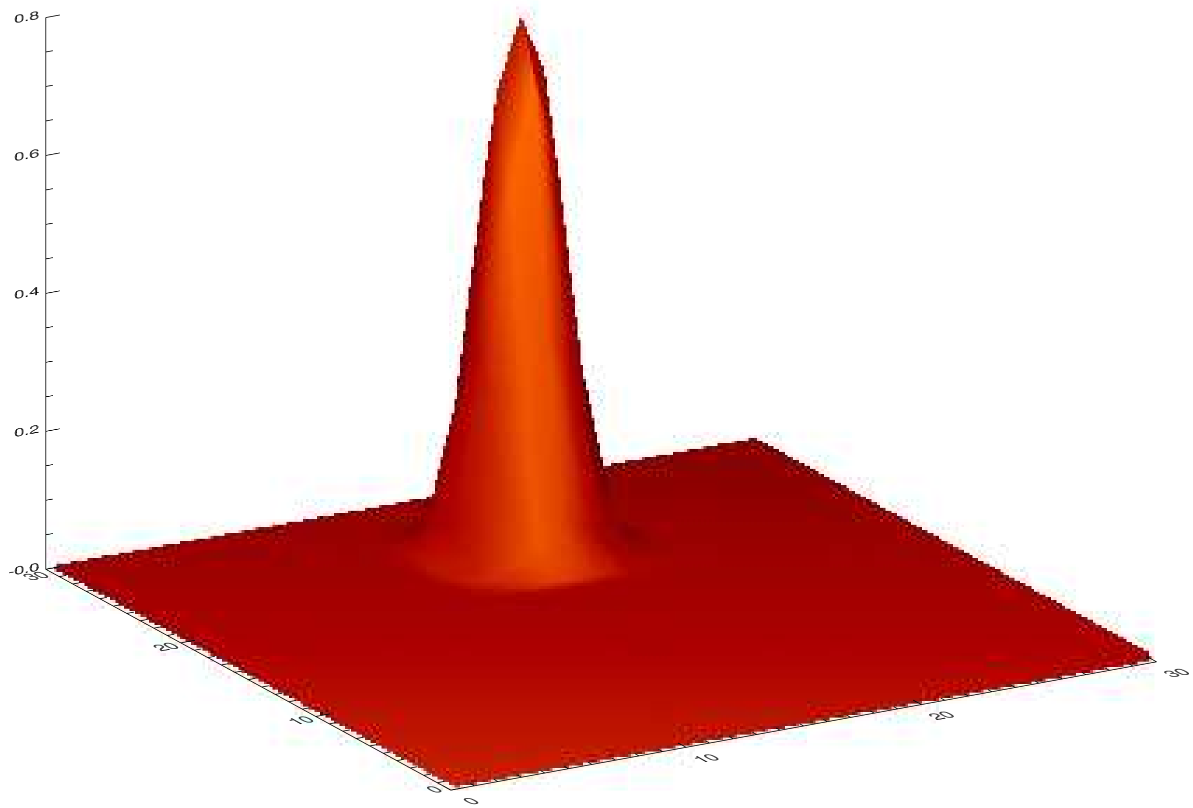
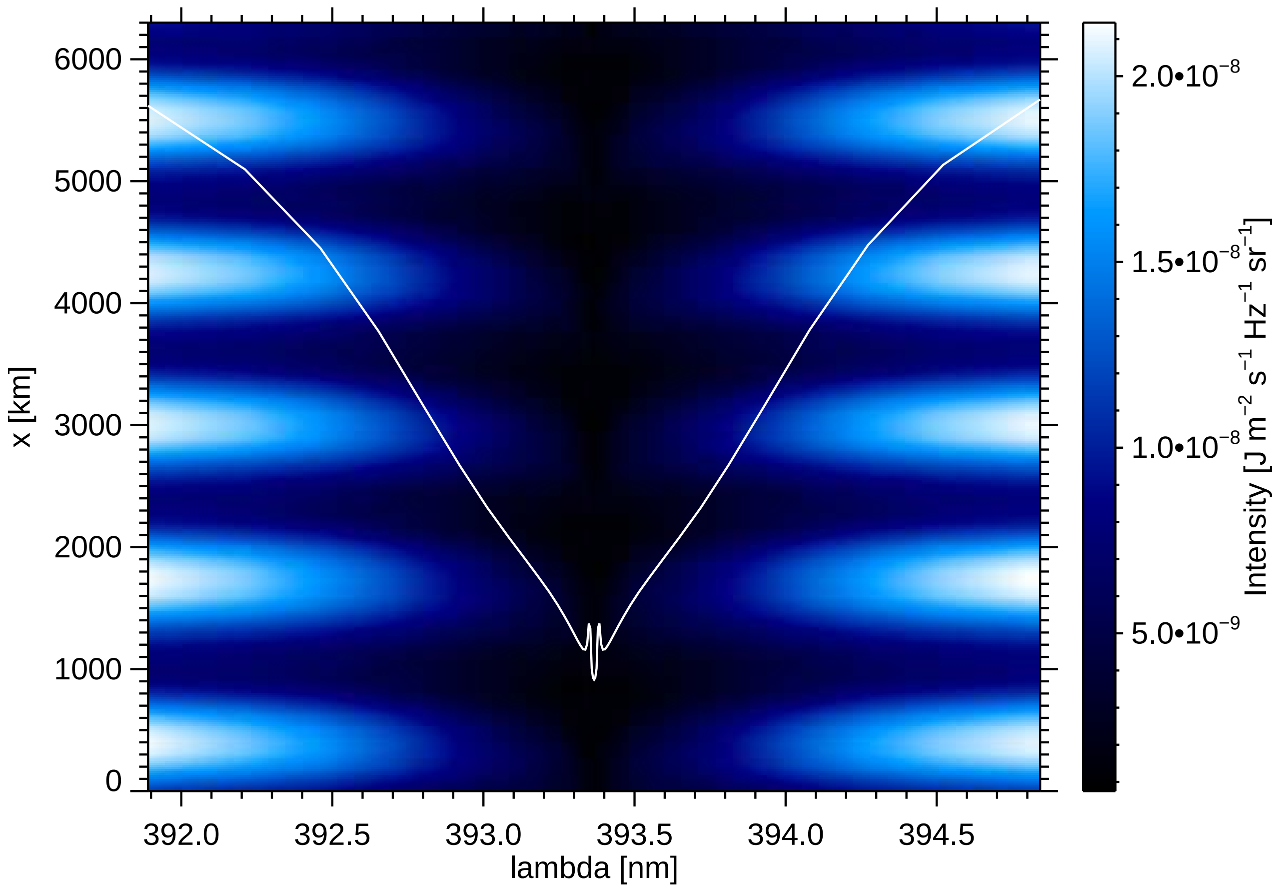
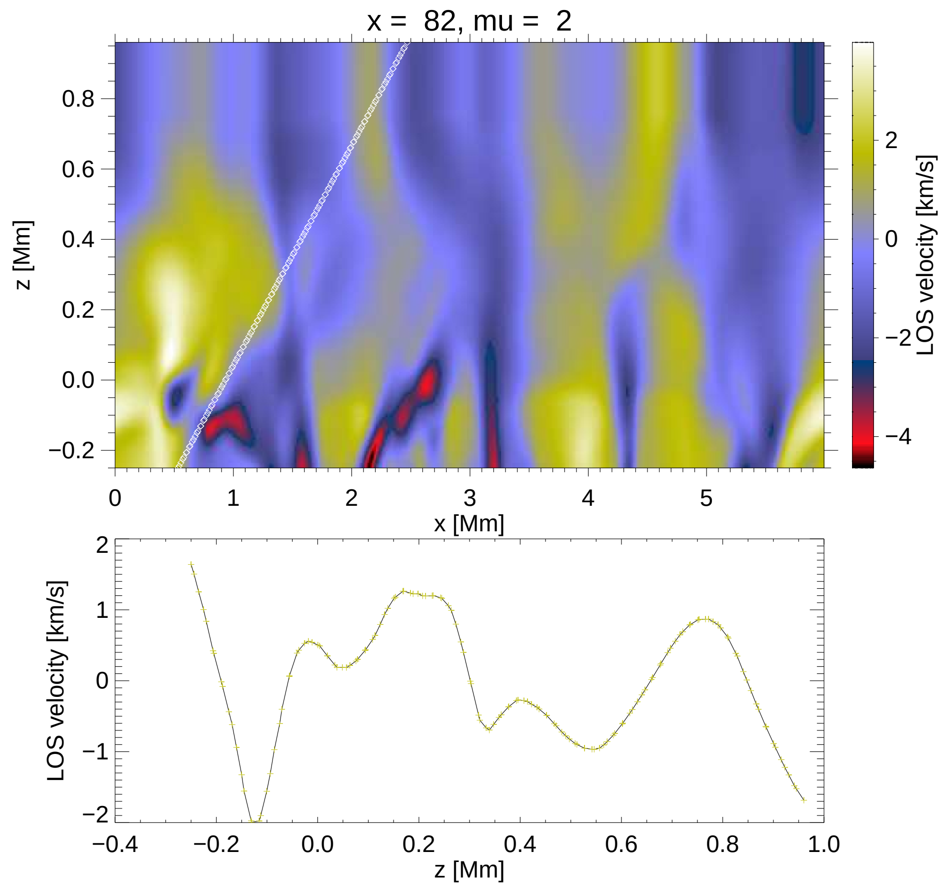


Interpretation of spectra for a slanted ray



Variation of properties along straight ray



The Redistribution Function R_{ij}

References: Hummer (1962), Heinzel & Hubeny (1982)

The laboratory frame redistribution function:

$$R_{ij}(\nu, \mathbf{n}; \nu', \mathbf{n}') d\nu d\nu' \frac{d\Omega}{4\pi} \frac{d\Omega'}{4\pi}$$

Describes the conditional probablility that, when a photon in line (i, j) and solid angle $d\Omega'$ around direction $\mathbf{n}'$ and frequency range $(\nu', \nu' + d\nu')$ is scattered by that line, it will be emitted into angle $d\Omega$ around direction $\mathbf{n}$ and frequency range $(\nu, \nu + d\nu)$.

Complete frequency in the laboratory frame:

$$R_{ij}(\nu, \mathbf{n}; \nu', \mathbf{n}') = \phi_{ij}(\nu, \mathbf{n}) \phi_{ij}(\nu', \mathbf{n}')$$

The Redistribution Function R_{ij} (2)

Normalization:

$$\oint\oint \frac{d\Omega}{4\pi}\frac{d\Omega'}{4\pi}\int\int d\nu' d\nu \; R_{ij}(\nu, \mathbf{n}; \nu', \mathbf{n}') \equiv 1.$$

$$\oint \frac{d\Omega'}{4\pi}\int d\nu' \; R_{ij}(\nu, \mathbf{n}; \nu', \mathbf{n}') \equiv \phi_{ij}(\nu, \mathbf{n})$$

Coherency fraction:

$$R_{ij} = \gamma R_{ij}^{V} + (1-\gamma) R_{ij}^{III}$$

$$\gamma = P_j/(P_j + Q_{\mathrm{E}})$$

Partial Frequency Redistribution in the emission profile

$$\psi_{ij}^{\mathrm{PRD}}(\nu) = \phi_{ij}(\nu)\left\{1 + \gamma\frac{n_i B_{ij}}{n_j P_j}\int\left[\frac{R_{iji}^{II}(\nu,\nu')}{\phi_{ij}(\nu)} - \phi_{ij}(\nu')\right]J(\nu')d\nu'\right\}$$

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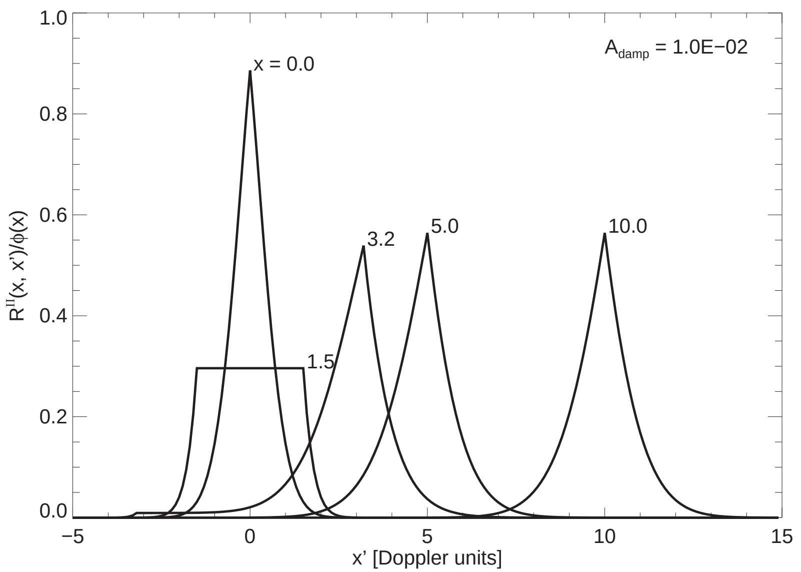

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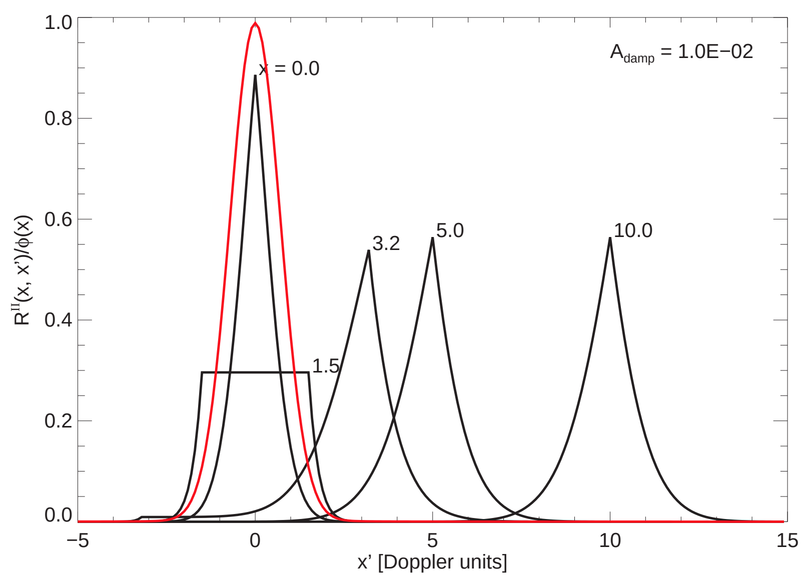

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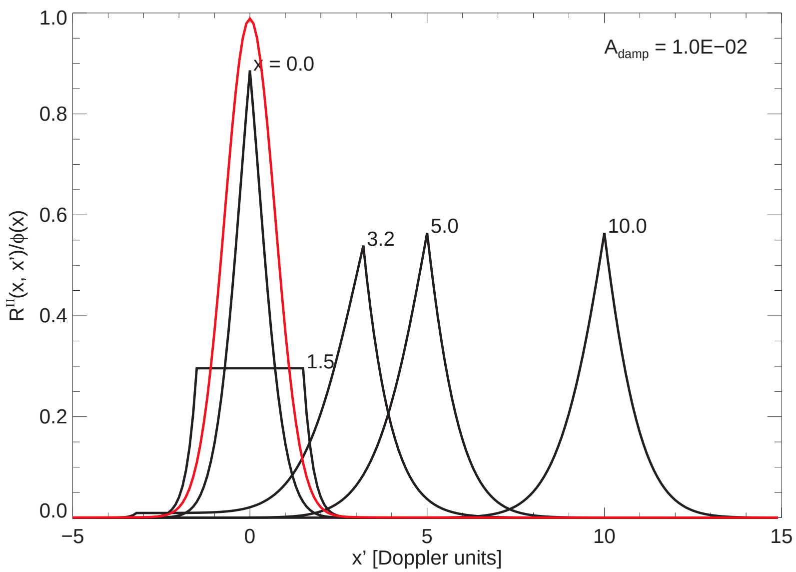

- Complete redistribution in core
- Coherent scattering in the wings
- Decoupling of wing source function

MALI with PRD

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- Employ Ng's convergence acceleration

The iterative method (Uitenbroek 2001, ApJ 557, 389)

Opacity and emissivity due to line (i, j):

$$\chi_{ij}(\nu, \mathbf{n}) = \frac{h\nu}{4\pi}\left(n_i B_{ij}\varphi_{ij}(\nu, \mathbf{n}) - n_j B_{ji}\psi_{ij}(\nu, \mathbf{n})\right) = n_i V_{ij}(\nu, \mathbf{n}) - n_j V_{ji}(\nu, \mathbf{n})$$

$$\eta_{ij}(\nu, \mathbf{n}) = \frac{h\nu}{4\pi} A_{ji}\psi_{ij}(\nu, \mathbf{n}) = n_j U_{ji}(\nu, \mathbf{n})$$

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Radiative rates:

$$R_{ij} = \oint d\Omega \int \frac{d\nu}{h\nu} V_{ij}(\nu, \mathbf{n}) I(\nu, \mathbf{n})$$

$$R_{ji} = \oint d\Omega \int \frac{d\nu}{h\nu} \left\{U_{ji}(\nu, \mathbf{n}) + V_{ji}(\nu, \mathbf{n}) I(\nu, \mathbf{n})\right\}$$

The iterative method (2)

Equation of statistical equilibrium:

$$\sum_k n_k \left(C_{kl} + R_{kl} \right) = n_l \sum_k \left(C_{lk} + R_{lk} \right)$$

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Approximate formal solution:

$$I(\nu, \mathbf{n}) = \Psi^*_{\nu,\mathbf{n}} \left[\eta_{\rm tot}\right] + \left(\Psi_{\nu,\mathbf{n}} - \Psi^*_{\nu,\mathbf{n}}\right) \left[\eta^\dagger_{\rm tot}\right]$$

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Profile ratio:

$$\psi(\nu, \mathbf{n}) = \rho(\nu, \mathbf{n})\phi(\nu, \mathbf{n})$$

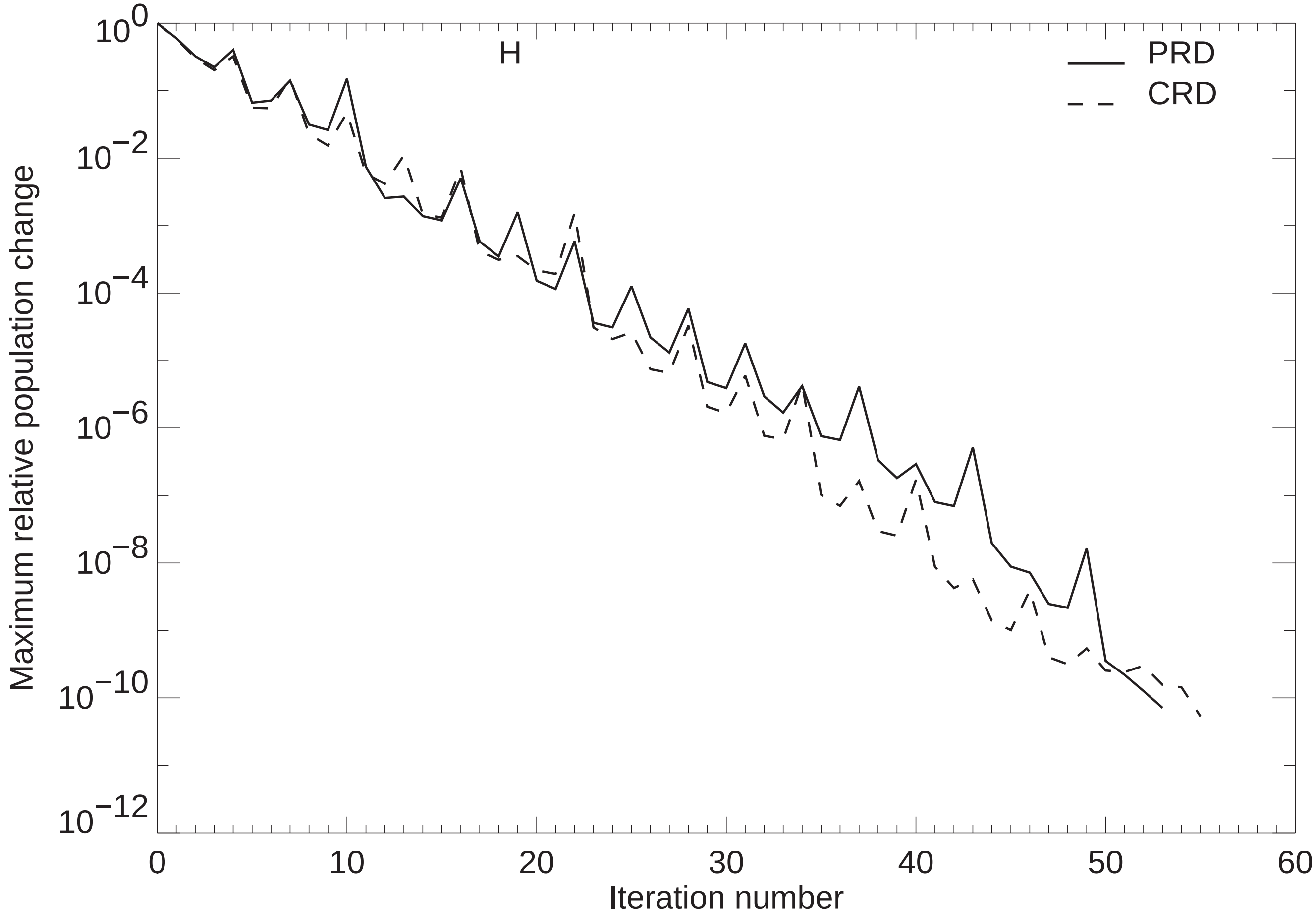
To include Partial Frequency Redistribution in scheme

$$\rho_{ij} = \frac{\psi_{ij}^{\mathrm{PRD}}(\nu)}{\phi_{ij}(\nu)} = 1 + \gamma \frac{n_i B_{ij}}{n_j P_j} \int \left[\frac{R_{iji}^{II}(\nu, \nu')}{\phi_{ij}(\nu)} - \phi_{ij}(\nu') \right] J(\nu') d\nu'$$

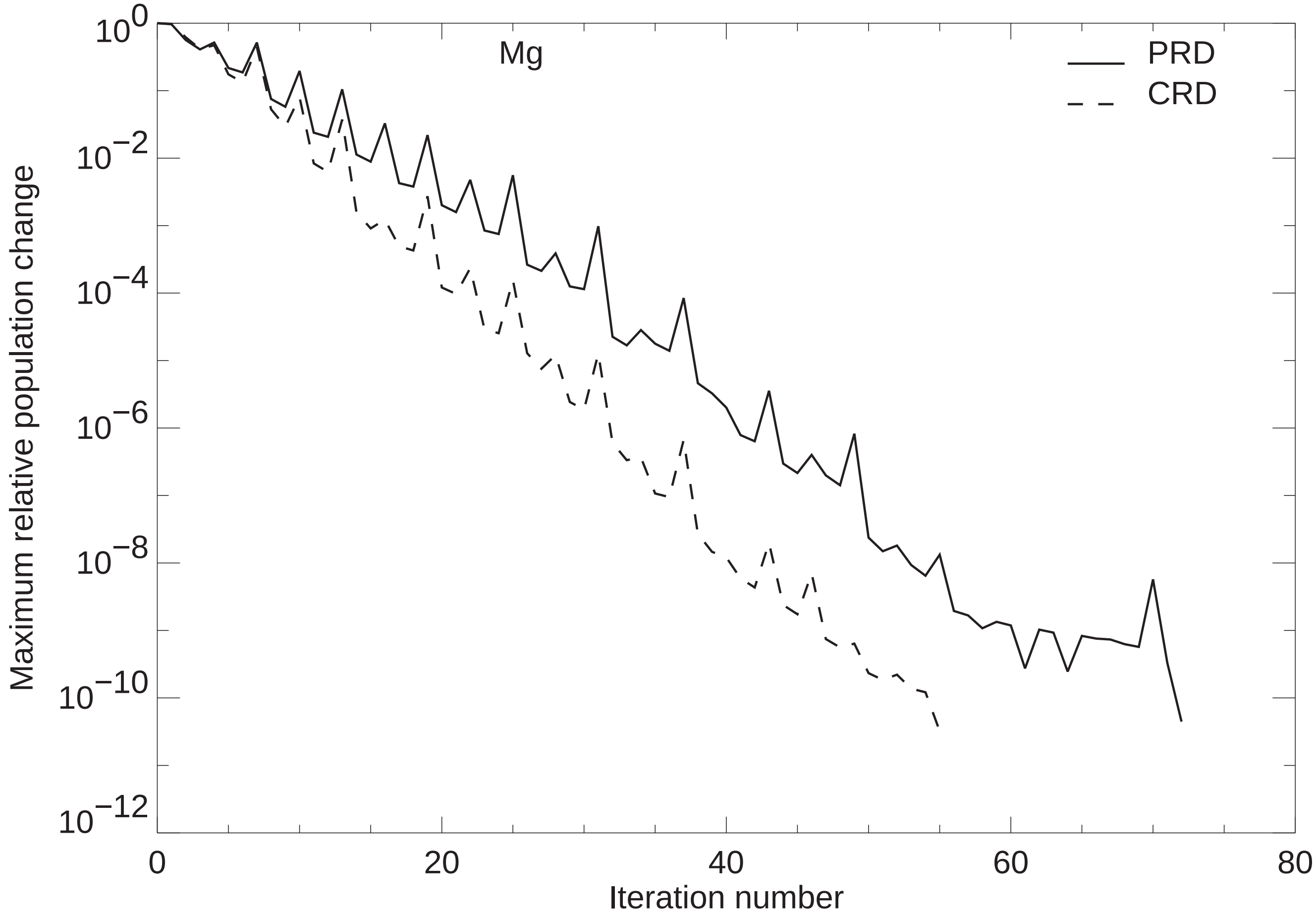
To extend the MALI formalism to include PRD we need:

- Calculate redistribution function for angular grid.
- Evaluate profile ratio ρ with scattering integral in inner loop while populations are kept fixed. This converges rapidly as it does **not** introduce new photons into the atmosphere.
- Correct profile ϕ for each PRD line with factor ρ. This involves only one additional line of code.

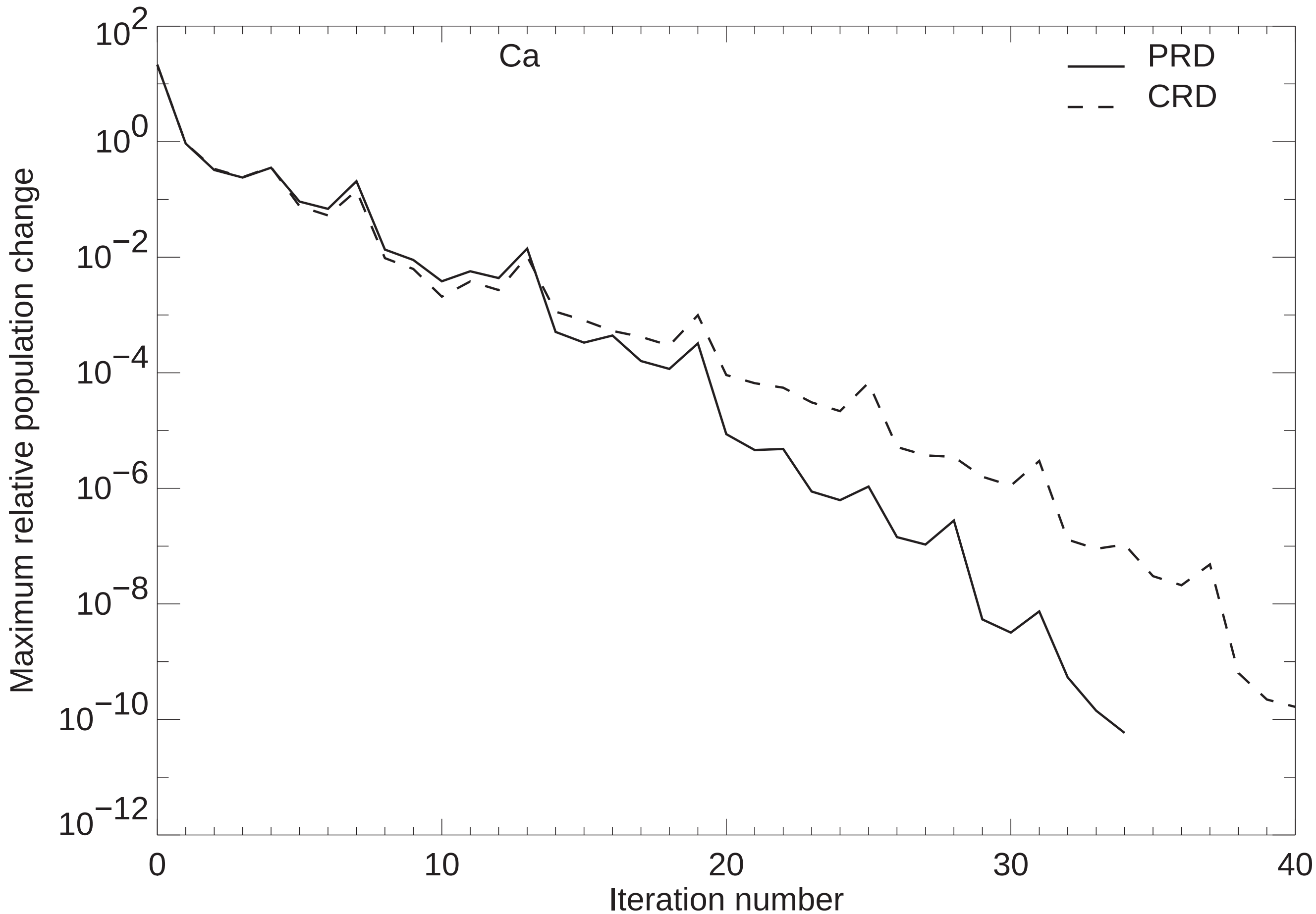
Convergence of Hydrogen atom in 1-D atmosphere



Convergence of Magnesium atom in 1-D atmosphere



Convergence of Calcium atom in 1-D atmosphere

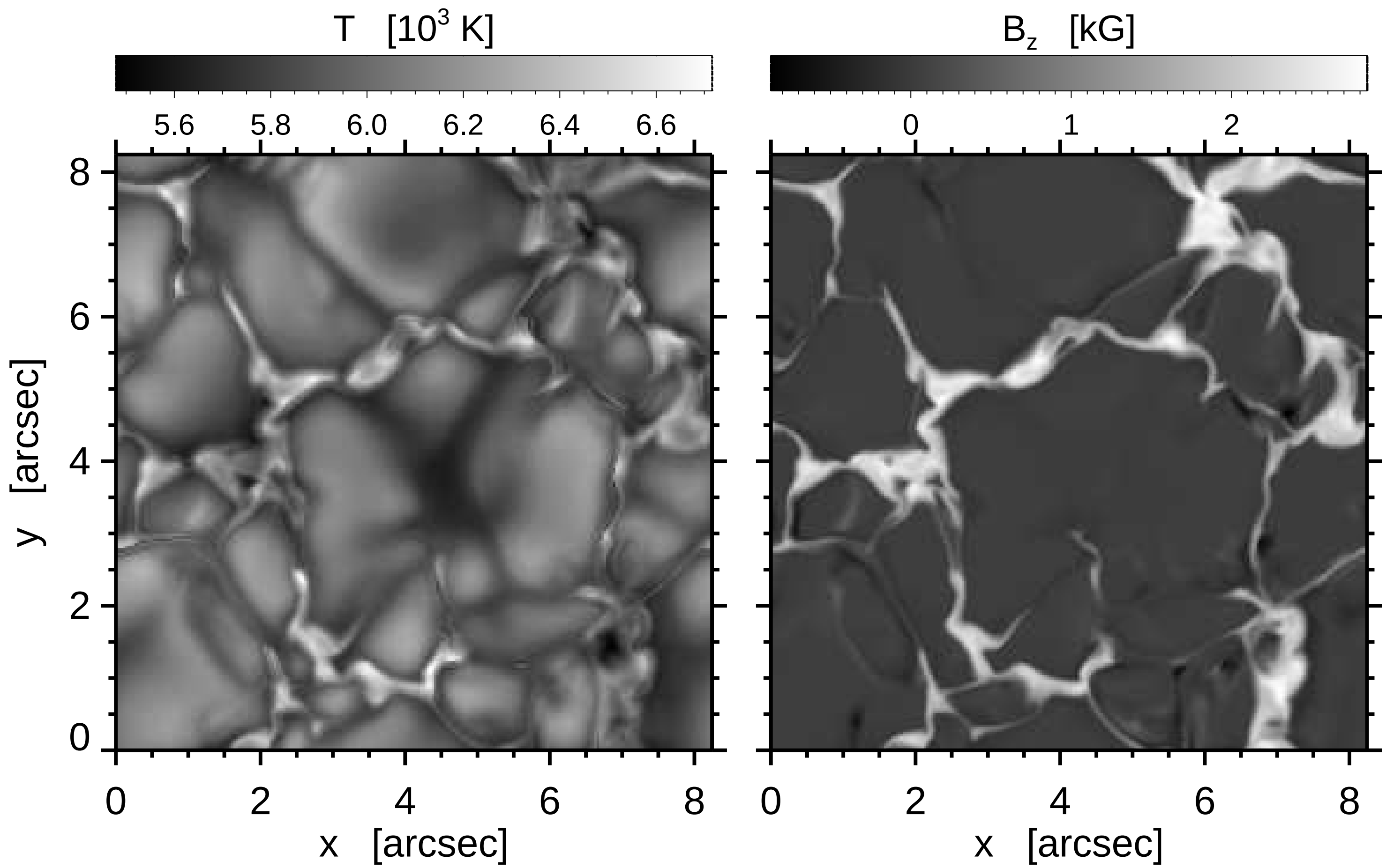


Reducing computation times with Multi-threading

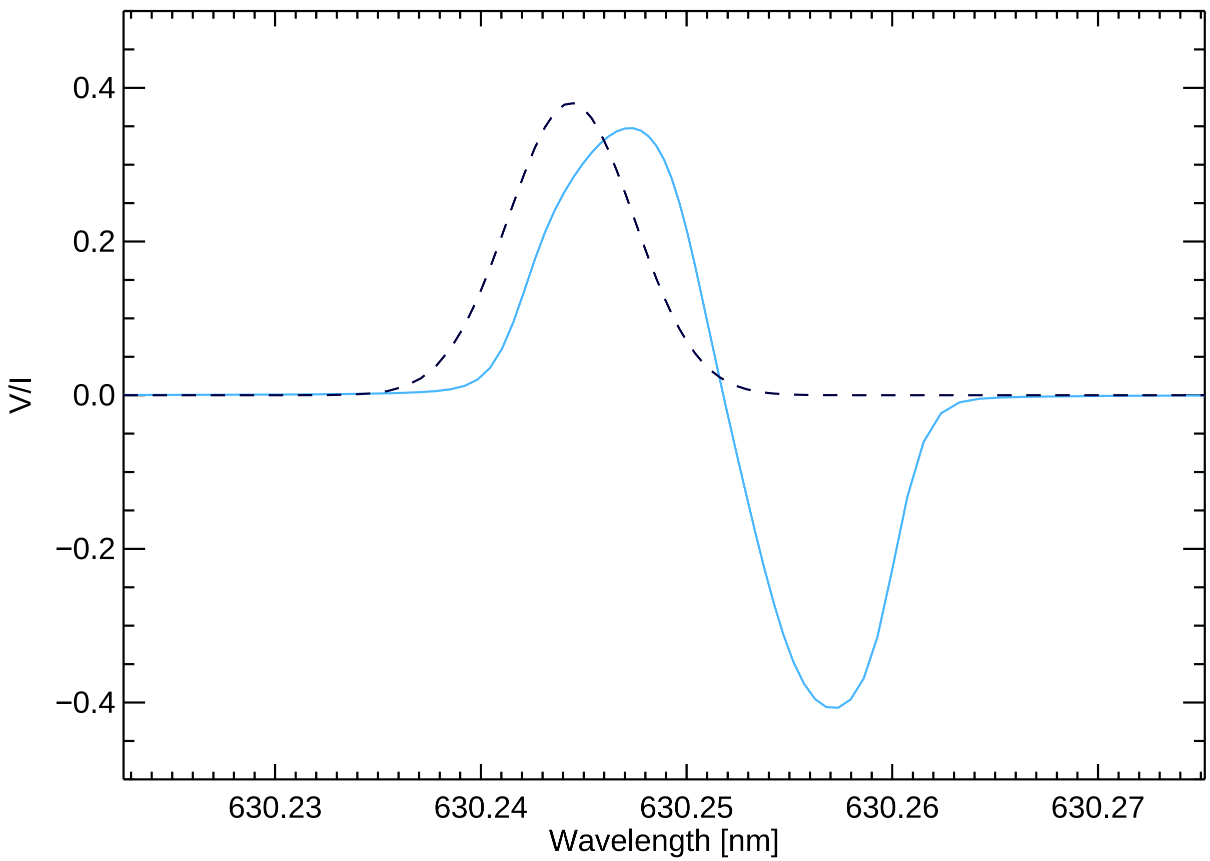
- Formal solutions at different wavelengths can be done in parallel if populations are given.
- We can exploit this by running multiple threads on a multi-processor machine to distribute the work.
- Use mutual exclusion (mutex) locks to lock wavelength-integrated quantities.
- This approach scales very well on multi-processor machines with shared memory architecture.

LOS Magnetograms

Three-dimensional Transfer in MHD simulation



SOUP filter pass band



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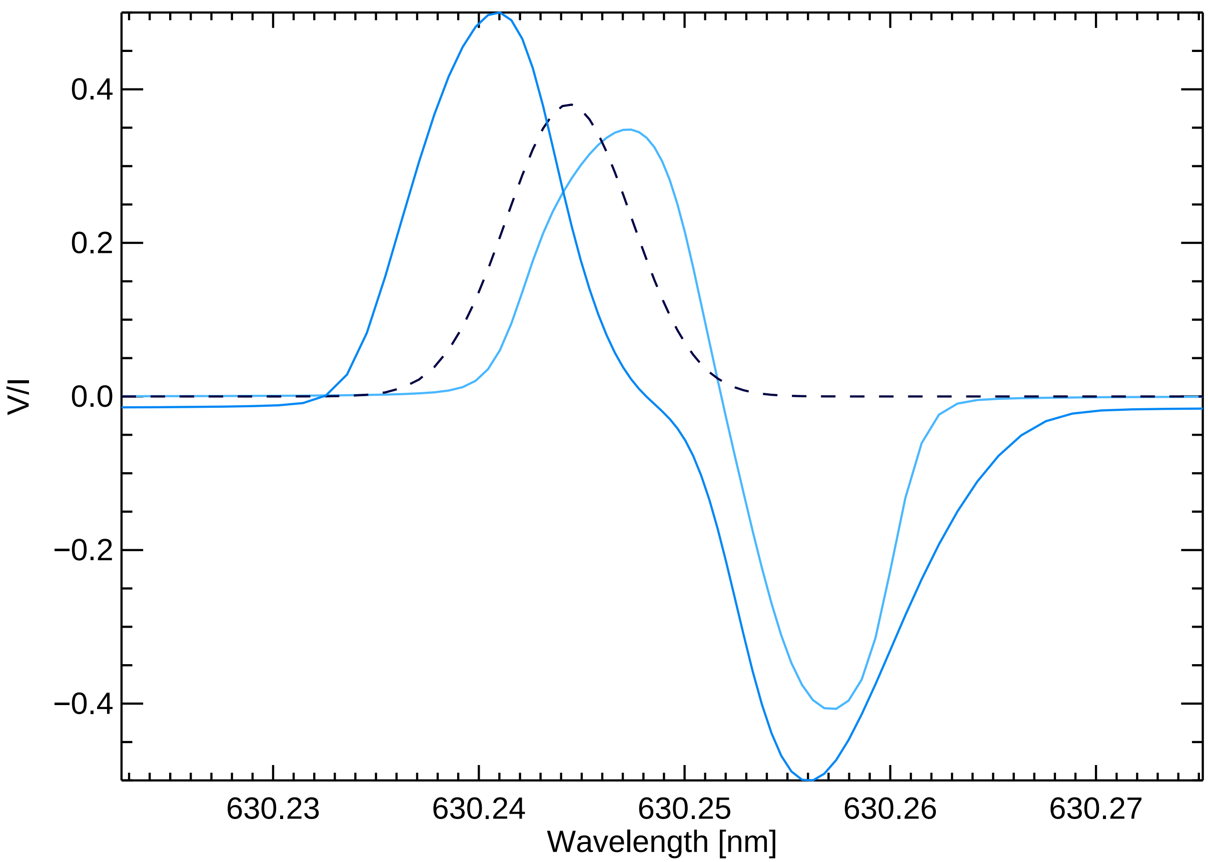
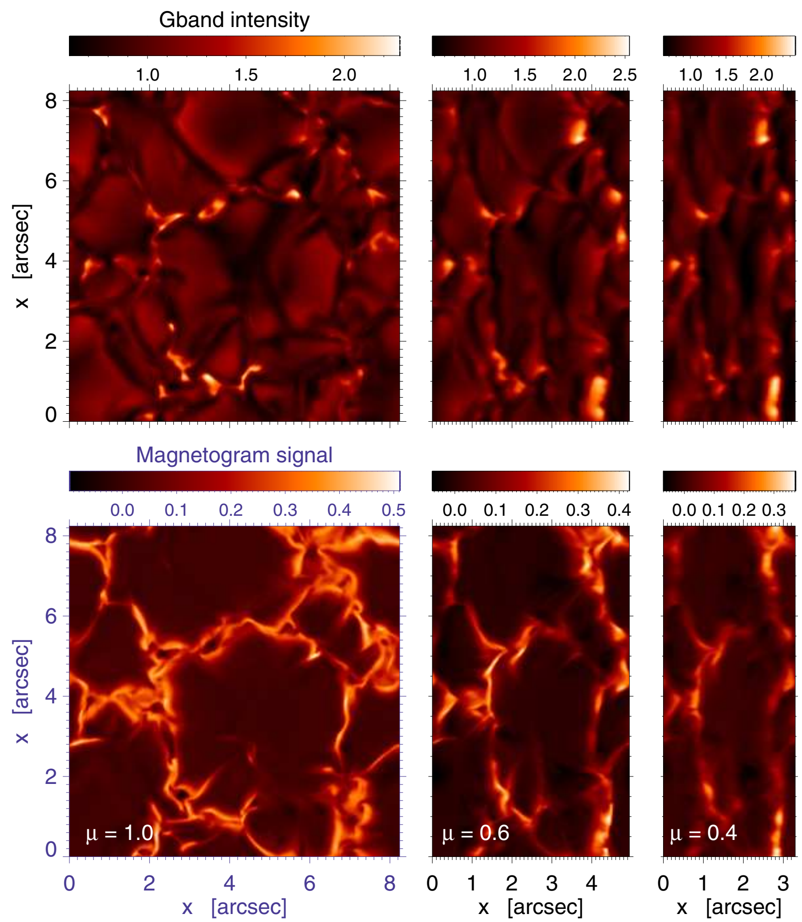


Comparing G-band imaging and SOUP magnetograms



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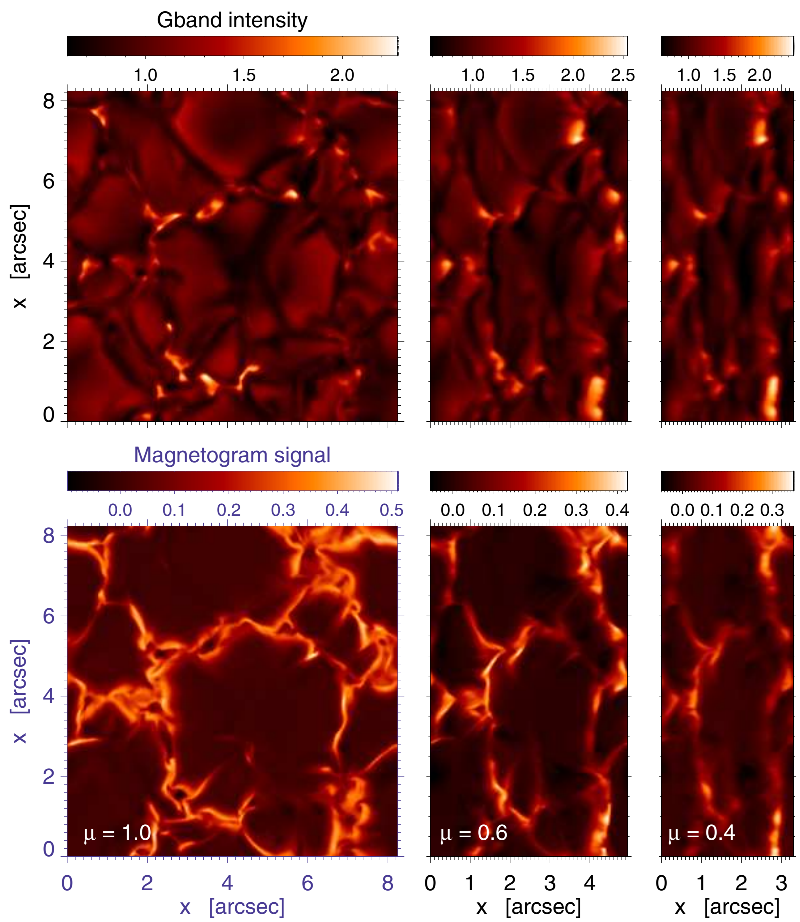
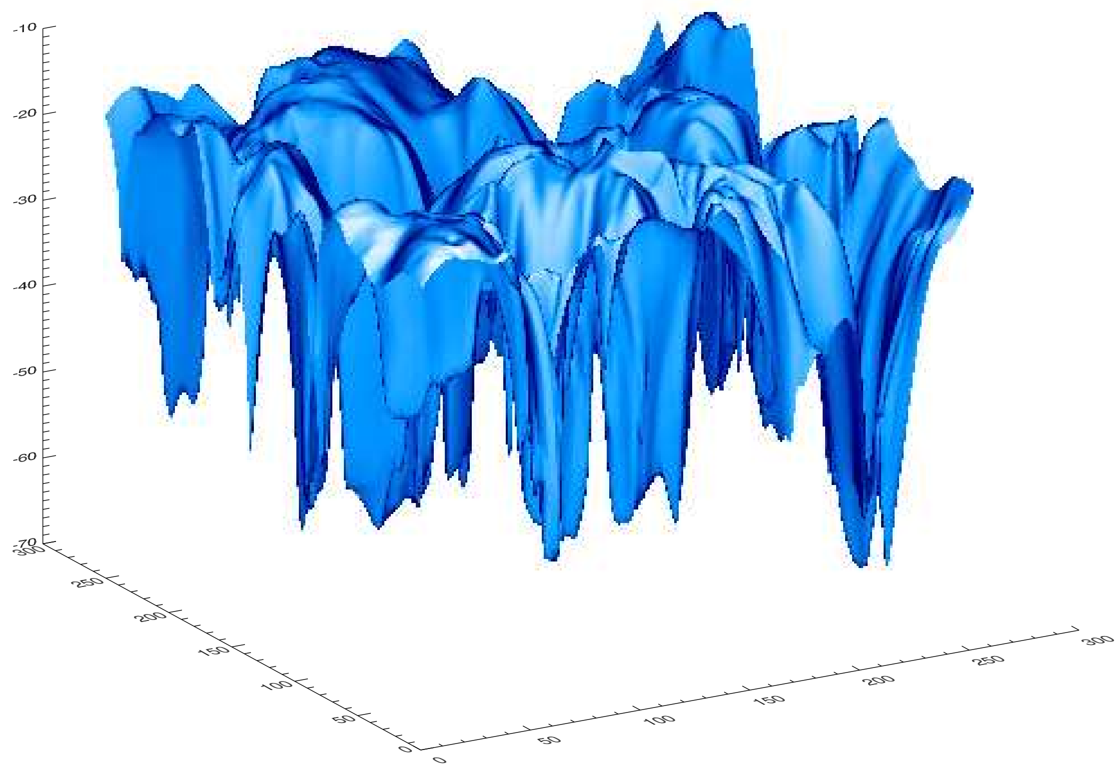
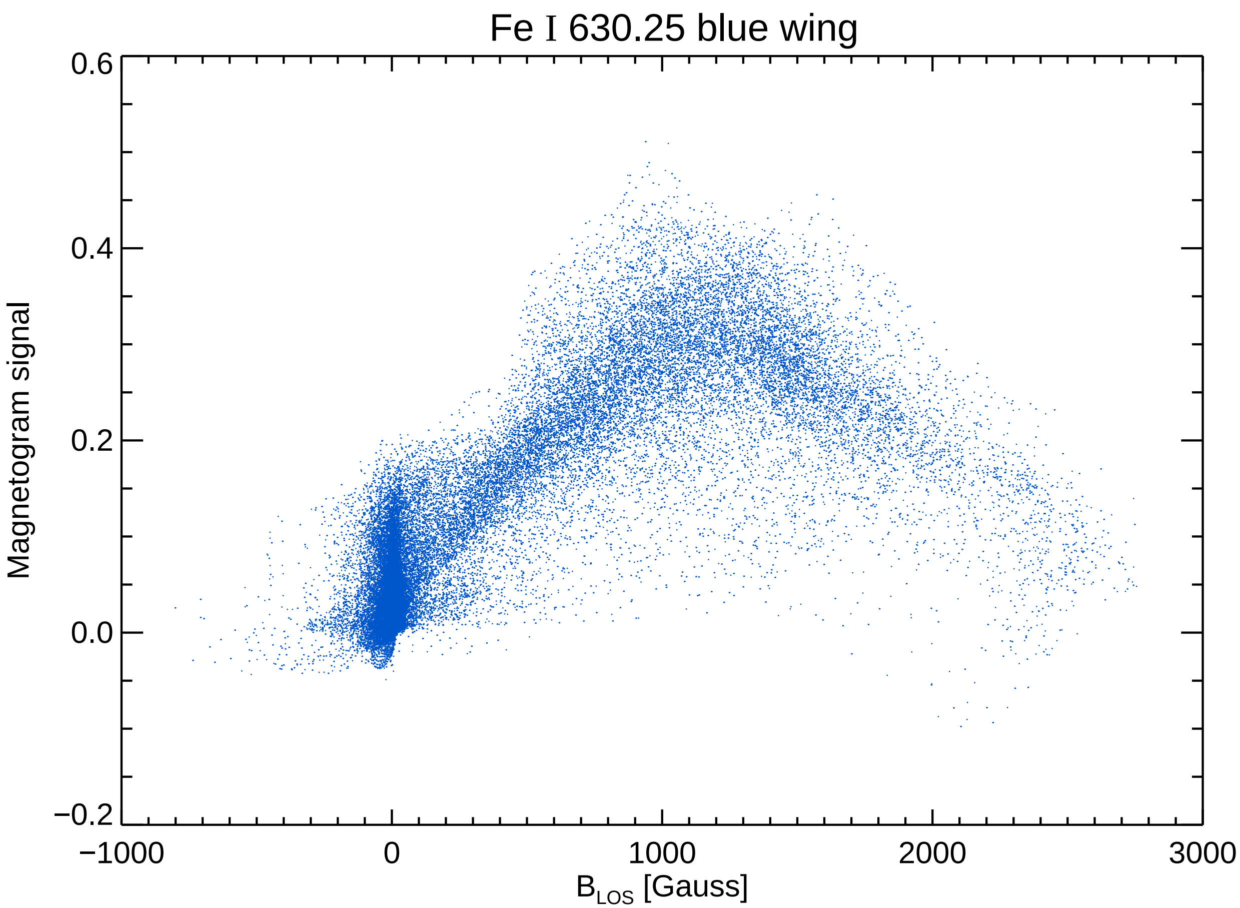


Variation in formation height of the Fe I 630.25 nm line



Can we measure B_{LOS} from magnetograms



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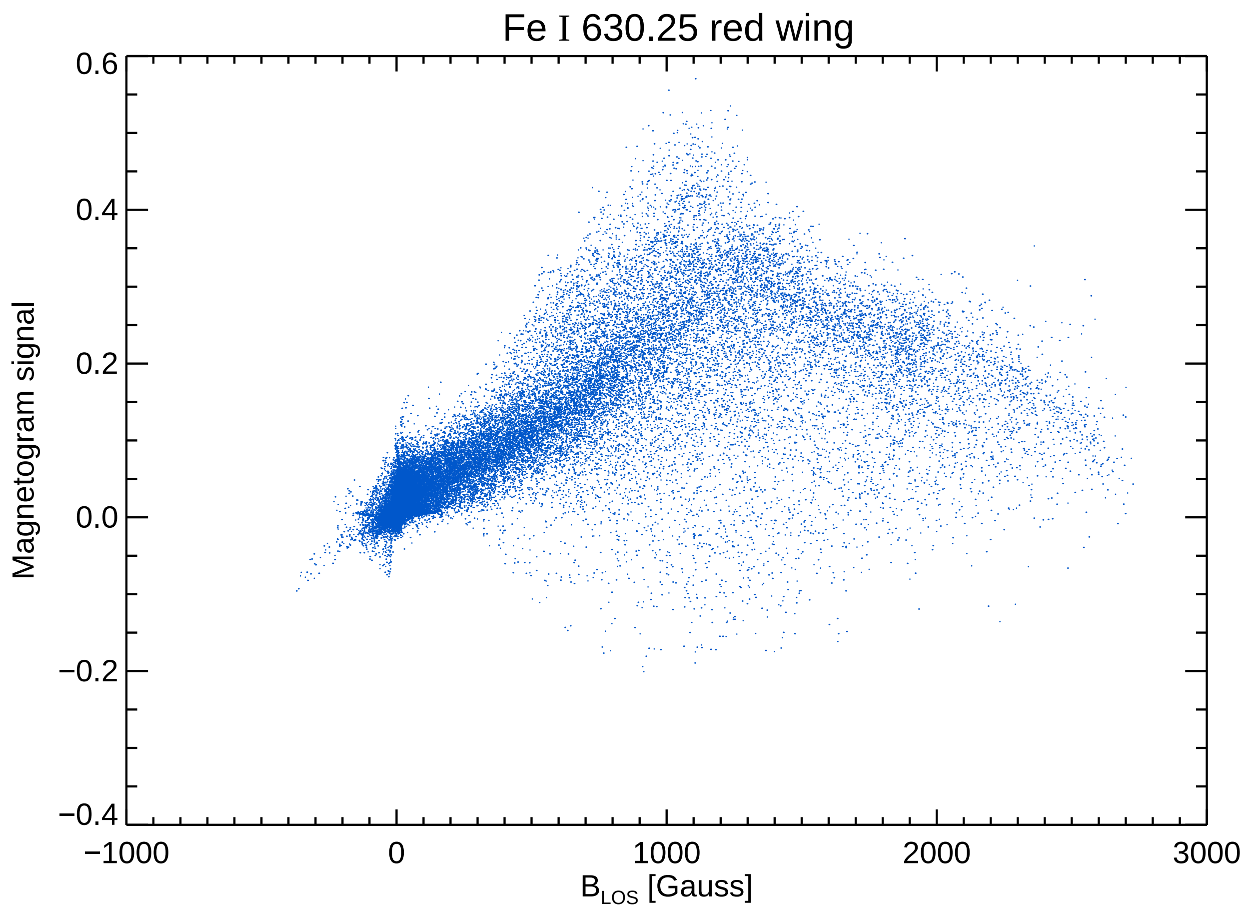

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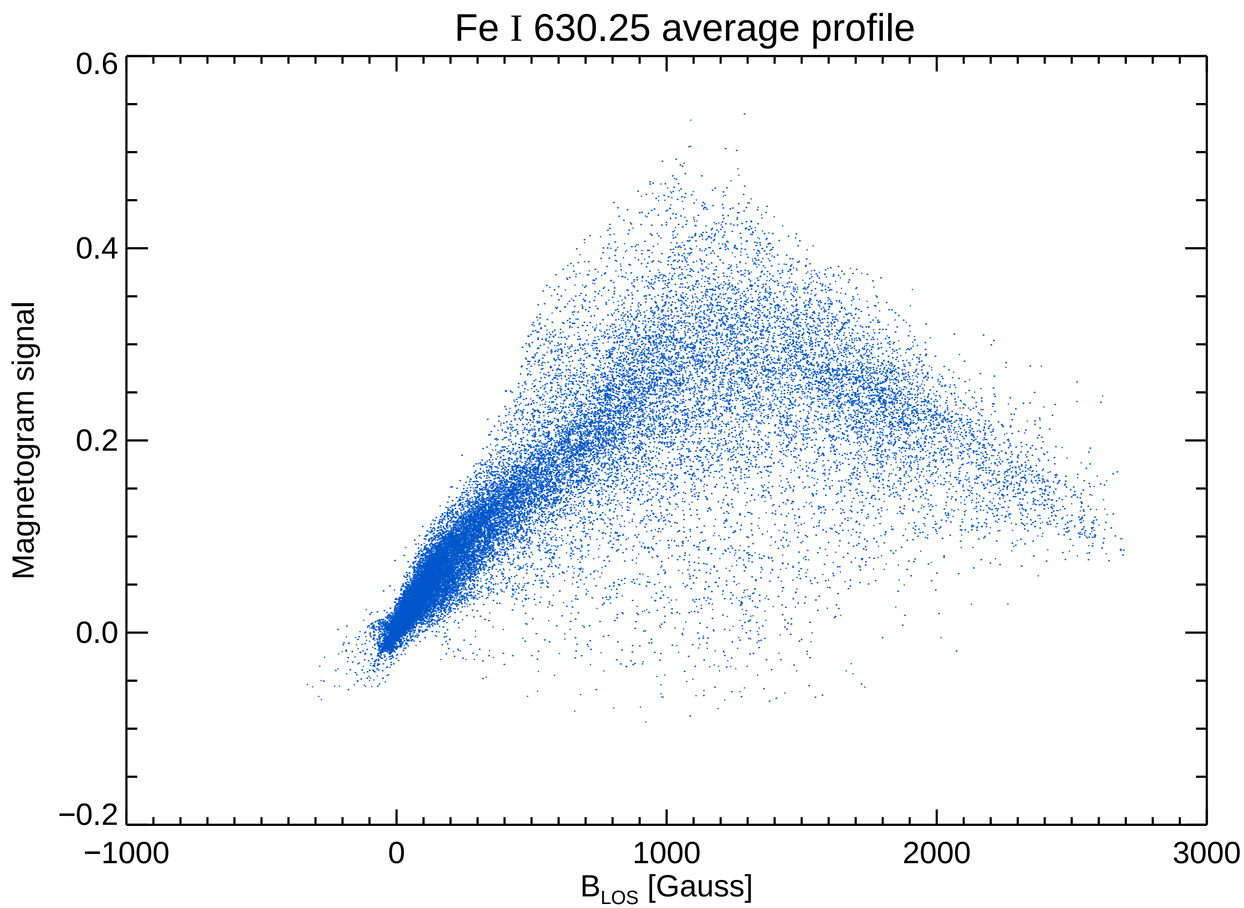
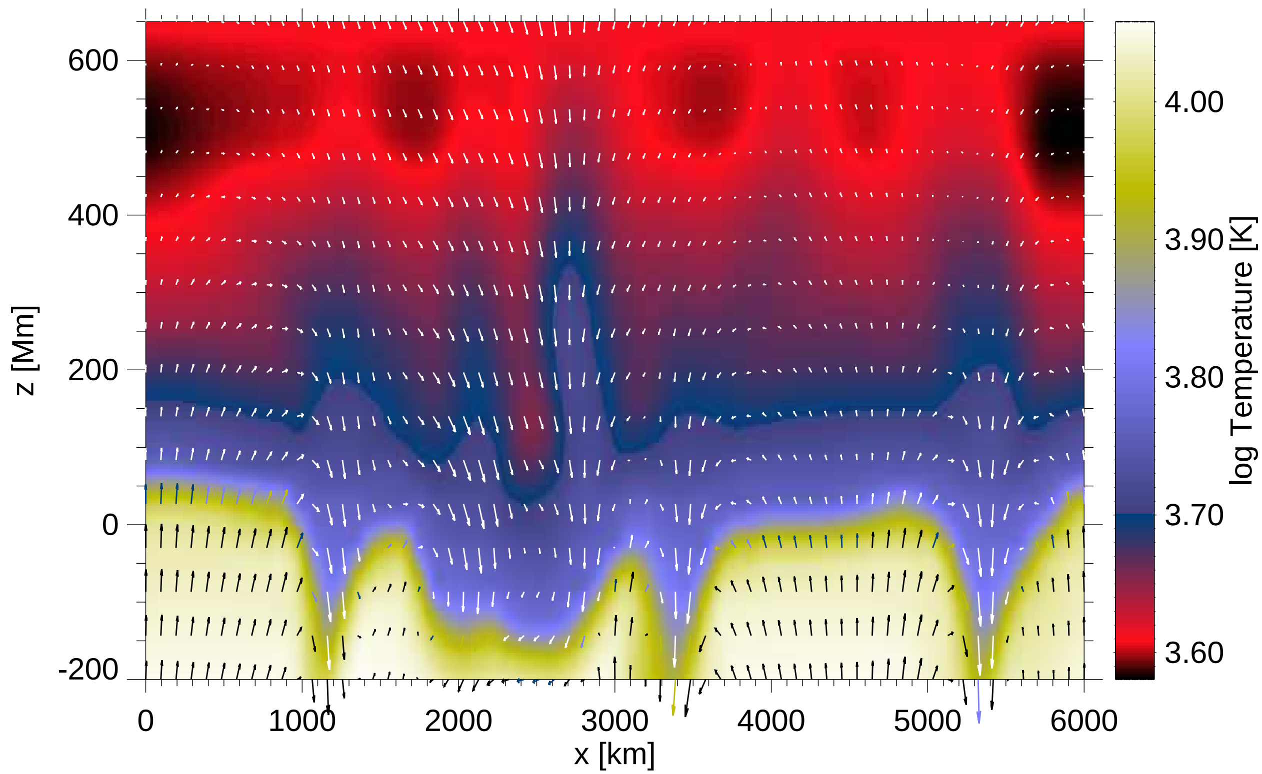


Lithium Abundance

Lithium abundance determination with inhomogeneities



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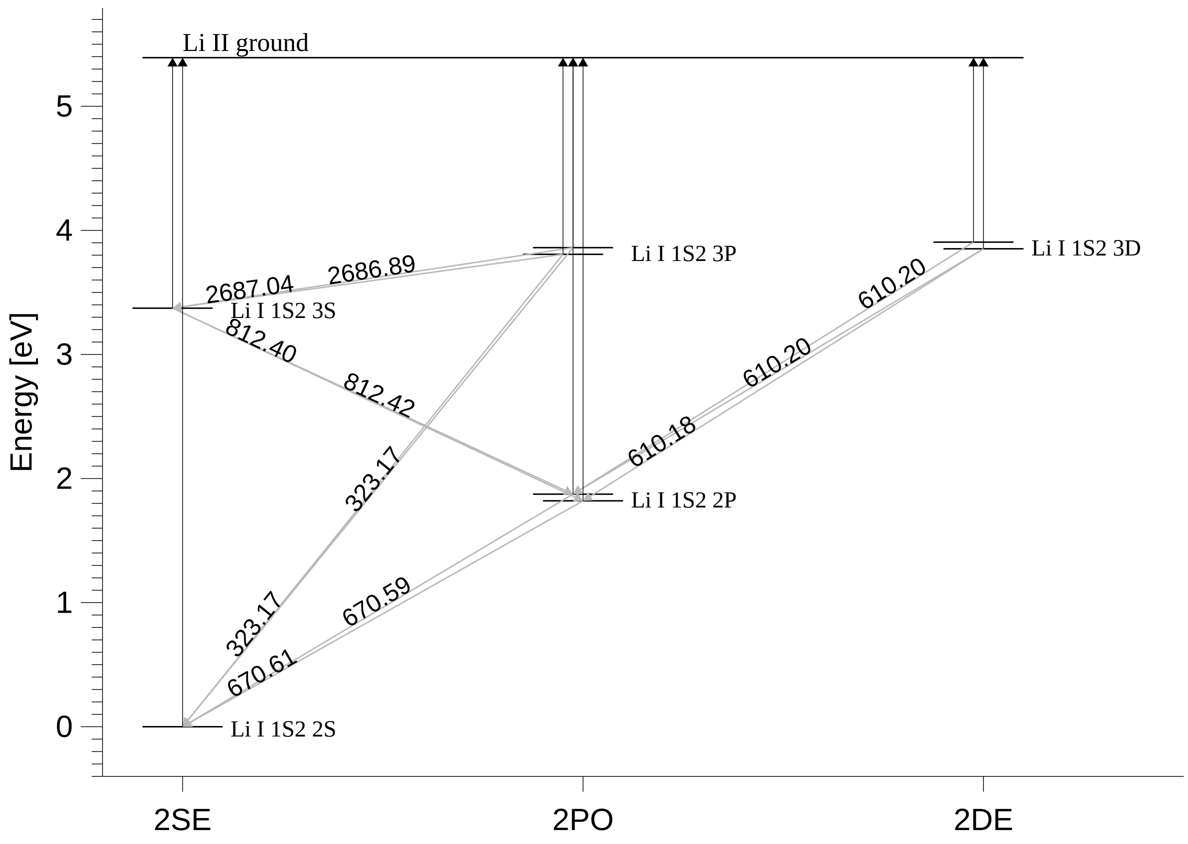

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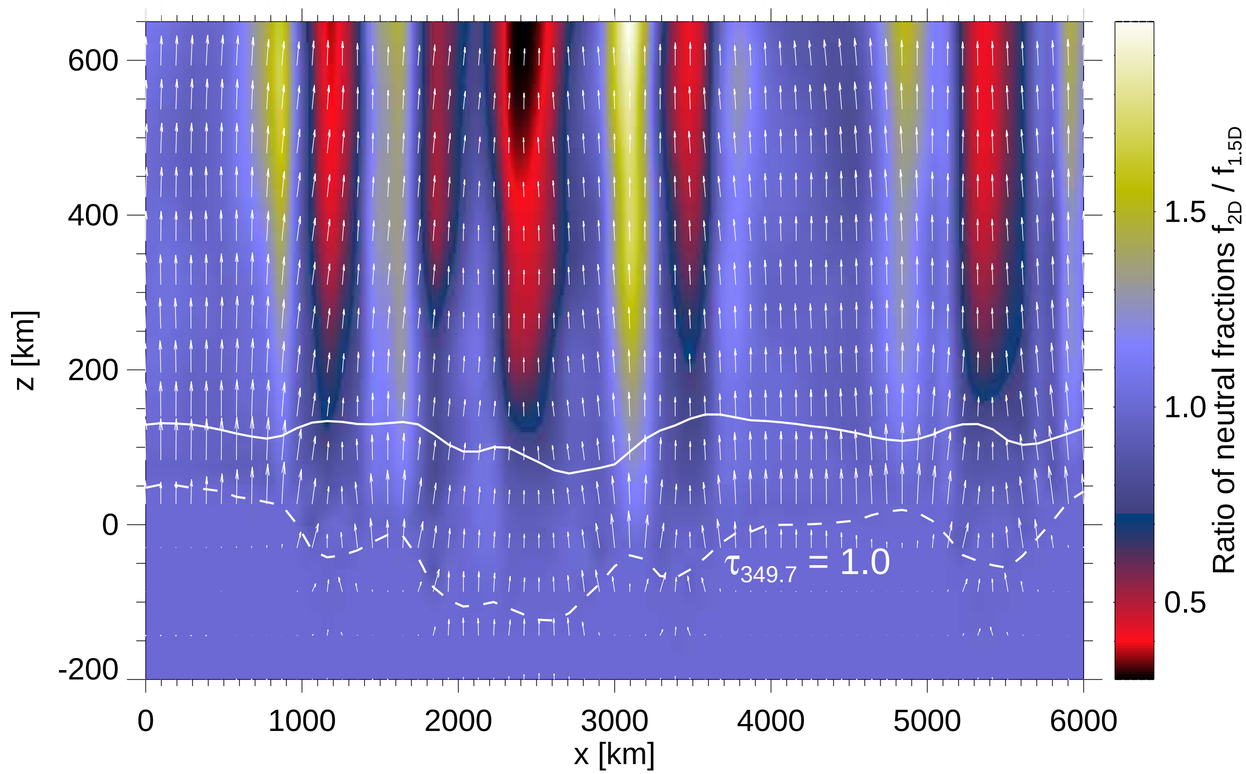

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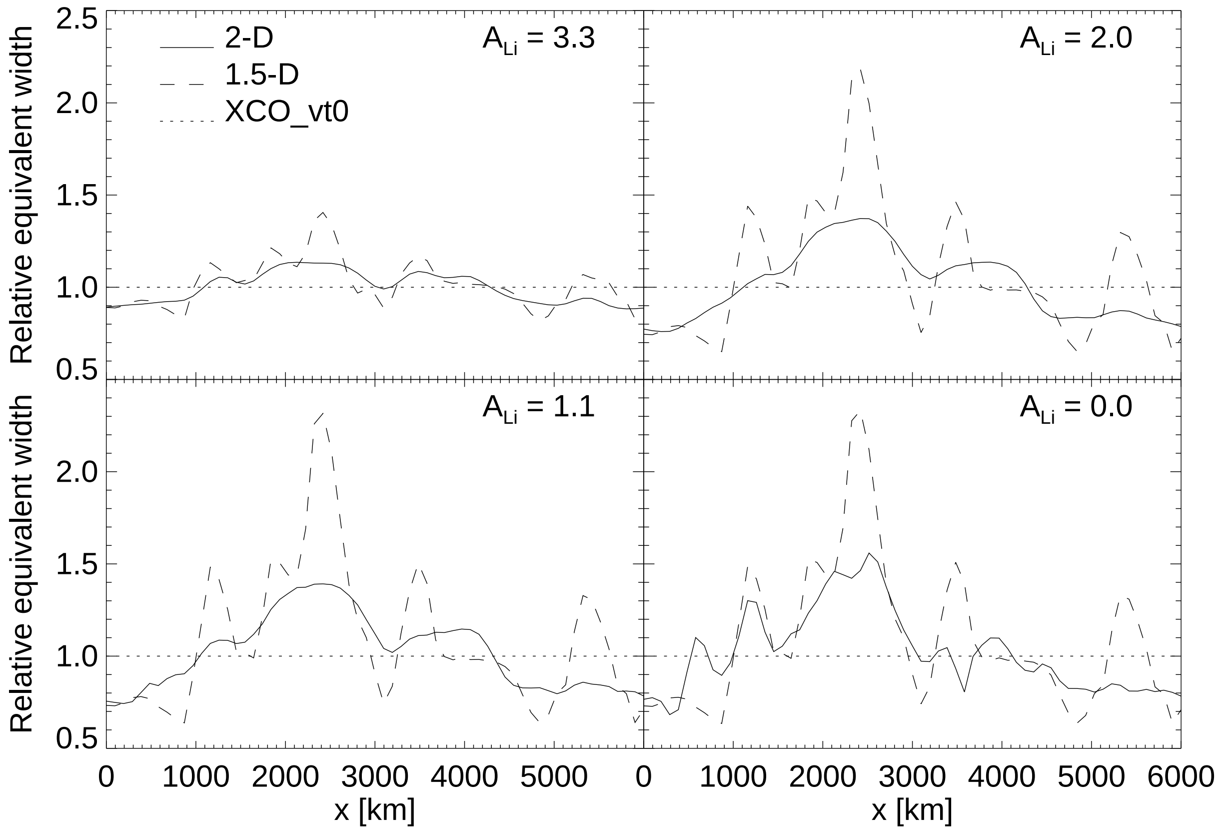
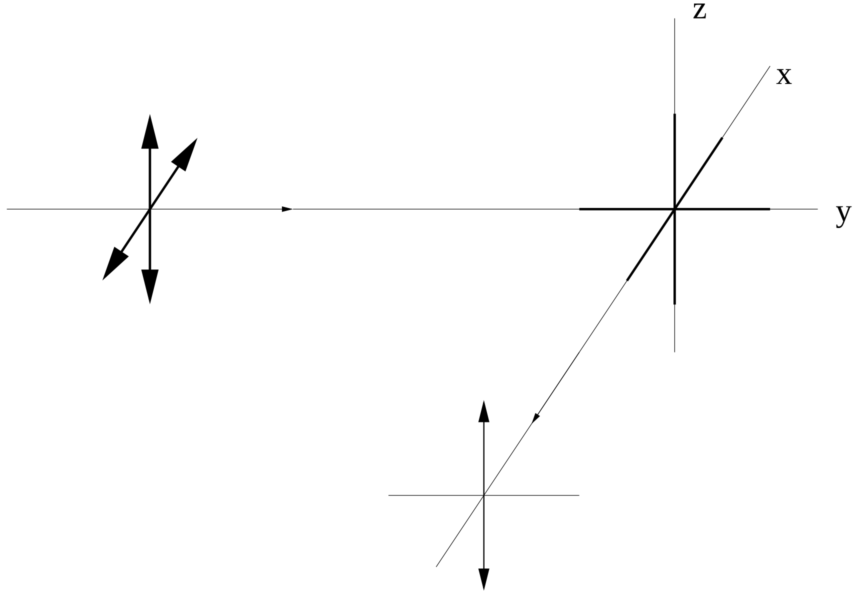


Continuum Polarization

Linear Polarization through Continuum Scattering

Thermal emission/absorption plus scattering: $\alpha = \alpha_c + \sigma_R + \sigma_T$



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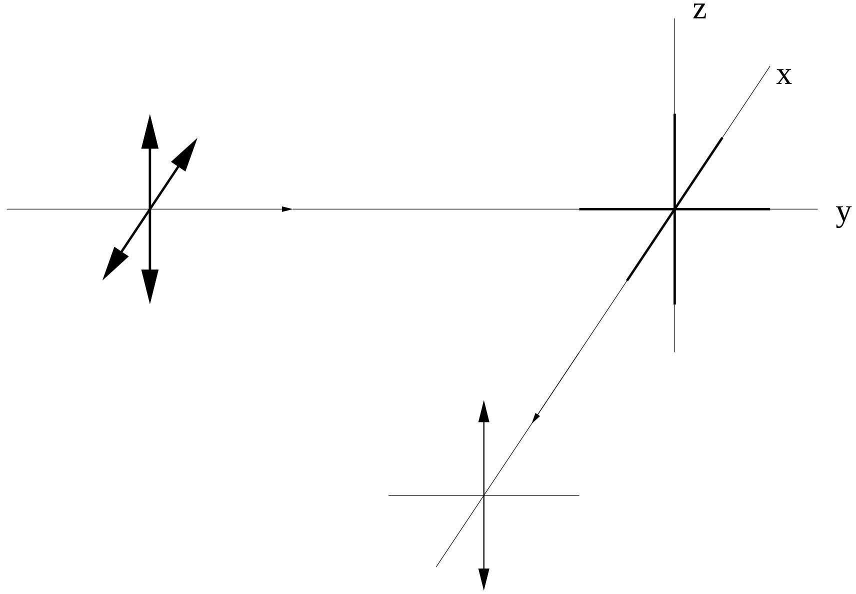


$$S_I = \frac{\sigma}{\alpha_c + \sigma} S_I^c + \frac{\alpha_c}{\alpha_c + \sigma} B$$

$$S_Q = \frac{\sigma}{\alpha_c + \sigma} S_Q^c$$

$$S_I^c = J_0^0 + \frac{1}{2\sqrt{2}} (3\mu^2 - 1) J_0^2$$

$$S_Q^c = \frac{3}{2\sqrt{2}} (\mu^2 - 1) J_0^2$$

The Radiation Field Tensors

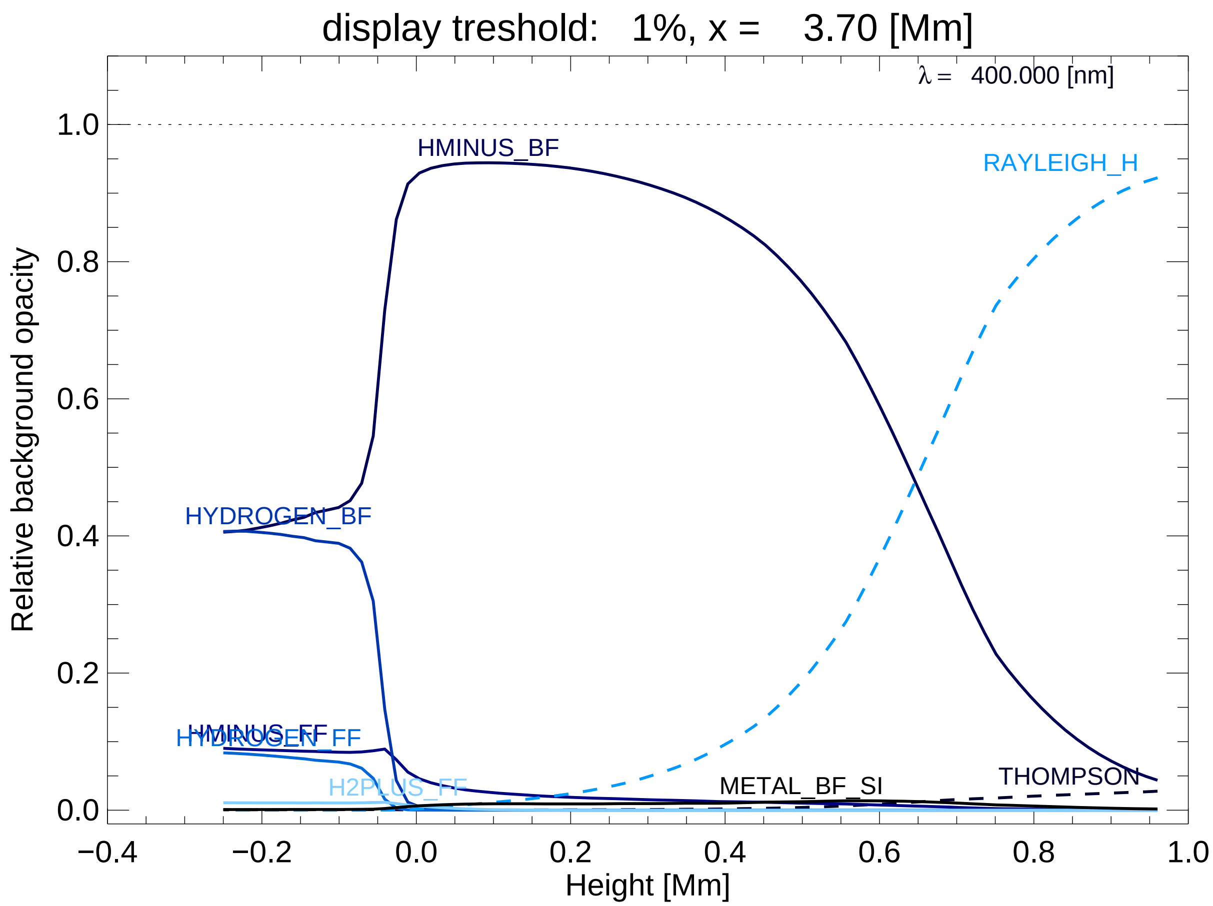
Angle-averaged mean intensity:

$$J_0^0 = \frac{1}{4\pi} \int \mathrm{d}\Omega \, I$$

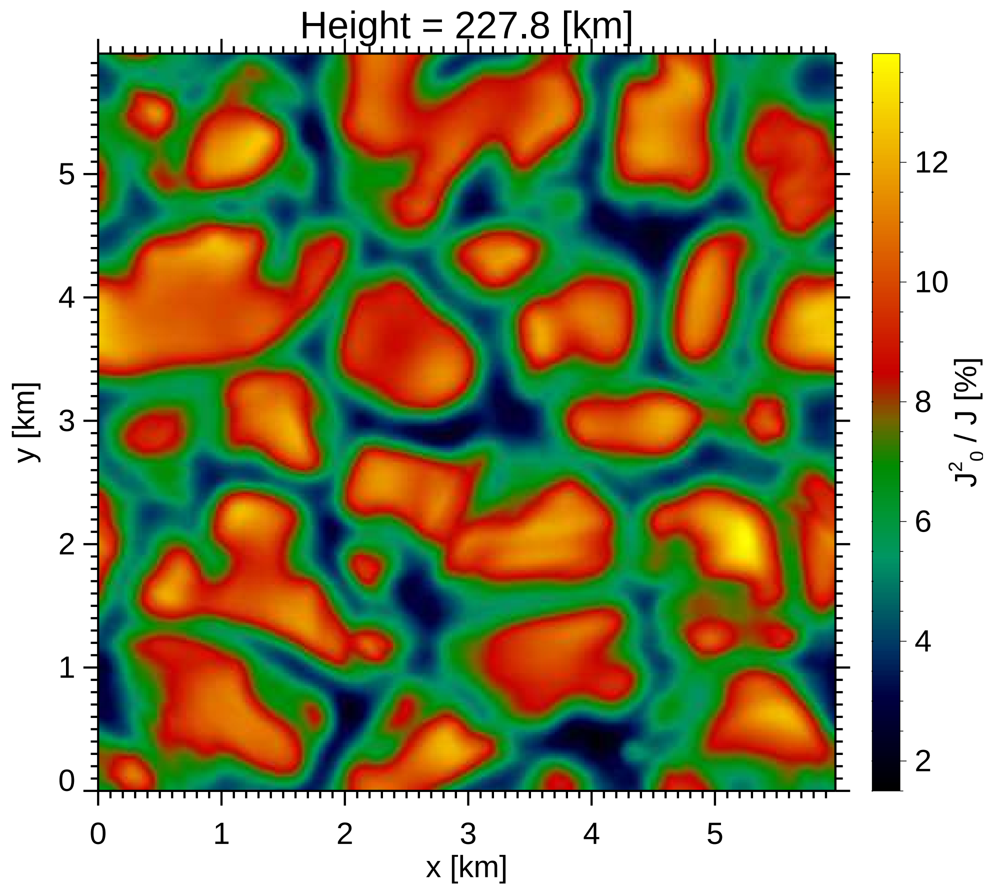
Second monochromatic Radiation Field Tensor:

$$J_0^2 = \frac{1}{4\pi} \frac{1}{2\sqrt{2}} \int \mathrm{d}\Omega \, \left[(3\mu^2 - 1) I + 3(\mu^2 - 1) Q \right]$$

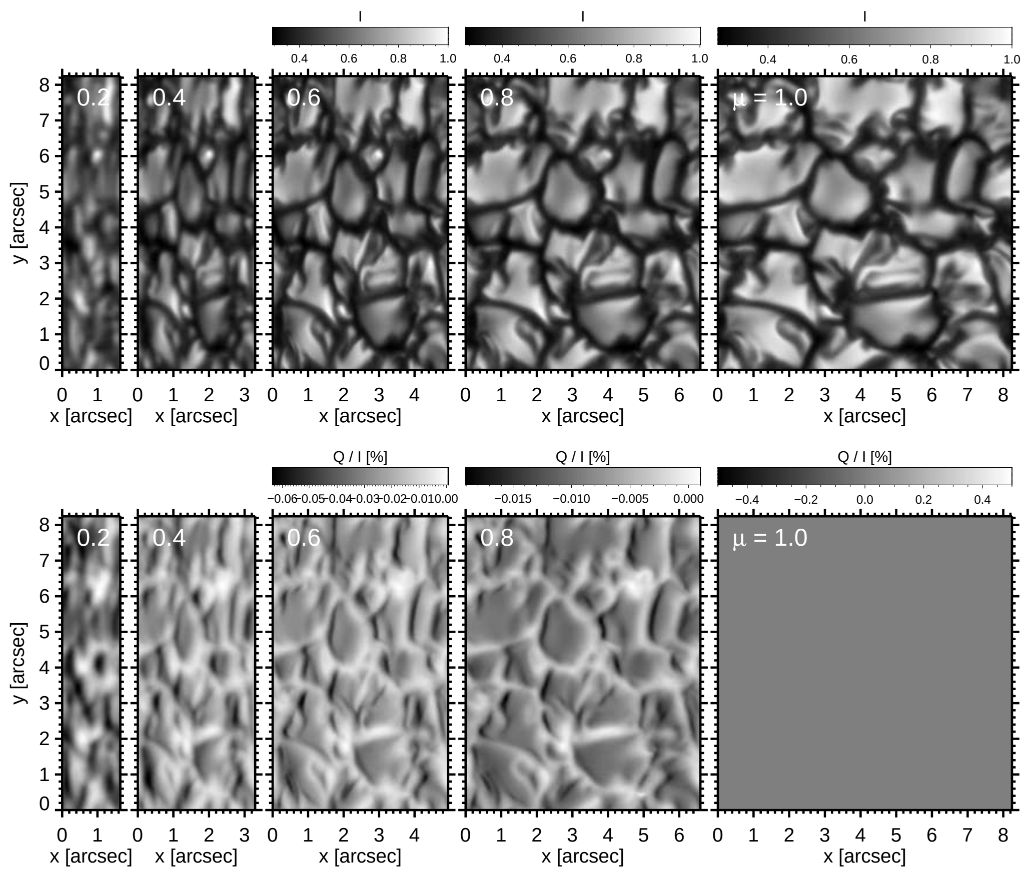
Relative Opacity Contributions in Granule



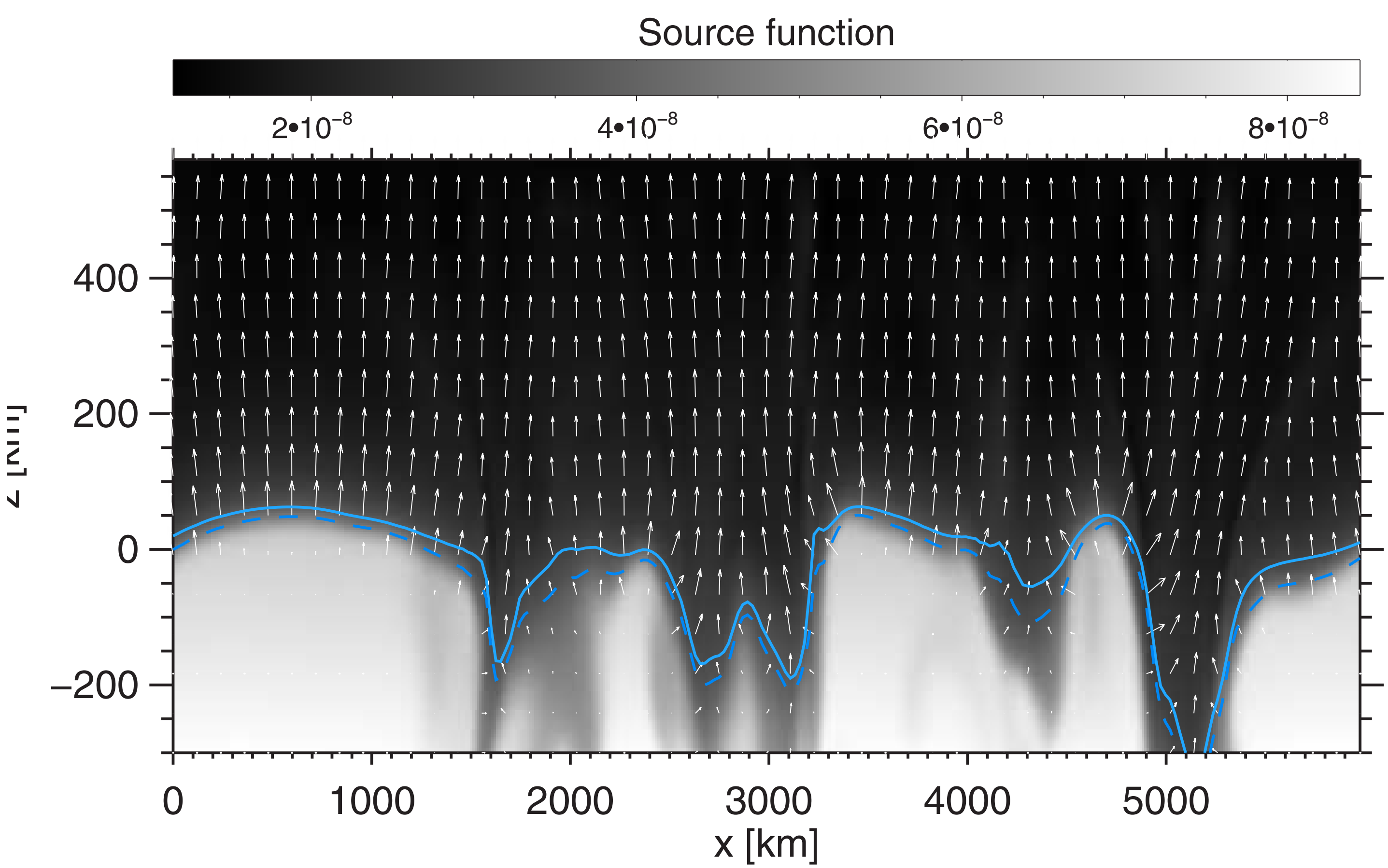
Anisotropy and continuum polarization



Radiation Anisotropy and continuum polarization



Horizontal radiative inflow



Conclusions

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Conclusions

- The solar atmosphere is really structured in three dimensions
- Relatistic radiative transfer serves both as diagnostic tool, interpreting observed radiation, and is needed to properly estimate radiative contributions to the energy budget
- However, exponential stratification imposes an anisotropy in the vertical direction on the radiation field. Only structures smaller than a few scale heights need to be treated with truly multi-dimensional radiative transport.
- Non-LTE multi-dimensional radiative transfer including polarization and partial redistribution is feasible on modest computers. Often effects can be studied very well in two dimensions.

Advertisement

The MALI-PRD code for 1-, 2-, 3-dimensional, and spherical geometry is avaliable from http://www.nso.edu/~uitenbr.

- Multi-level, overlapping lines.
- Molecular Non-LTE with "superlevels" (rotation-vibration lines).
- Zeeman Polarization.
- Extensive point-and-click IDL analysis routines.
- Short characteristics with monotonic interpolation.

Thank You