Accurate measurements and calibrations of the MICROSCOPE mission

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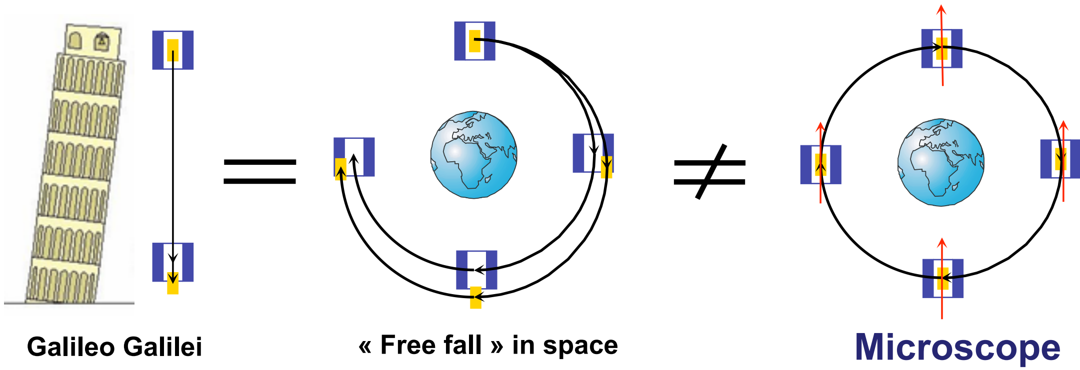
on behalf the MICRSCOPE Team

Testing the universality of free fall by means of differential accelerometers in space

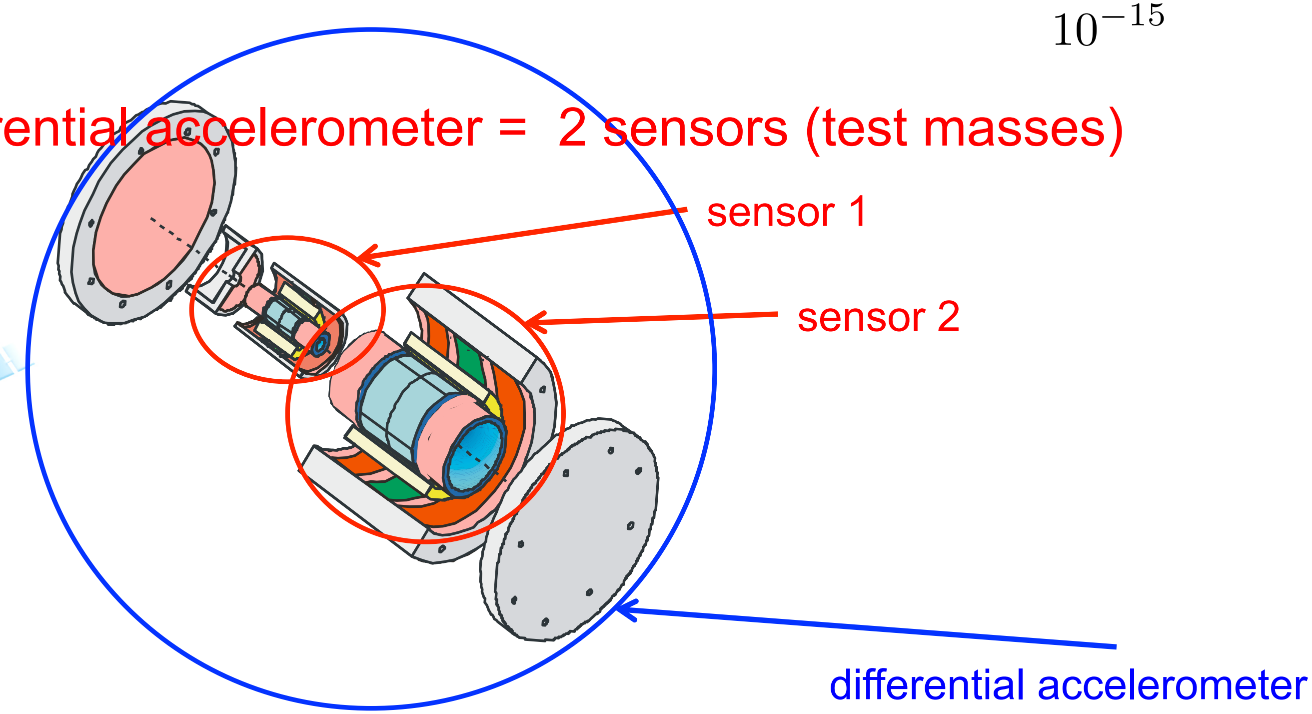


$$\delta_{1,2} = \delta_1 - \delta_2 = \frac{m_{G_1}}{m_{I_1}} - \frac{m_{G_2}}{m_{I_2}} \simeq \eta_{1,2}$$

$$\Downarrow$$

$$10^{-15}$$

1 differential accelerometer = 2 sensors (test masses)



Applied acceleration

$$\overrightarrow{\gamma}_i = -\,[\mathbf{T}]\,\overrightarrow{GO}_i$$ *gravity gradient*

$$+(\delta_S - \delta_i)\,\overrightarrow{g}$$ *EP violation*

$$+\,[\mathbf{In}]\,\overrightarrow{GO}_i + 2\,[\mathbf{\Omega}]\,\overset{\circ}{\overrightarrow{GO}}_i + \overset{\circ\circ}{\overrightarrow{GO}}_i$$ *inertial accel. and relative motion*

$$+\frac{\overrightarrow{F}}{M} + \overrightarrow{p}$$ *non grav. forces including propulsion*

$$-\frac{\overrightarrow{fp}_i}{m_i}$$ *NG perturbations on the test mass*

$$-\overrightarrow{g}_S(m_i)$$ *local gravity (satellite and the rest of the payload)*

$$-\sum_{j \neq i} \frac{\overrightarrow{fe}_j}{M}$$ *electrostatic forces on other proof mass*

« applied » accelerations…

… equilibrated by the measured « electostatic » acceleration

Differential acceleration between two test masses

@*fep* = *forb* + *fspin* in the instrument frame

$$
\begin{aligned}
2\vec{\gamma}^{(d)} = & \left([\mathbf{T}](O_{12}) - [\mathbf{In}]\right)\overrightarrow{O_1O_2} && \textit{gradients: gravity and inertia} \\
& +(\delta_2-\delta_1)\vec{g}(O_{12}) && \textit{EP violation} \\
& -2[\mathbf{\Omega}]\overset{\circ}{\overrightarrow{O_1O_2}} - \overset{\circ\circ}{\overrightarrow{O_1O_2}} && \textit{relative motion of the test masses} \\
& -2\vec{\gamma_p}^{(d)} - 2\vec{g}_S^{(d)} && \textit{differential perturbations on the masses}
\end{aligned}
$$

The potential EP violation signal is their but:

- We do not measure the difference of acceleration but we compute the difference of two measurements !
- Each of this measurement is affected by the sensor characteristics

Sensor model

- sensor (test mass) k
- theoretical acceleration (input): $\vec{\gamma}^{(k)}$
- measured acceleration (output): $\vec{\Gamma}^{(k)}$

$$\vec{\Gamma}^{(k)} = \vec{B}^{(k)} + [\mathbf{M}]^{(k)} [\mathbf{\Theta}]^{(k)} \vec{\gamma}^{(k)} + \vec{Q}^{(k)} + \left[\mathbf{N^{(k)}}\right] \dot{\vec{\Omega}}^{(k)}$$

Biais → DC part of the acceleration not known precisely

Scale factors + axis coupling (symmetric matrix)

Test mass rotation

Quadratic terms Negligible (will be checked by in flight calibration)

Angular to linear acceleration coupling Negligible because dΩ/dt≅0

$$[\mathbf{M}]^{(k)} [\mathbf{\Theta}]^{(k)} = [\mathbf{Id}] + [\mathbf{dA}]^{(k)}$$

The differential measured acceleration

$$
\begin{aligned}
2\Gamma_x^{(d)} &= 2B_x^{(d)} \\
&+ \delta_x g_x + \delta_y g_y + \delta_z g_z \\
&+ \Delta_x S_{xx} + \Delta_y S_{xy} + \Delta_z S_{xz} + (ac_{13}\Delta_y + ac_{12}\Delta_z) S_{yz} + ac_{12}\Delta_y S_{yy} + ac_{13}\Delta_z S_{zz} \\
&+ (-ac_{13}\Delta_y + ac_{12}\Delta_z + 2nd_{11})\,\dot{\Omega}_x - (\Delta_z - 2ac_{13}\Delta_x + 2nd_{12})\,\dot{\Omega}_y + (\Delta_y - 2ac_{12}\Delta_x + 2nd_{13})\,\dot{\Omega}_z \\
&+ 2\left(-ac_{13}\dot{\Delta}_y + ac_{12}\dot{\Delta}_z\right)\Omega_x - 2\left(\dot{\Delta}_z - 2ac_{13}\dot{\Delta}_x\right)\Omega_y + 2\left(\dot{\Delta}_y - 2ac_{12}\dot{\Delta}_x\right)\Omega_z \\
&- mc_{11}\ddot{\Delta}_{x,inst} - mc_{12}\ddot{\Delta}_{y,inst} - mc_{13}\ddot{\Delta}_{z,inst} \\
&+ 2\left(ad_{11}\Gamma_x^{(c)} + ad_{12}\Gamma_y^{(c)} + ad_{13}\Gamma_z^{(c)}\right) \\
&+ K_{2xx}^{(1)}\left(\frac{\Gamma_x^{(1)} - {b_{0x}}^{(1)}}{{K_{1x}}^{(1)}}\right)^2 - K_{2xx}^{(2)}\left(\frac{\Gamma_x^{(2)} - {b_{0x}}^{(2)}}{{K_{1x}}^{(2)}}\right)^2
\end{aligned}
$$

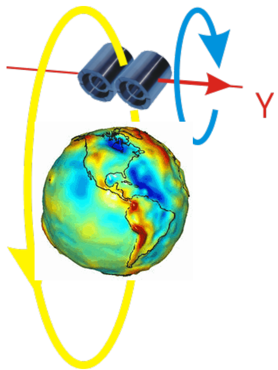
estimated by calibration
observed or/and computed
negligible at Fep

Target: 10^{-15} accuracy ⇔ 8 10^{-15} ms^{-2} in acceleration

The fep frequency

- The EP signal, collinear to the gravity vector, follows the direction of the Earth centre, seen from the satellite.

- This direction rotates at the orbital frequency and,
  if the satellite is not rotating, the EP signal has the frequency
  fep = forb which will be well determined



- If the satellite rotates, the signal is modulated
  and the EP signal has the frequency *fep = forb + fspin*

- This fact is used to
  - Optimize the accuracy of the experiment around the *fep* frequency
  - To discriminate the EP signal from some other perturbing signals

Needs for calibration

Gravity gradient

Terme perturbateur	Paramètre limitant la précision de l'estimation	Contribution à la mesure EP à f_{EP} (ms^{-2})	Impact sur l'estimation de δ_{EP}
$\frac{1}{2}a_{c11} \cdot T_{xx} \cdot \Delta_x$	$a_{c11} \cdot \Delta_x < 20,2\,\mu m$	$4,2 \cdot 10^{-14}$	$10,6 \cdot 10^{-15}$
$\frac{1}{2}a_{c11} \cdot T_{xz} \cdot \Delta_z$	$a_{c11} \cdot \Delta_z < 20,2\,\mu m$	$4,3 \cdot 10^{-14}$	$10,9 \cdot 10^{-15}$
$\frac{1}{2}a_{c11} \cdot T_{xy} \cdot \Delta_y$	$a_{c11} \cdot \Delta_y < 20,2\,\mu m$	$3,0 \cdot 10^{-16}$	$0,08 \cdot 10^{-15}$
$\frac{1}{2}a_{c12} \cdot T_{yy} \cdot \Delta_y$	$a_{c12} < 2,6 \cdot 10^{-3}$ rad $\Delta_y < 20\,\mu m$	$4,4 \cdot 10^{-16}$	$0,11 \cdot 10^{-15}$
$\frac{1}{2}a_{c13} \cdot T_{zz} \cdot \Delta_z$	$a_{c13} < 2,6 \cdot 10^{-3}$ rad $\Delta_z < 20\,\mu m$	$3,4 \cdot 10^{-16}$	$0,09 \cdot 10^{-15}$
$a_{d11} \cdot \Gamma_{res_{df},x}$	$a_{d11} < 10^{-2}$	$1,0 \cdot 10^{-14}$	$2,5 \cdot 10^{-15}$
$a_{d12} \cdot \Gamma_{res_{df},y}$	$a_{d12} < 1,6 \cdot 10^{-3}$ rad	$1,5 \cdot 10^{-15}$	$0,38 \cdot 10^{-15}$
$a_{d13} \cdot \Gamma_{res_{df},z}$	$a_{d13} < 1,6 \cdot 10^{-3}$ rad	$1,5 \cdot 10^{-15}$	$0,38 \cdot 10^{-15}$
$2 \cdot K_{2,cxx} \cdot \Gamma_{app,dx} \cdot \Gamma_{res_{df},x}$	$K_{2,cxx} < 14000\,s^2/m$	$4,0 \cdot 10^{-16}$	$0,10 \cdot 10^{-15}$
$K_{2,dxx} \cdot (\Gamma^2_{app,dx} + \Gamma^2_{res_{df},x})$	$K_{2,dxx} < 14000\,s^2/m$	$4,0 \cdot 10^{-16}$	$0,10 \cdot 10^{-15}$
Total		$2 \cdot 10^{-13}$	$25 \cdot 10^{-15}$

Auxiliary data required

g : gravity acceleration
at the instrument position
projected in the instrument frame

S : gravity gradient projected in the instrument frame
+ symetric part of the gradient of inertia

Ω : angular velocity

Ω : angular acceleration

Requires a gravity model (mainly Earth)

Requires the instrument position

Requires the instrument orientation

Orbite restitution

$$T_{ij}\Delta_j = \frac{\partial^2 V_g}{\partial x_i \partial x_j}\Delta_j$$ must be substracted to the measured acceleration

Motivation:

$$\text{Err}\,(T_{ij}\Delta_j) = T_{ij}\text{Err}(\Delta_j) + \frac{\partial^3 V_g}{\partial x_i \partial x_j \partial x_k}\Delta_j \text{Err (position)}$$

$\Delta_j = O(20\mu m)$ and frequency considerations lead to the specifications :

Frequency	*Radial*	*Tangent*	*Normal*
DC	$100\ m$	$100\ m$	$2\ m$
f_{ep}	**7 m**	**14 m**	$100\ m$
$2\ f_{ep}$	$100\ m$	$100\ m$	$2\ m$
$3\ f_{ep}$	$2\ m$	$2\ m$	$100\ m$

Not too stringent, in principle, with GPS but…
… the computation must take into account the thrust on the satellite commended by the drag-free system.

Performance evaluation ➔ no problem

Attitude

Motivation:

- Projection of the gravity gradient in the instrument frame ➔ attitude [A]
- Computation of the gradient of inertia ➔ [In] =[A] [Ä]^T=d/dt([Ω])+[Ω] [Ω]

$$\dot{\vec{\Omega}}_{@\mathrm{Fep}} < 1.10^{-11}\ \mathrm{rd/s^2}\quad \text{(inertial \& rotating modes)}$$

$$\vec{\Omega}_{@\mathrm{Fep}} < 1.10^{-9}\ \mathrm{rd/s}\quad \text{(rotating mode)}$$

$$\text{so that : } \dot{\vec{\Omega}} \wedge \vec{\Delta} \text{ and } \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{\Delta}) \leq 2.10^{-16}\,\mathrm{m/s^2}$$

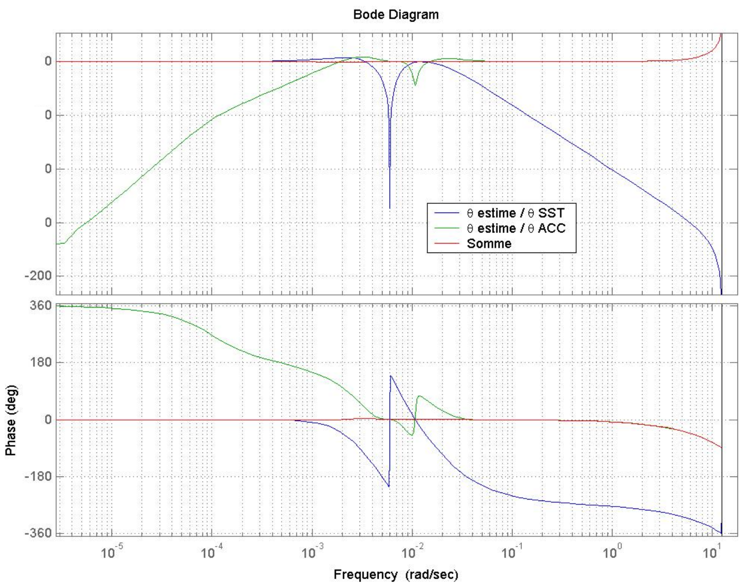
1.10^{-9}rd/s @Fepr is equivalent to an attitude stability of the instrument better than 0.16µrad

This is a real challenge

Solution (courtesy P. Prieur, CNES)

1 : on board attitude and angular acceleration control

Hybridation of
the star trackers (low frequencies) and
the accelorementer (high frequencies)



Red : real time attitude performance
Blue : a posteriori attitude knowledge performance
Above : versus time over 20 orbits
Under : versus frequency

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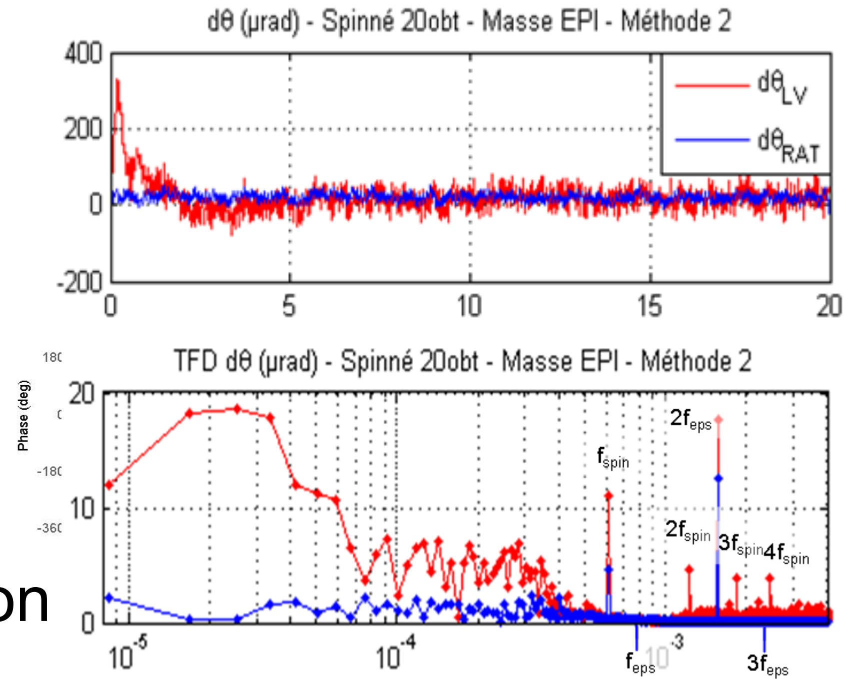

2 : on ground attitude restitution

Hybridation 'frequency by frequency'

Red : real time attitude performance
Blue : a posteriori attitude knowledge performance
Above : versus time over 20 orbits
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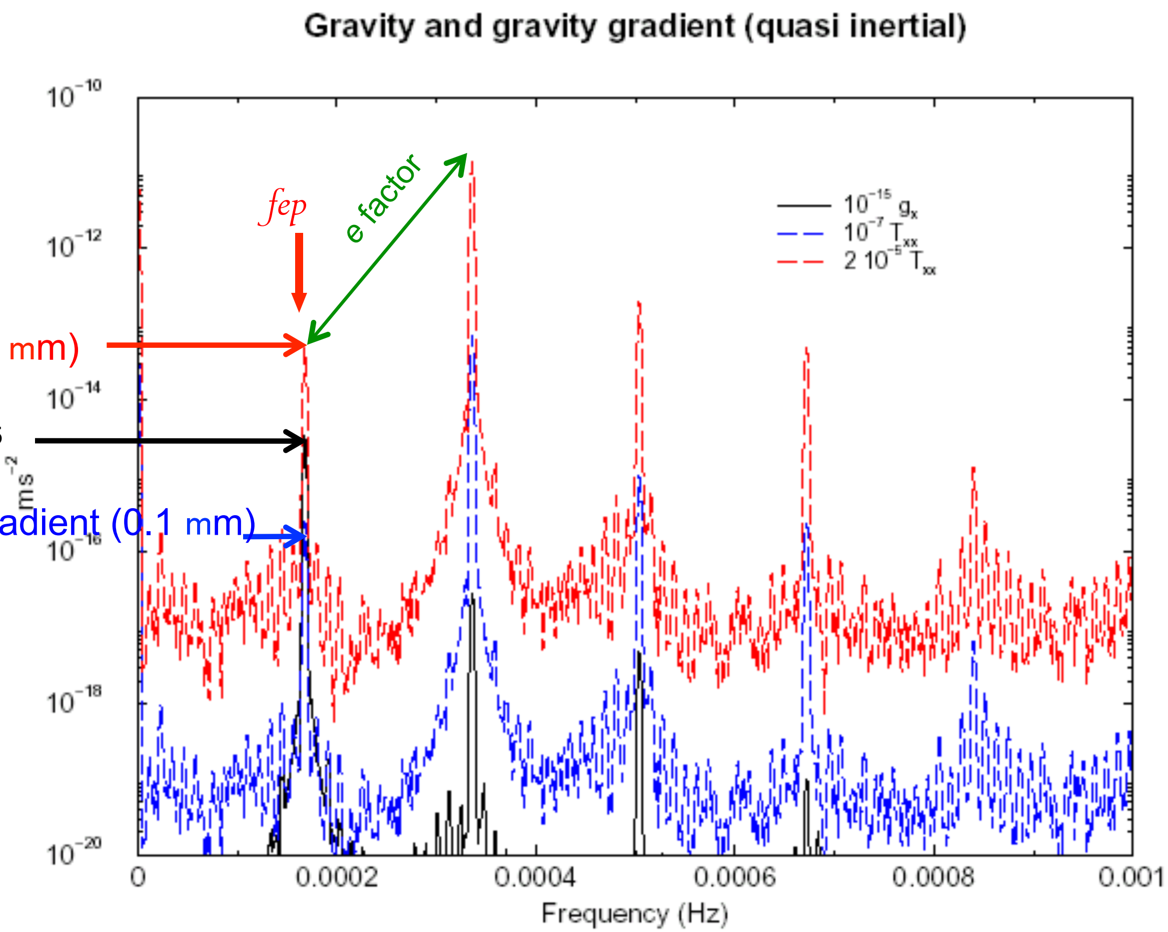
Exemple of calibration

Correction of the gravity gradient effects

e = 0.005



Gravity gradient (20 mm)

EP violation @ 10^{-15}

Corrected gravity gradient (0.1 mm)

Calibration of the other parameters

Paramètre à étalonner	Spécification sur la précision de l'estimation	Erreur d'estimation sans retraitement de la mesure	Erreur d'estimation après retraitement de la mesure
$a_{c11} \cdot \Delta_x$	$0,1\,\mu$m	$0,03\,\mu$m	$0,03\,\mu$m
$a_{c11} \cdot \Delta_z$	$0,1\,\mu$m	$0,05\,\mu$m	$0,03\,\mu$m
$a_{c11} \cdot \Delta_y$	$2,0\,\mu$m	$0,13\,\mu$m	$0,002\,\mu$m
a_{c12}	$9,0 \times 10^{-4}$ rad	$4,9 \times 10^{-4}$ rad	$1,2 \times 10^{-4}$ rad
a_{c13}	$9,0 \times 10^{-4}$ rad	$7,8 \times 10^{-4}$ rad	$5,9 \times 10^{-4}$ rad
a'_{d11}	$1,5 \times 10^{-4}$	$6,0 \times 10^{-4}$	$8,0 \times 10^{-5}$
a_{d12}	5×10^{-5} rad	$1,6 \times 10^{-6}$ rad	$9,0 \times 10^{-7}$ rad
a_{d13}	5×10^{-5} rad	$5,8 \times 10^{-6}$ rad	$6,6 \times 10^{-6}$ rad
$\mathrm{K}_{2dxx}/\mathrm{K}_{1cx}^2$	$250\,\mathrm{s}^2/\mathrm{m}$	$132\,\mathrm{s}^2/\mathrm{m}$	$18\,\mathrm{s}^2/\mathrm{m}$
$\mathrm{K}_{2cxx}/\mathrm{K}_{1cx}^2$	$1000\,\mathrm{s}^2/\mathrm{m}$	$147\,\mathrm{s}^2/\mathrm{m}$	$147\,\mathrm{s}^2/\mathrm{m}$

E. Hardy thesis

Conclusion

- The MICROSCOPE mission is optimized to discriminate an EP signal at the *fep* frequency
- Measurement of environment data (position of the masses, position of the satellite, attitude, temperature…) are planed
- Dedicated calibration sessions have been designed and included in the mission scenario
- The performances are verified:
  - At the sub-system level :
    - Return from previous missions
    - On ground tests
    - Simulations
  - At the global level :
    - Analytical error budget => worst case (see P. Touboul)

      Spin mode : 1,12 10^{-15} over 20 orbits and 0,66 10^{-15} over 120 orbits
      Inertial mode : 1,42 10^{-15} over 120 orbits
    - Numerical simulations (for calibration and EP test) ➔ 0,3 10^{-15}
      => Monte Carlo simulations are planed

Thank you for your attention

Treatment of the data gaps

- Lack of data can exist due to accelerometer saturations or transmission problems
- The duration of the gaps could extend from 1s (frequently) to 1mn (up to once per orbit) and even more (rarely).
- This can increase the projection of some perturbations on the *Fep* frequency (cf presentation by E. Hardy)
- To limit this effect, different actions are planed (cf presentation by E. Hardy)
  - To fill the short gaps by reconstructing the lacking data
  - To “remove” a whole number of orbits in case of large gap
- The corresponding data will be well flagged

How to handle the differential signal ?

Example of equation to solve :

Design matrix A — parameters X — observations Y

$$\begin{pmatrix} g_x(t_1) & T_{xx}(t_1) & T_{xz}(t_1) \\ . & . & . \\ . & . & . \\ . & . & . \\ g_x(t_N) & T_{xx}(t_N) & T_{xz}(t_N) \end{pmatrix} \begin{pmatrix} \delta \\ \Delta x \\ \Delta z \end{pmatrix} = \begin{pmatrix} \Delta\gamma_x(t_1) \\ . \\ . \\ . \\ \Delta\gamma_x(t_N) \end{pmatrix}$$

$\Rightarrow$

$$X = (A^T P A)^{-1} A^T P Y$$

P : weight matrix = inverse of the covariance matrix

Difficulties :

- P non diagonal for non white noise
- Covariance matrix difficult to know accurately
- Even if known, heavy inversion (typical dimension = 1 000 000)

Transformation of the linear system

First method

A X = Y

Discret filter
(e.g. Butterworth passe-band pass, order 4)

A_F X = Y_F

➡ Inversion

$$X = (A_F^T A_F)^{-1} A_F^T Y_F$$

Second method

A X = Y

FFT + band selection

$$\tilde{A}\ X = \tilde{Y}$$

➡ Inversion in the frequency domain

$$X = (\tilde{A}^T \tilde{A})^{-1} \tilde{A}^T \tilde{Y}$$

Simulations