Turbulence Modelling

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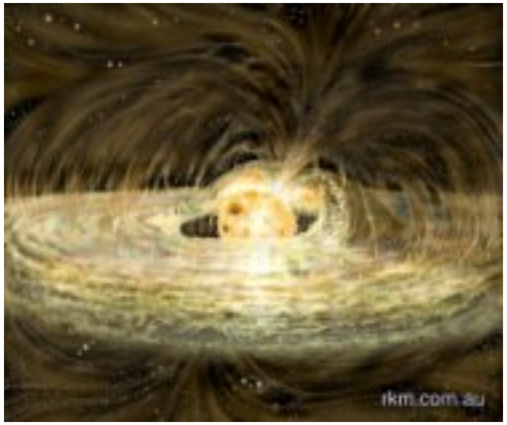
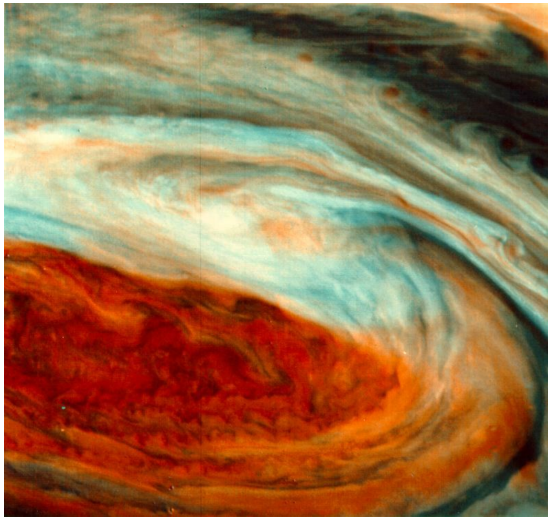
Astrophysical Flows



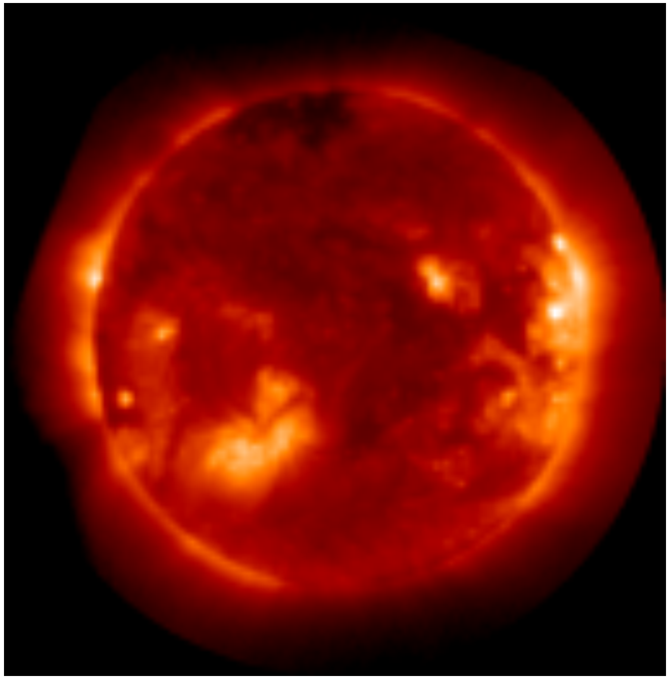
Disks/Galaxies

$\mathrm{Re} \approx 10^{13}$



Planetary Atmosphere

$\mathrm{Re} = 10^{9}$



Stars $\mathrm{Re} = 10^{8}$

Navier-Stokes Equations:

$$\partial_t u + u \bullet \nabla u = - \nabla p + \nu \Delta u + f$$

Control Parameter: $\mathrm{Re} = \frac{LU}{\nu}$

$Re = 10^2$

$Re = 10^7$

$Re = 10^{11}$

$Re = 10^{12}$

$Re = 10^{21}$

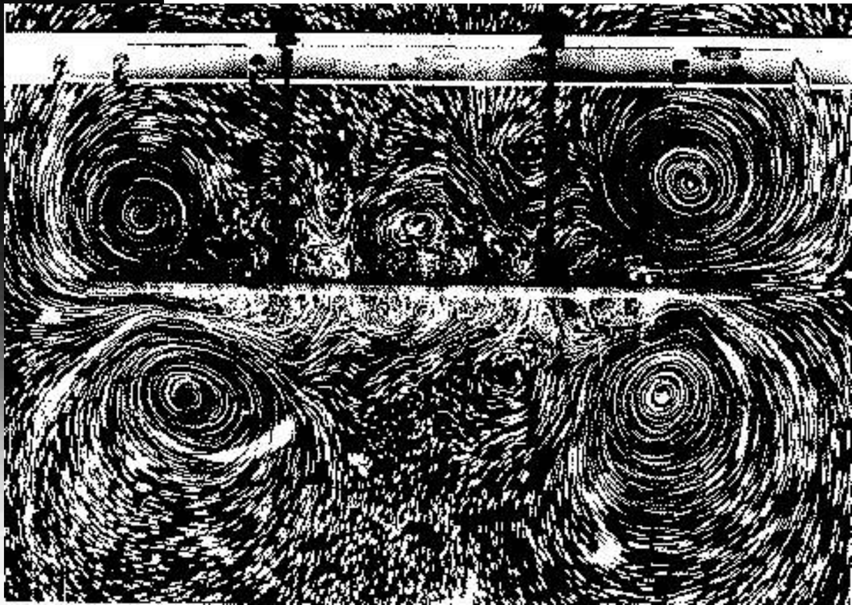
Laboratory Flows

Von Karman

$$\mathrm{Re} = 10^6$$

Taylor-Couette

$$\mathrm{Re} = 10^5$$

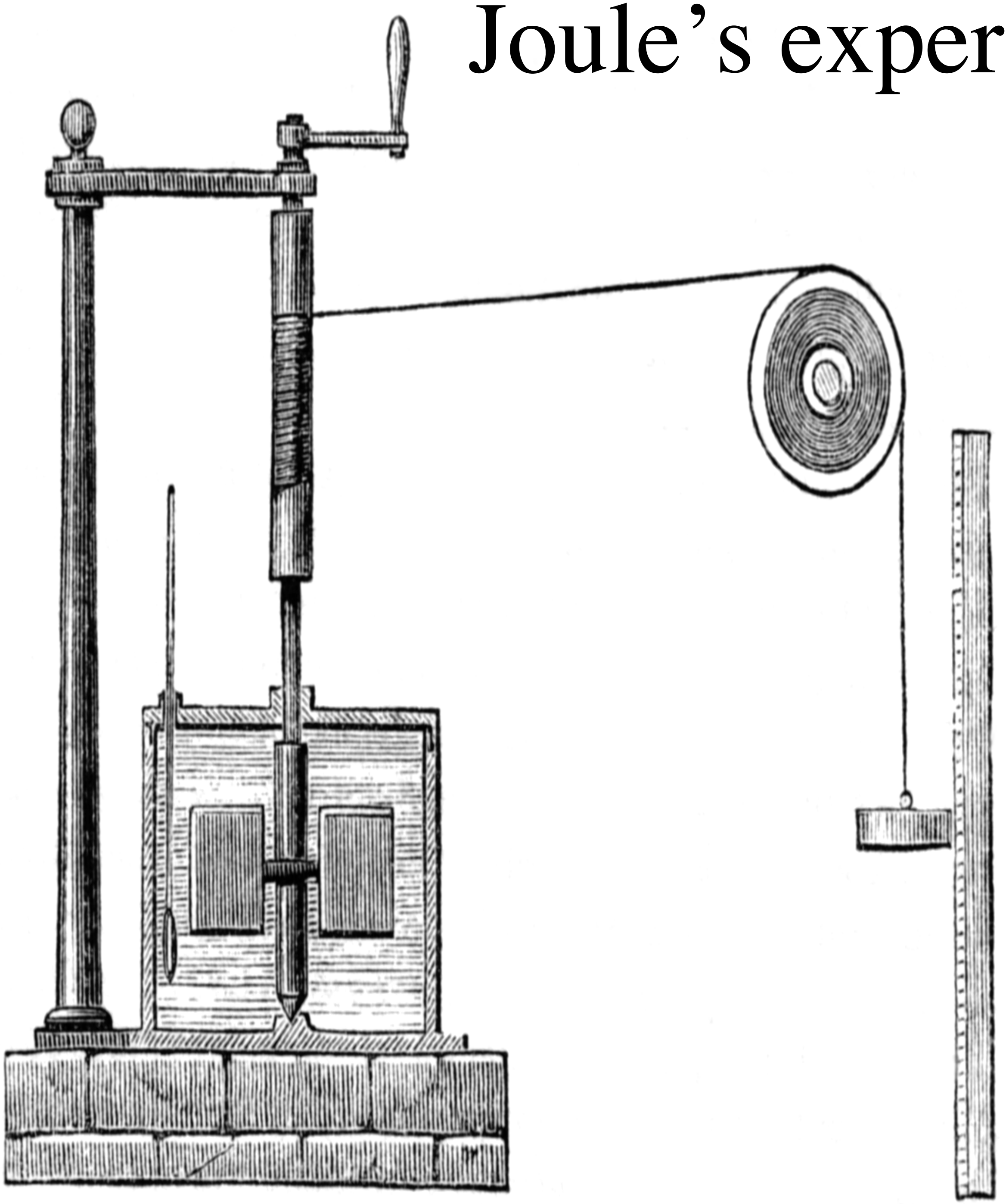


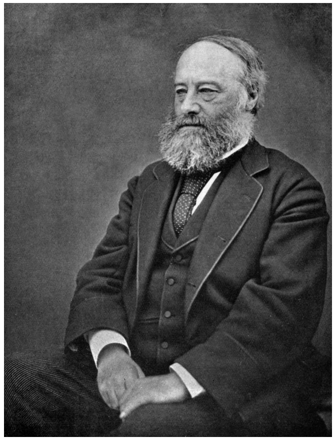
Grid Turbulence

$$\mathrm{Re} = 10^3$$

Similarity Principle:same boundray conditions, same Reynolds->same behaviour

Joule's experiment (1840)





Application of
772.24 foot pound force
(4.1550 J.cal−1)
results in elevation of
temperature of a pound of
water by one degree
Fahrenheit

Fluid good converters of Mechanical enery into heat

Aside about Joule's experiment

NEQFLUIDS2016: Classical and Quantum Fluids Out of Equilibrium
13-16 Jul 2016 Grand Hotel de Paris, Villard de Lans (France)

Robert&Collins

Lessiveuse: n.m. Traduction: washing machine

Ils utilisent le club de cartes comme leur **lessiveuse**. ->They use the card club as their **washing machine**.

Aside about Joule's experiment

1840

1991

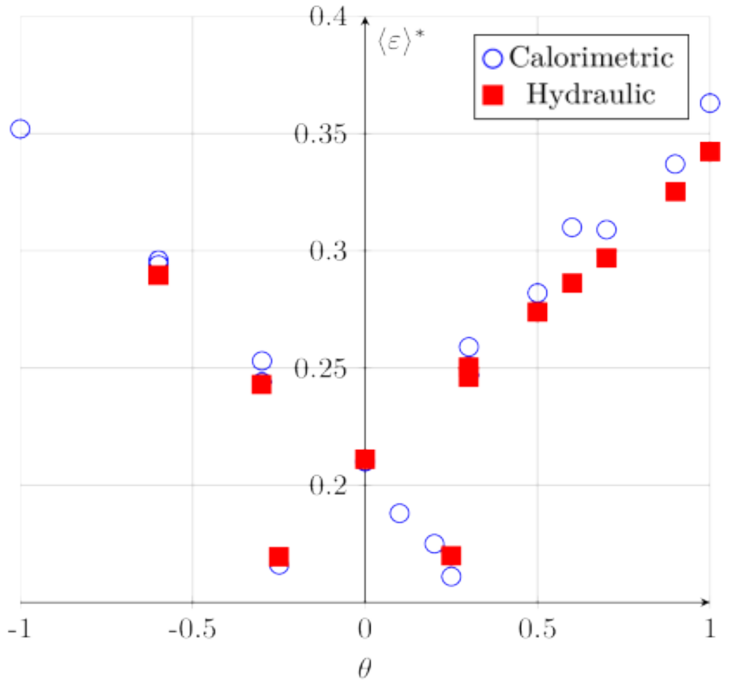
Robert&Collins

Von Karman Flow: n.m. Traduction: french washing machine

They use their **french washing machine** as experimental set up-> Ils utilisent leur **lessiveuse** comme experience.

Douady S, Couder Y and Brachet M E 1991 Direct observation of the intermittency of intense vorticity filaments in turbulence Phys. Rev. Lett. 67 983

A modern version: VK flow

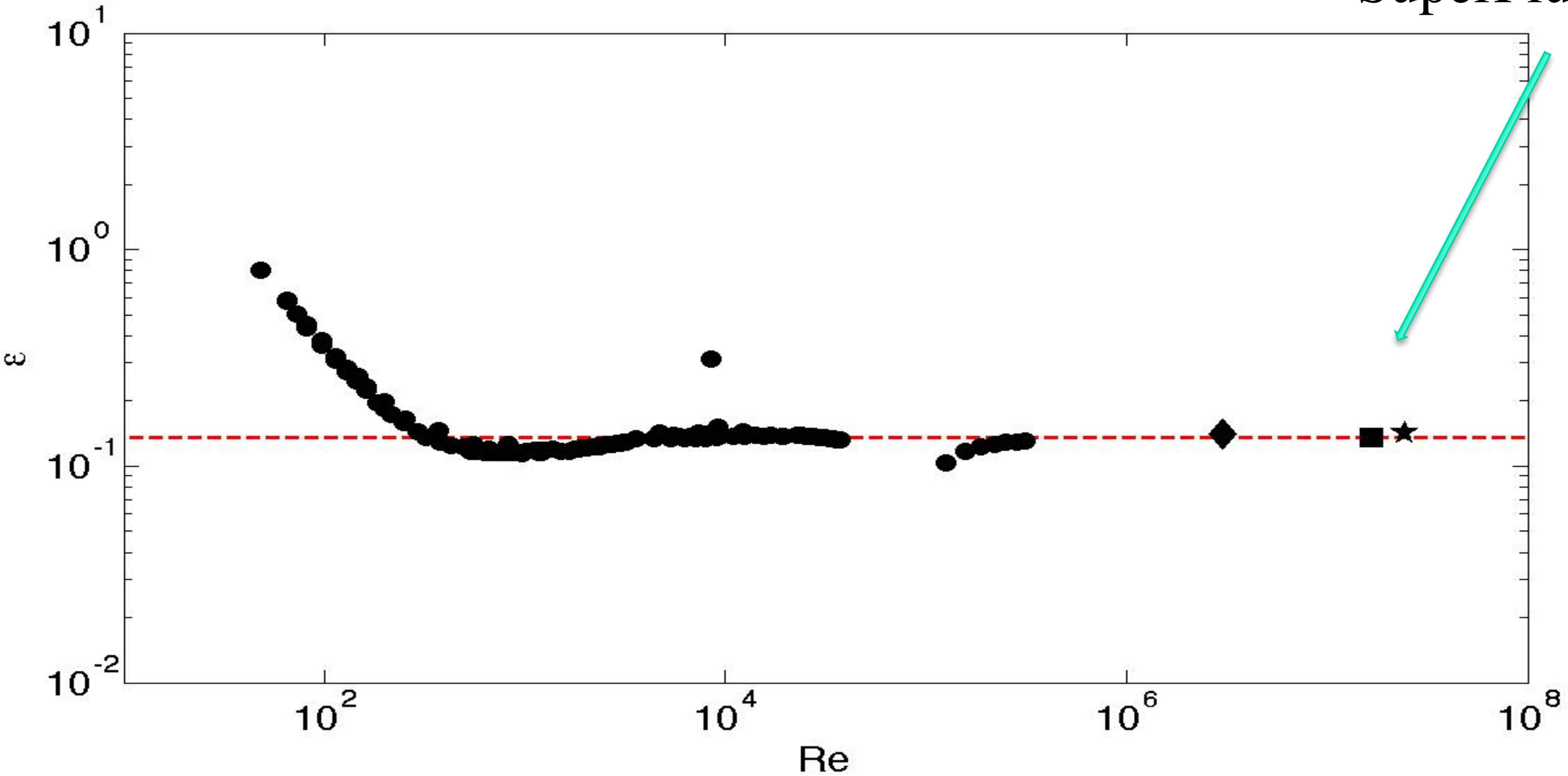


Work measured by
Torques applied at
Shafts
Heat flux measured
By keeping T constant

Rousset et al, RSI 85, 103908 (2014);

Zeroth law of turbulence

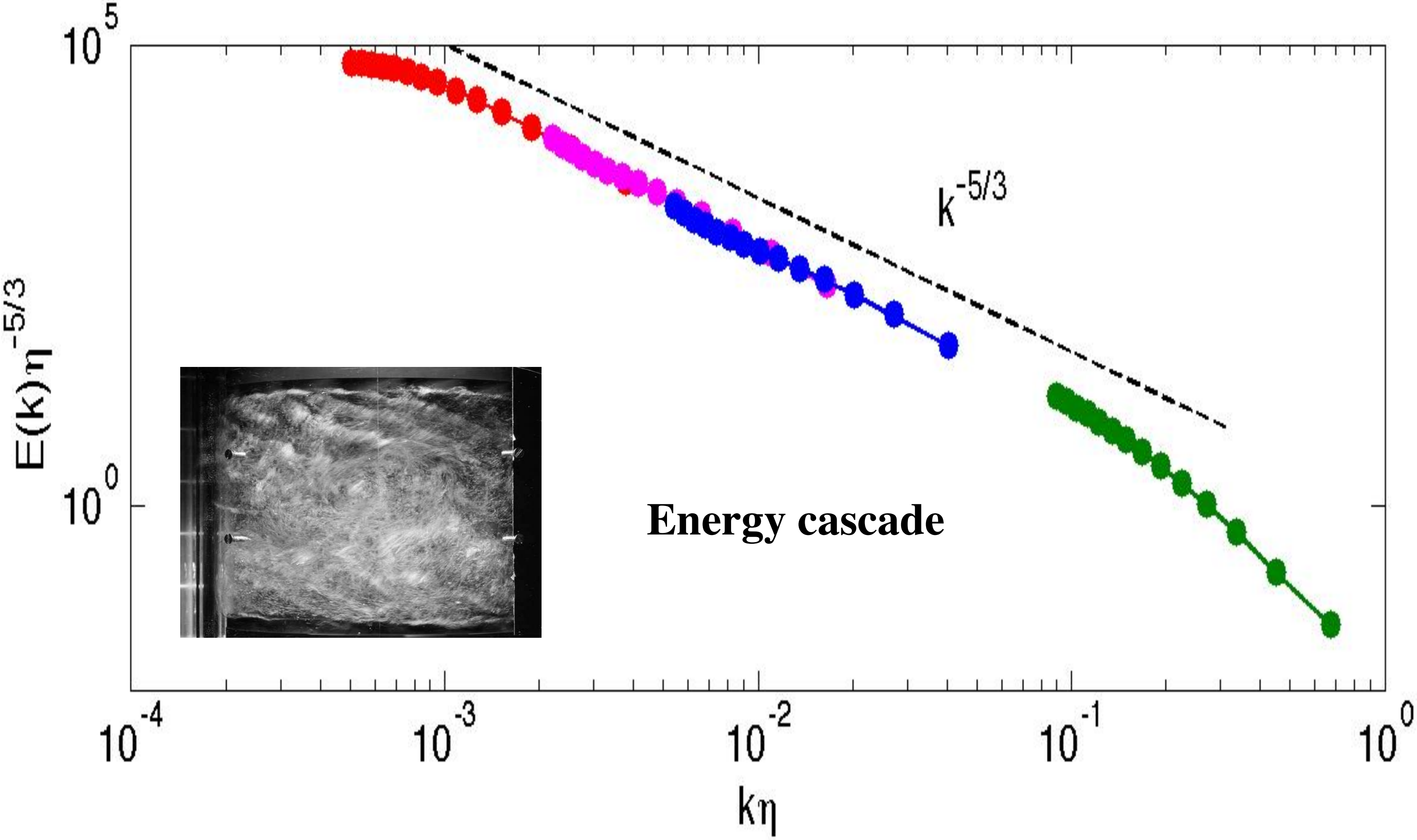
SuperFluid!



Non-dimensional Energy dissipation per unit mass is independent of viscosity!!!!!

Ravelet PhD, Saint-Michel PhD, VKS collaboration, SHREK collaboration

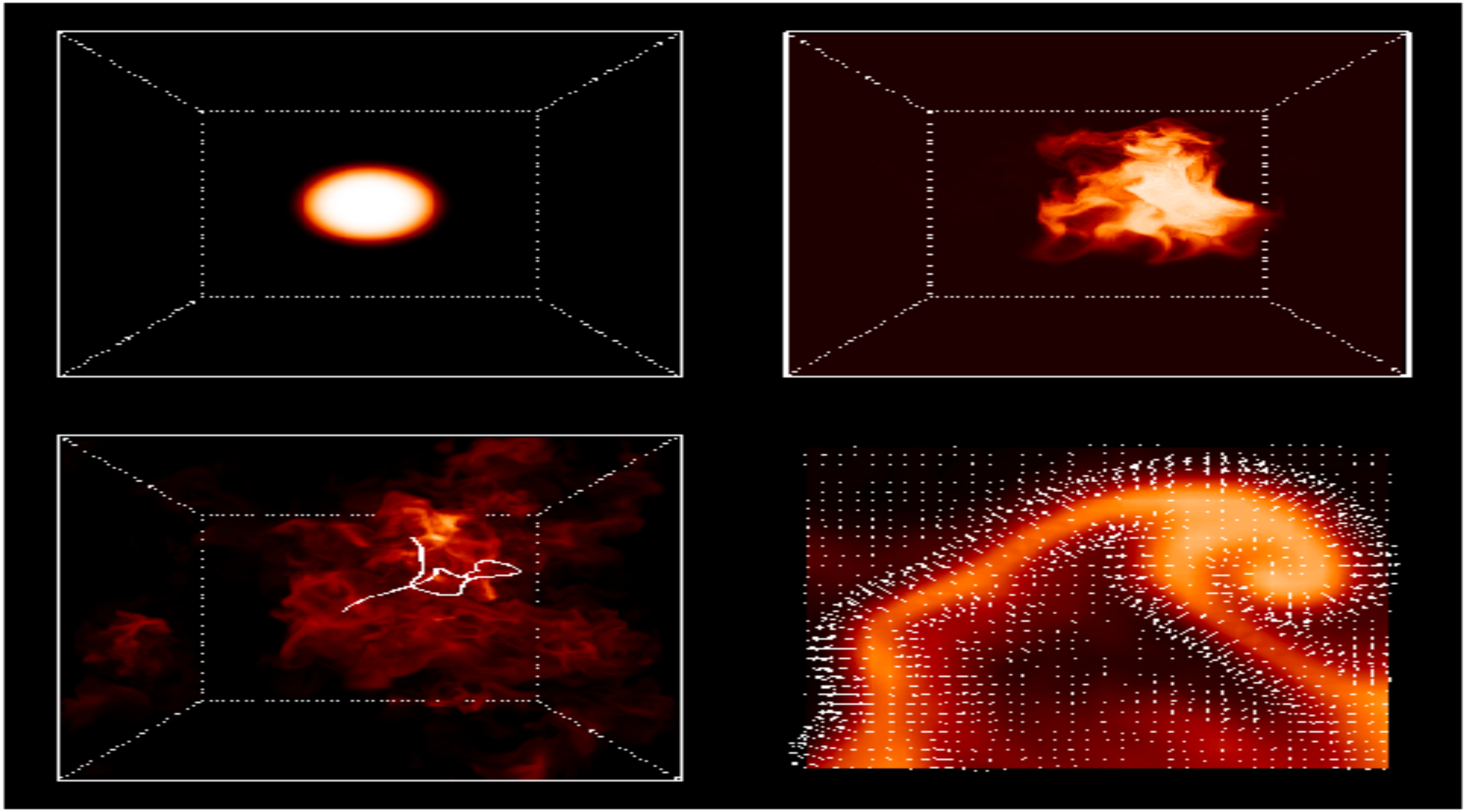
Observation: power-law spectrum



Energy cascade

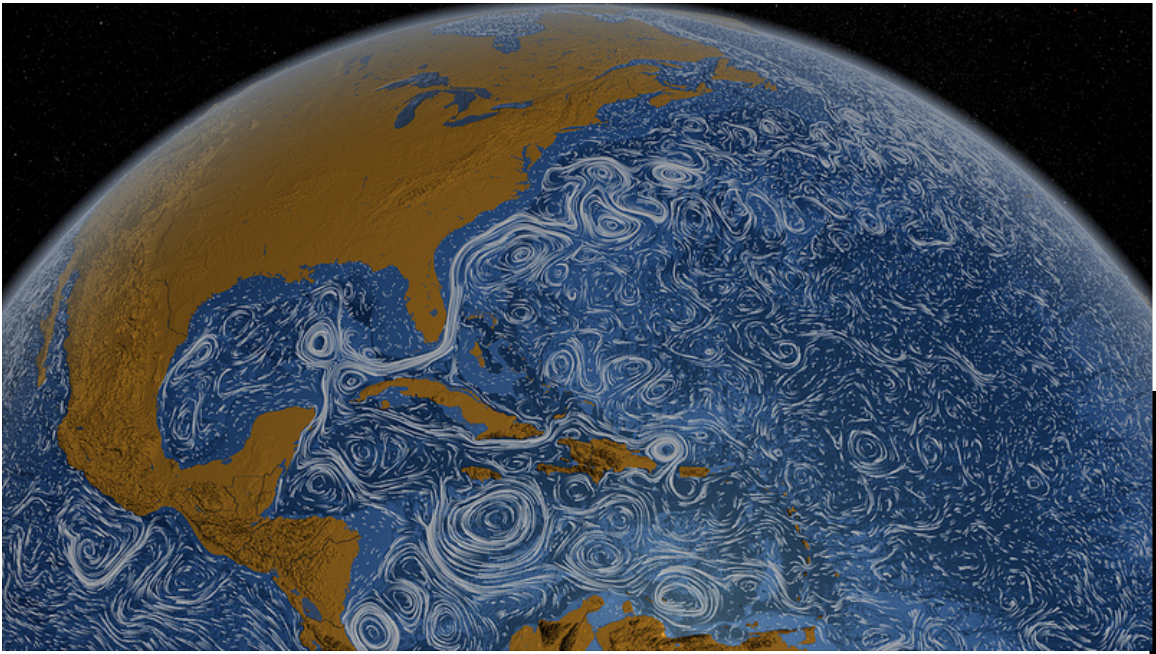
Saclay team: Debue, Kuzzay, Faranda, Saw, Daviaud, Dubrulle et al, (2016)

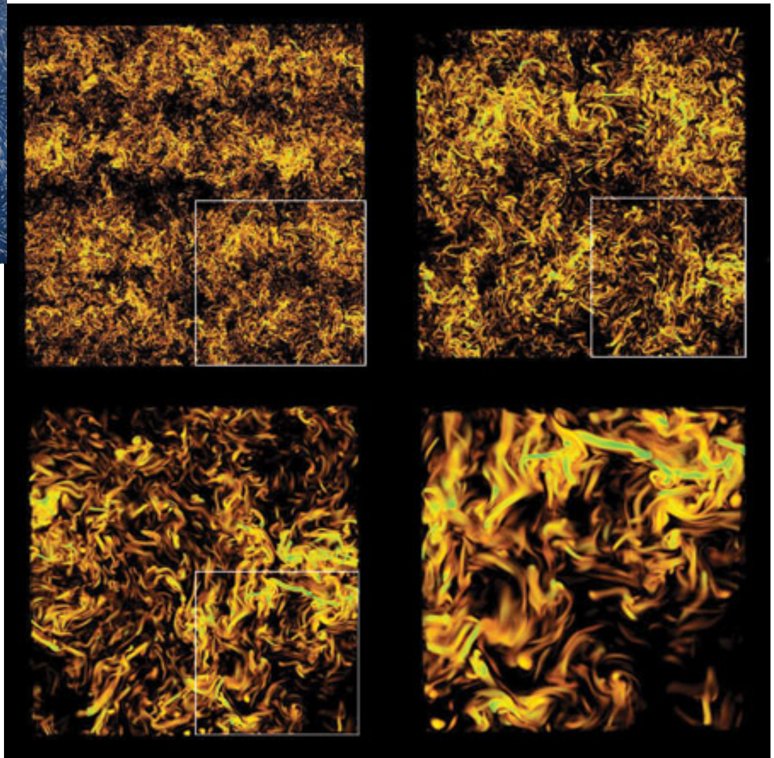
Observation of Fluid Movements



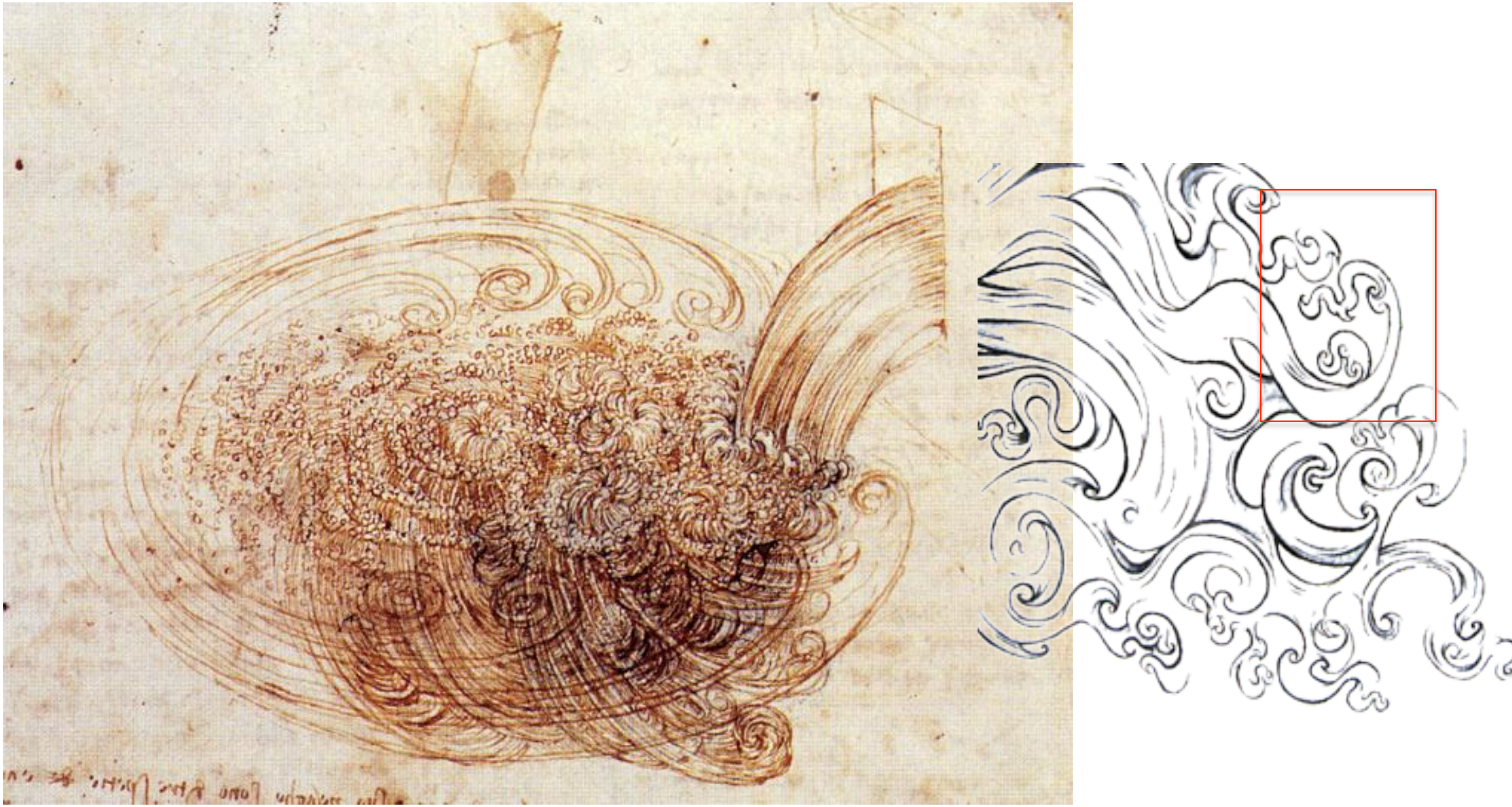
Creation of structures finer and finer till dissipation by viscosity-> scale hierachy

Scale hierarchy



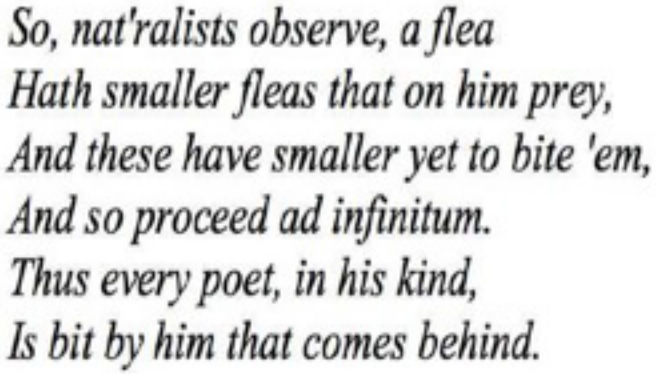


Scale hierarchy



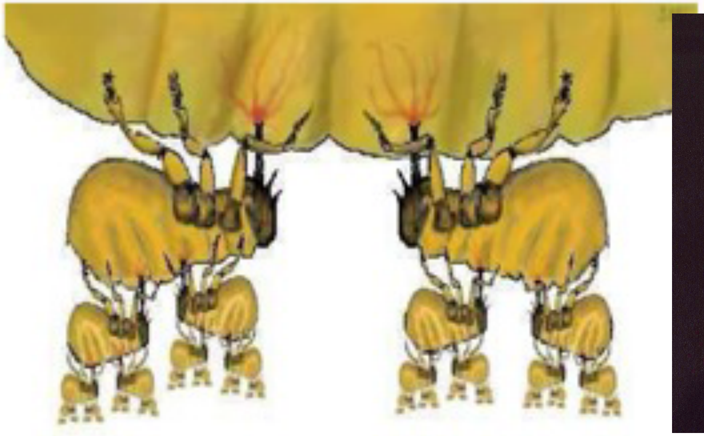
Leonard de Vinci

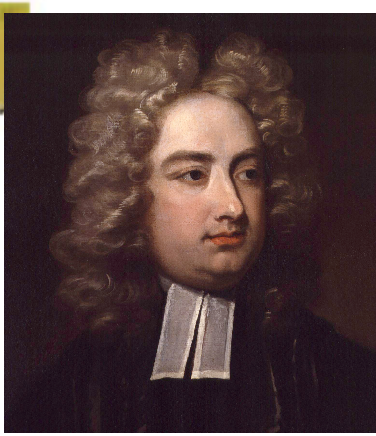
Richardson’s Cascade



So, nat'ralists observe, a flea
Hath smaller fleas that on him prey,
And these have smaller yet to bite 'em,
And so proceed ad infinitum.
Thus every poet, in his kind,
Is bit by him that comes behind.

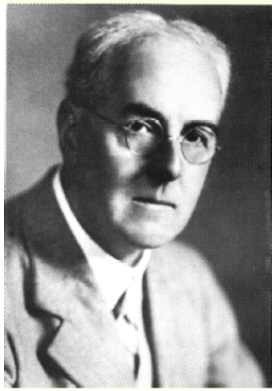
(Jonathan Swift)



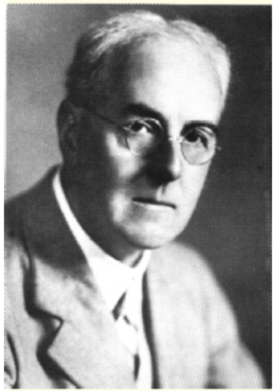


Big whirls have little whirls,
Which feed on their velocity,
And little whirls have lesser whirls,
And so on to viscosity.”

Lewis Fry Richardson 1881-1953



Kolmogorov Cascade (1941)



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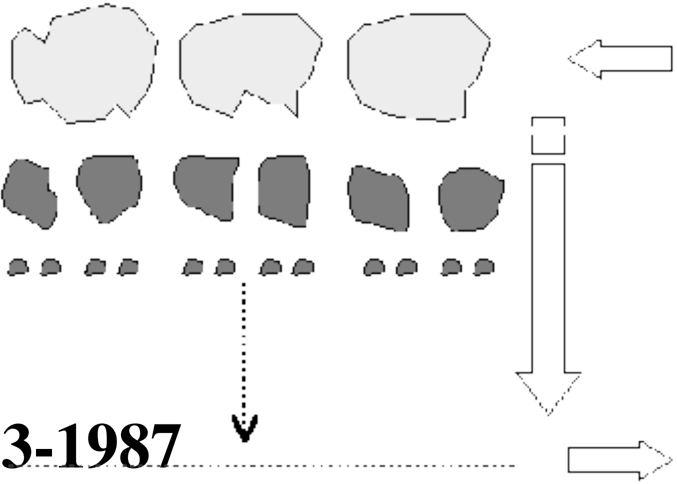


Andrey Kolmogorov 1903-1987

Kolmogorov Theory (1)

NSE+homogeneity

⇓

$$\frac{1}{2}\partial_t\left\langle\left(\delta u_\ell\right)^2\right\rangle+\varepsilon=-\frac{1}{4}\nabla_\ell\left\langle\left(\delta u_\ell\right)^3\right\rangle+\nu\Delta_\ell\left\langle\left(\delta u_\ell\right)^2\right\rangle$$

$$\delta u_\ell=u\left(x+\ell\right)-u\left(x\right)$$

Karman Howarth equation

Kolmogorov Theory (2)

KH equation + self-similarity+stationarity

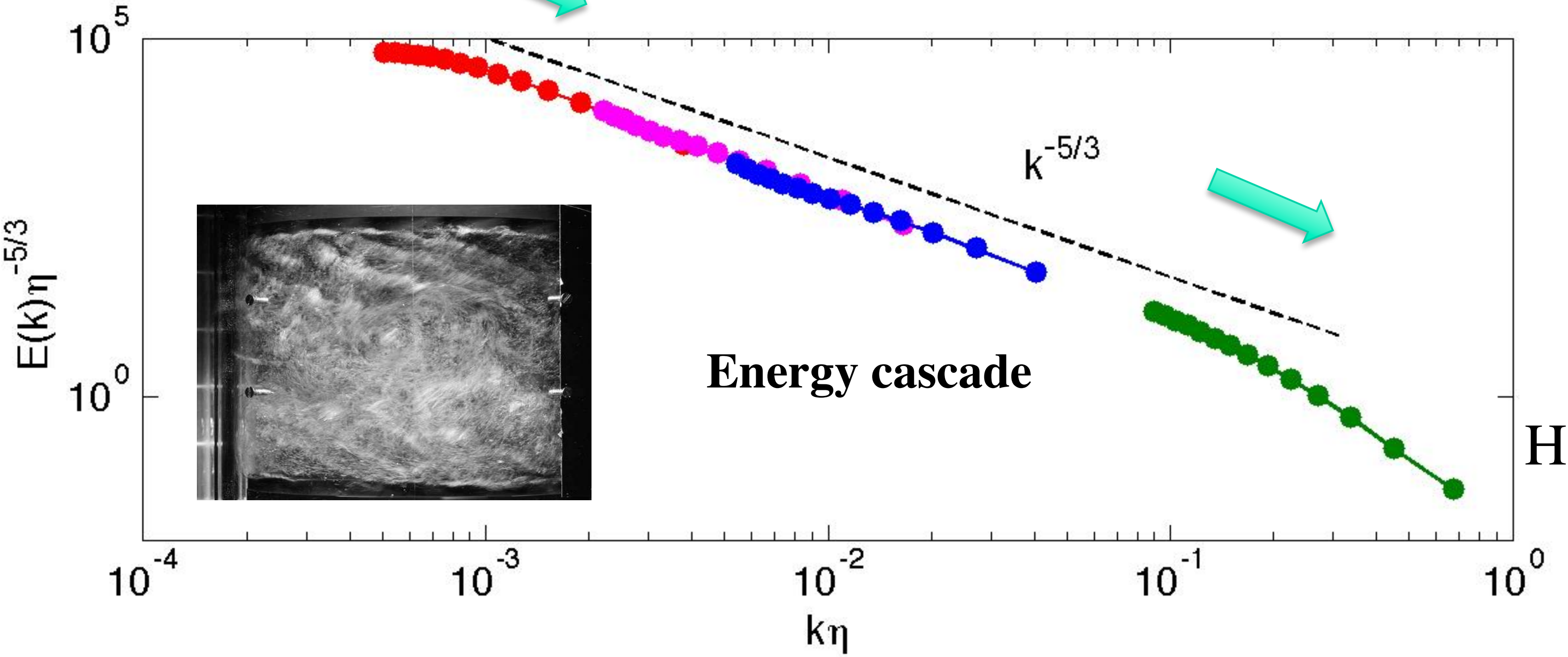
$$\frac{1}{2}\partial_t \xcancel{\left\langle\left(\delta u_\ell\right)^2\right\rangle}+\varepsilon=-\frac{1}{4}\nabla_\ell\left\langle\left(\delta u_\ell\right)^3\right\rangle+\nu\xcancel{\Delta_\ell\left\langle\left(\delta u_\ell\right)^2\right\rangle}$$

$$\left\langle\left(\delta u_\ell\right)^3\right\rangle \propto -\frac{4}{3}\varepsilon\ell$$

$$\left\langle\left(\delta u_\ell\right)^2\right\rangle \propto \left(\varepsilon\ell\right)^{2/3} \Rightarrow E\left(k\right)=C\,\varepsilon^{2/3}k^{-5/3}$$

Interpretation:The energy cascade

Work



Energy cascade

Heat

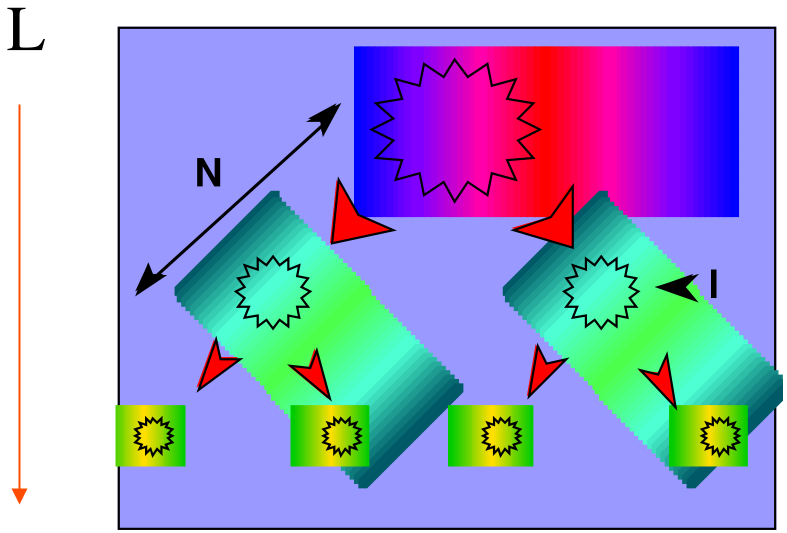
Injection scale

Dissipative scale
(viscosity)

Turbulence phenomenology

Robust result
Kolmogorov spectrum

Interpretation (Kolmogorov 1941)
Energy cascade



$$\eta = \left(\nu^3 / \varepsilon \right)^{1/4}$$

Constant energy transfer

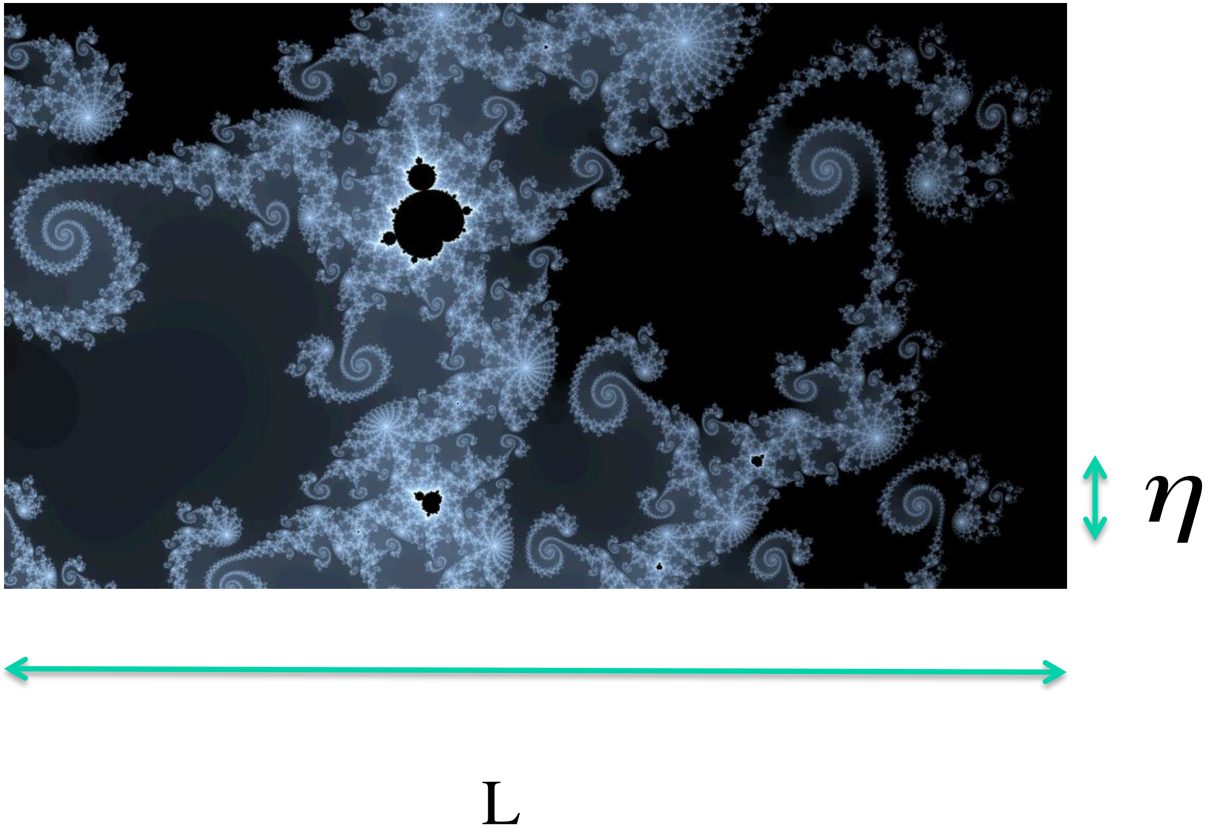
$$\frac{du^2}{dt} = \varepsilon \cong \frac{u^3}{l} = cte$$

$$\Rightarrow u \propto (\varepsilon l)^{1/3}$$

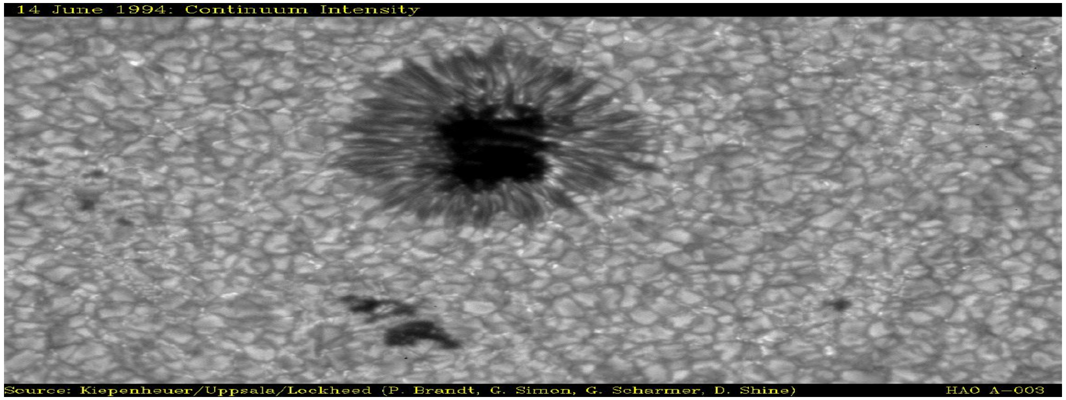
Degrees of freedom

$$N = \left(\frac{L}{\eta} \right)^3 \propto \mathrm{Re}^{9/4}$$

The scales of turbulence



Example: the sun



14 June 1994: Continuum Intensity

Source: Kiepenheuer/Uppsala/Lockheed (P. Brandt, G. Simon, G. Scharmer, D. Shine)

Dissipation scale	Granule	Solar spot	Giant Convective cell
0.1 km	$10^3 km$	$3 \cdot 10^4 km$	$2 \cdot 10^5 km$

$$N = \left(10^6\right)^3 = 10^{18}$$

Many degrees of freedom!

Scale of turbulence

L=5cm
η=1mm

L=10m
η=0.06mm

L=6000 km
η=6mm

Simulation of turbulence

We have the basic equations…
Can we simulate all these systeme?

$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

Computer Blue Gene

Simulation of turbulence

Simulation:

6 10^9 nodes --> 1 week of CPU
on Blue Gene

Storage:
10^8 nodes- -> 2Tb= 1 disk

Simulation of turbulence

$$N = \left(\frac{L}{\eta}\right)^3$$

L=5cm
η=1mm

N=10^5

L=10m
η=0.06mm

N=10^{16}

L=6000 km
η=6mm

N=10^{27}

Simulation of turbulence

L=5cm
η=1mm

1 mn of cpu
Less than a disk

L=10m
η=0.06mm

$N=10^{16}$

L=6000
km
η=6mm

$N=10^{27}$

Simulation of turbulence

L=5cm
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L=10m
η=0.06mm

20 000 years of cpu
100 billions disks

L=6000 km
η=6mm

$N=10^{27}$

Simulation of turbulence

L=5cm
η=1mm

1 mn of cpu
Less than a disk

L=10m
η=0.06mm

20 000 years of cpu
100 billions disks

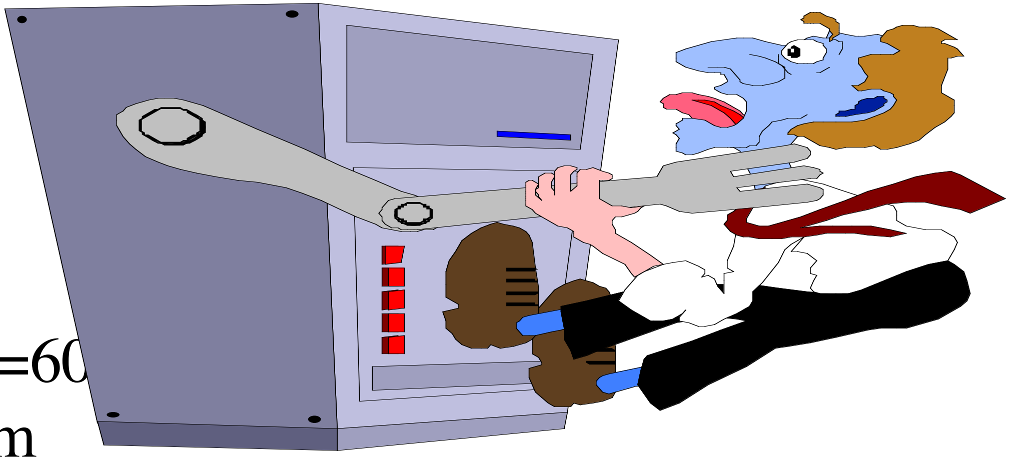
L=6000
km
η=6mm

10^{15} years of cpu
10^{19} disks

Simulation of turbulence

L=5cm
η=1mm

1 mn of cpu
Less than a disk

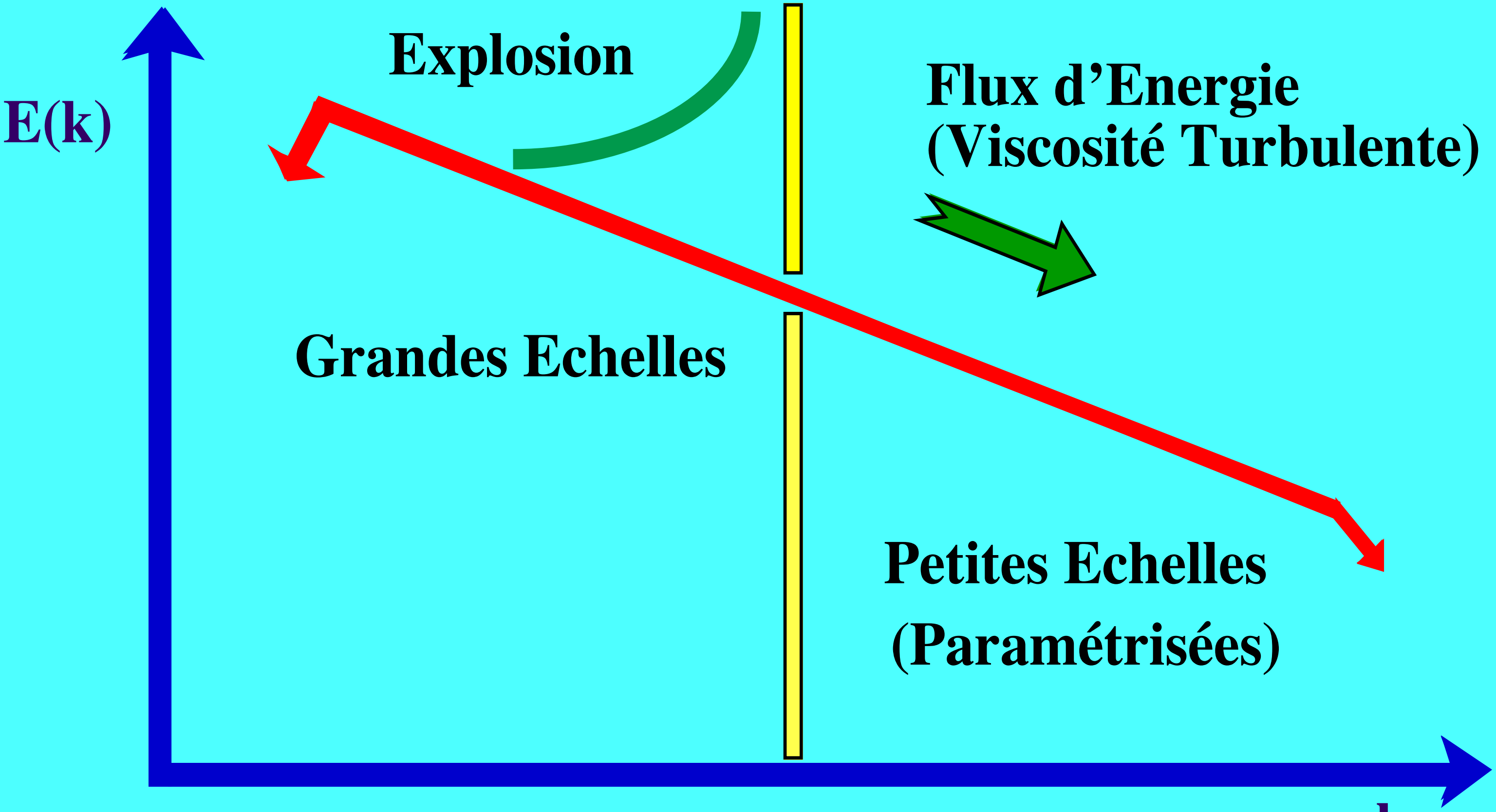


20 000 years of cpu
100 billions disks

L=60
km
η=6mm

10^{15} years of cpu
10^{19} disks

What can be done?



E(k)

Explosion

Flux d’Energie
(Viscosité Turbulente)

Grandes Echelles

Petites Echelles

(Paramétrisées)

k

90 % des ressources informatiques

Building on Kolmogorov

$$\frac{1}{2}\partial_t\left\langle\left(\delta u_\ell\right)^2\right\rangle+\varepsilon=-\frac{1}{4}\nabla_\ell\left\langle\left(\delta u_\ell\right)^3\right\rangle+\nu\Delta_\ell\left\langle\left(\delta u_\ell\right)^2\right\rangle$$

Energy injection

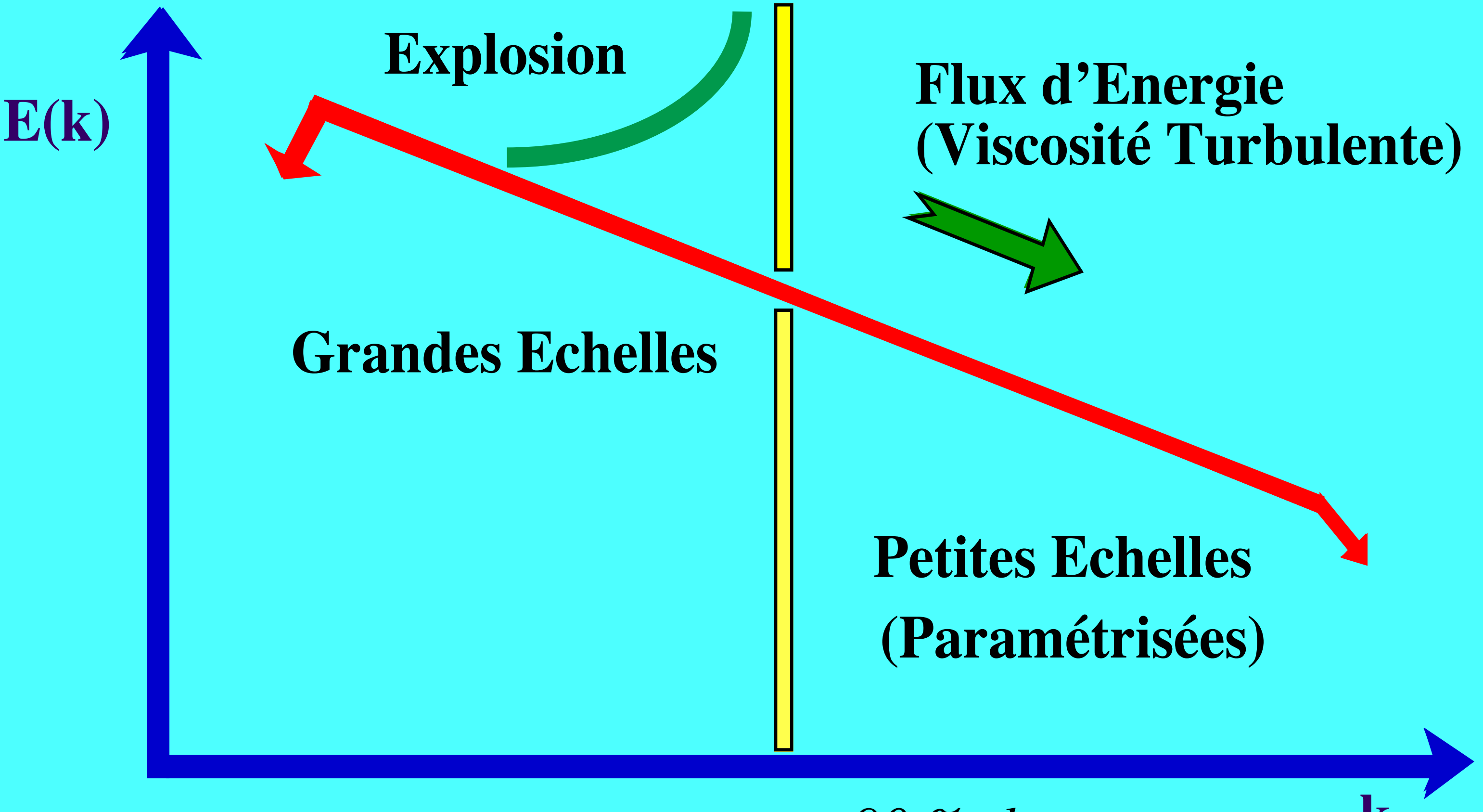
Energy transfer

Viscous
dissipation

$$\frac{1}{2}\partial_t\left\langle\left(\delta u_\ell\right)^2\right\rangle+\varepsilon=\nabla_\ell(\nu_T\nabla_\ell\left\langle\left(\delta u_\ell\right)^2\right\rangle)+\nu\Delta_\ell\left\langle\left(\delta u_\ell\right)^2\right\rangle$$

Turbulent Viscosity

What can be done?



E(k)

Explosion

Flux d'Energie (Viscosité Turbulente)

Grandes Echelles

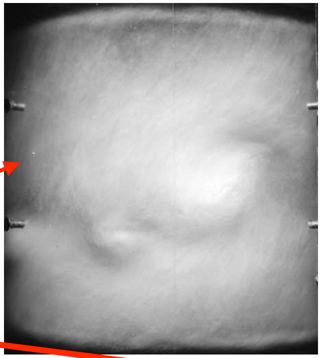
Petites Echelles

(Paramétrisées)

k

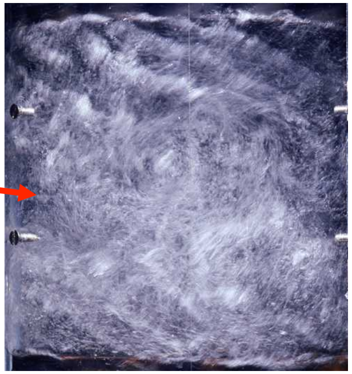
90 % des ressources informatiques

Two ways to cut the scale space

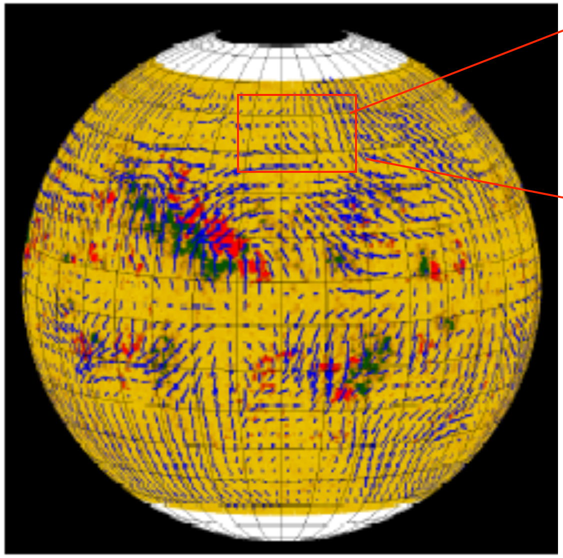


Mean Flow

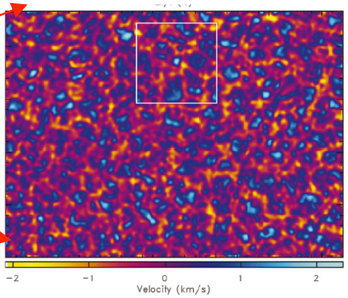
Fluctuations



RANS:
You keep the mean Flow
Parametrize fluctuations



Large scale



Small scales

LES:
You keep the large scale
Parametrize the small

Mathematical translation

$$\partial_t u_i + u_j \nabla_j u_i = -\nabla_i p + \frac{1}{\mathrm{Re}} \Delta u_i + f_i$$

$$u = \bar{u} + u'$$

- Spatial filter for LES
- Ensemble average for RANS

$$\partial_t \bar{u}_i + \bar{u}_j \nabla_j \bar{u}_i = -\nabla_i \bar{p} + \frac{1}{\mathrm{Re}} \Delta \bar{u}_i + \bar{f}_i - \nabla_j \tau_{ij}$$

Reynolds stress

$$\tau_{ij} = \overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j + \overline{\bar{u}_i u'_j} + \overline{u'_i \bar{u}_j} + \overline{u'_i u'_j}$$

LES

$$\tau_{ij} = +\overline{u'_i u'_j}$$

RANS

Influence of decimated scales

Typical time at scale l: $\delta t \approx \frac{l}{u} \propto l^{2/3}$

Decimated scales (small scales) vary very rapidly
We may replace them by a noise with short time scale

$$u = \overline{u} + u'$$

$$D_t u'_i = A_{ij} u'_j + \xi_j$$

$$\left\langle \xi_i(x,t)\xi_j(x',t') \right\rangle = \kappa_{ij}(x,x')\delta(t-t')$$

Generalized Langevin equation

Obukhov Model

Simplest case

$$\bar{u} = 0$$

$$A_{ij} = -\gamma\delta_{ij}, \quad \gamma >> \delta t$$

$$\kappa_{ij}(x,x') \propto \gamma\delta_{ij}$$

No mean flow

Large isotropic friction

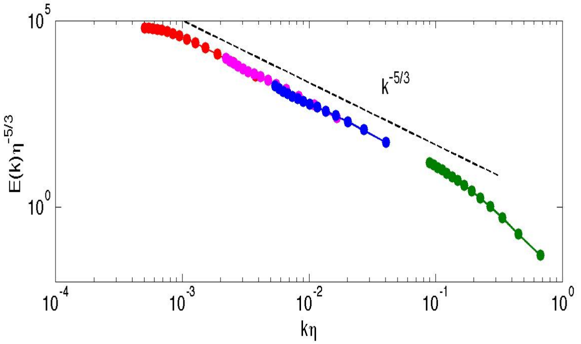
No spatial correlations

$$P(\vec{x},\vec{u},t) = \left(\frac{\sqrt{3}}{2\pi\varepsilon t^2}\right)\exp\left(-\frac{3x^2}{\varepsilon t^3} - \frac{3\vec{x}\bullet\vec{u}}{\varepsilon t^2} - \frac{u^2}{\varepsilon t}\right)$$

$u \propto \sqrt{\varepsilon t}$ Gaussian velocities

$x \propto \varepsilon^{2/3} t^{3/2}$ Richardson's law

$u \propto x^{1/3}$ Kolmogorov's spectra



LES: Langevin

Influence of decimated scales: transport

$$\dot{\vec{x}} = \overline{\vec{u}} + \vec{u}\,'$$

$$\vec{\Omega} = \vec{\nabla} \times \vec{u}$$

$$\dot{\vec{\Omega}} = (\vec{\Omega} \bullet \vec{\nabla})\overline{\vec{u}} + (\vec{\Omega} \bullet \vec{\nabla})\vec{u}\,'$$

Leprovost&Dubrulle

Stochastic computation

$$\beta_{kl} = \left\langle u^{'}_{k} u^{'}_{l} \right\rangle$$

$$\alpha_{ijk} = \left\langle u^{'}_{i} \partial_k u^{'}_{j} \right\rangle$$

$$\partial_t \overline{\Omega_i} + \overline{u}_k \nabla_k \overline{\Omega_i} = \overline{\Omega_k} \nabla_k \overline{u_i} + \nabla_k \left[\beta_{kl} \nabla_l \overline{\Omega_i} \right] + 2\alpha_{kil} \nabla_k \overline{\Omega_l}$$

Turbulent viscosity

AKA effect

Parametrization: RANS

Issue: Reynolds stress parametrization

$$\boxed{\begin{aligned}\tau_{ij} &= +\overline{u'_i u'_j} \\ &= -\alpha_{ijk}\overline{u_k} - \beta_{ijkl}\nabla_k \overline{u_l}\end{aligned}}$$

AKA effect	Turbulent Viscosity
Helicity effect	4 order tensor
Influence on mean flow (breaks Galilean invariance)	Can be « negative » (instabilities)
Produces large scale-instabilities (cf dynamo effect)	
Sulem, Frisch, She	*Dubrulle&Frisch*

Parametrization: RANS
AKA effect

Use to explain: Solar Granulation (Kishan, MNRAS, 1991)

Galaxy Clustering (Kishan, MNRAS, 1993)

Large-scale vortices in disks
(Kitchatinov et al, A&A, 1994)

Liitle (not?) used in general turbulence

No general theory

Analogy with dynamo:

$$\alpha_{ijk} = \frac{1}{3}\frac{\overline{\vec{u}'\bullet(\nabla\times\vec{u}')}}{\tau}\varepsilon_{ijk}$$

3D isotropic

Parametrization: RANS Viscosity

Not necessarily isotrop(cf shear flows) (Dubrulle&Frisch,

Isotropic Case $\beta_{ijkl} = \nu_T \delta_{jk} \delta_{il}$

Characteristic lenght

Dimensional analysis $\boxed{\nu_T = K V L}$

Constant

Characteristic velocity

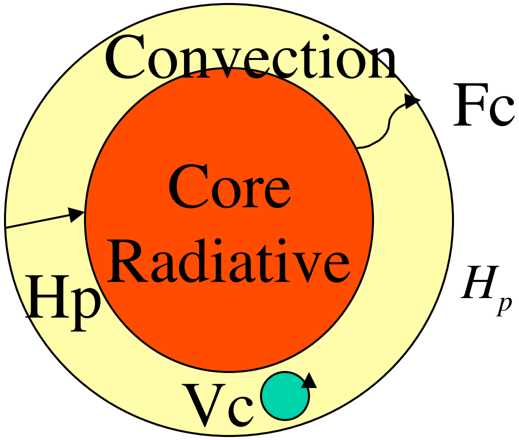
Kolmogorov theory $V = (\varepsilon L)^{1/3}$

$$\boxed{\nu_T = K \varepsilon^{1/3} L^{4/3}}$$

Example 1: Mixing length

$$\partial_t \vec{u} + \vec{u} \bullet \overrightarrow{\nabla} \vec{u} = -\frac{1}{\rho} \overrightarrow{\nabla p} + \beta \theta \vec{g} + \nu_T \Delta \vec{u}$$

Inertia = Buyancy



$$H_p = -\frac{dr}{d \ln p}$$

$$\frac{v_c^2}{L} \approx \beta g \theta$$

$$F_c = v_c \theta$$

$$v_c = \left(g \beta F_c L \right)^{1/3}$$

$$\nu_T = \left(g \beta F_c \right)^{1/3} L^{4/3}$$

$$L = \alpha\ H_p \qquad 1 < \alpha < 2$$

RANS: Viscosity

Example 2: Smagorinski

Viscosity written function of mean gradients

$$\nu_T = \left(c_s \Delta\right)^2 \sqrt{\nabla_j \bar{u}_i \nabla_j \bar{u}_i}$$

Adjustable
Constant

Mesh size

Example 3: RNG

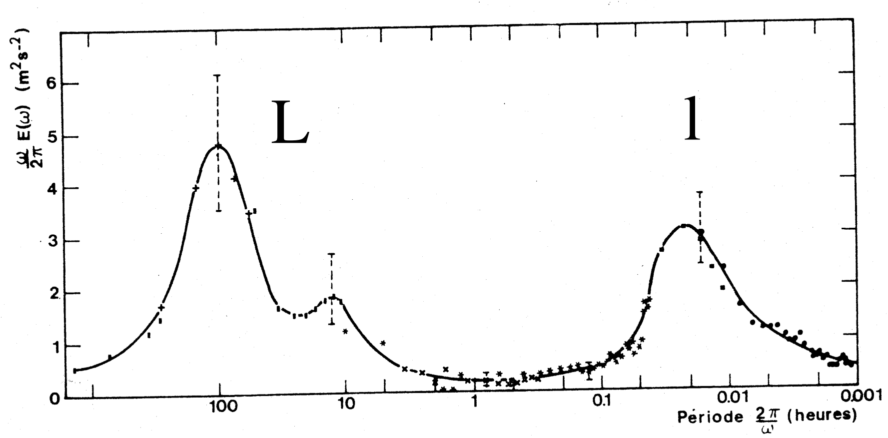
Viscosity in spectral space

$$\nu_T \Delta \overline{u}_i \rightarrow -\hat{\nu}_T k^2 \overline{\hat{u}_i}$$

$$\boxed{\hat{\nu}_T = K\left(\frac{\varepsilon}{k^4}\right)^{1/3}}$$

Computed with renormalization group

Multi-scale computation



$$\varepsilon = \frac{l}{L} << 1$$

Main idea: use scale separation

Method: a) Linearization equations
b) development of operators and fields in power of ε
c) Solve order by order
d) Solvability conditions provide coefficient

Example: Turbulent diffusivity

Frisch in « Lecture notes on turbulence » World Scientific, 1989

Before we start…

Is there a solution to ?

$$Ax = y$$

Solvability condition:

$$y \in \mathrm{Im}(A) \Leftrightarrow \forall z \in Ker(A^{+}), y \bullet z = 0$$

In space of periodical functions

$$f \bullet g = \int f(x,t) g(x,t) dxdt$$

$$Ker(A^{+}) = 1 : x \rightarrow 1$$

Solvability condition

$$1 \bullet y = \langle y \rangle = \int y(x) dx = 0$$

Turbulent diffusion (1)

Basic equation: passive scalar

$$\partial_T \Theta + \vec{u} \bullet \nabla_x \Theta = \kappa \nabla_x^2 \Theta$$

$$\langle \vec{u} \rangle = \vec{0}$$

Fast variables: x, t

Slow variables $X = \varepsilon x$, $T = \varepsilon^2 t$

expansion

$$\nabla_x \leftarrow \nabla_x + \varepsilon \nabla_X$$

$$\partial_t \leftarrow \partial_t + \varepsilon^2 \partial_T$$

$$\Theta(x, X, t, T) = \Theta^{(0)} + \varepsilon \Theta^{(1)} + \varepsilon^2 \Theta^{(2)} + \ldots$$

Frisch in « Lecture notes on turbulence » World Scientific, 1989

Turbulent diffusion (2)

Order 0

Equation
$$\partial_t \Theta^{(0)} + \vec{u}\nabla_x \Theta^{(0)} = \kappa \nabla_x^2 \Theta^{(0)}$$

Solvability
$$0=0$$

Solution
$$\Theta^{(0)}\left(x,t,X,T\right) = \Theta^{(0)}\left(X,T\right) = \left\langle \Theta^{(0)} \right\rangle$$

Order 1

Equation
$$\partial_t \Theta^{(1)} + \vec{u}\nabla_x \Theta^{(1)} = -\vec{u}\nabla_X \left\langle \Theta^{(0)} \right\rangle + \kappa \nabla_x^2 \Theta^{(1)}$$

Solvability
$$\left\langle \vec{u} \right\rangle = \vec{0}$$

Solution
$$\Theta^{(1)} = \vec{\chi}\nabla_X \left\langle \Theta^{(0)} \right\rangle$$

$$\partial_t \vec{\chi} + \vec{u}\nabla_x \vec{\chi} = -\vec{u} + \kappa \nabla_x^2 \vec{\chi}$$

Frisch in « Lecture notes on turbulence » World Scientific, 1989

Turbulent diffusion (3)

Order 2

Equation

$$\partial_t \Theta^{(2)} + \vec{u}\nabla_x \Theta^{(2)} + \partial_T \left\langle \Theta^{(0)} \right\rangle =$$

$$-\vec{u}\nabla_X \Theta^{(1)} + \kappa \nabla_x^2 \Theta^{(2)} + \kappa \nabla_X^2 \left\langle \Theta^{(0)} \right\rangle$$

$$+2\kappa \nabla_x \nabla_X \Theta^{(1)}$$

Solvability

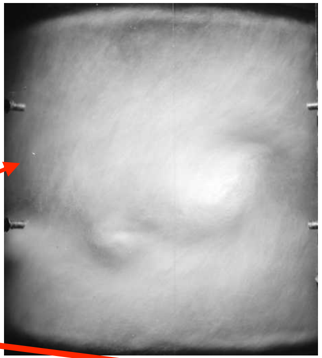
$$\partial_T \left\langle \Theta^{(0)} \right\rangle = \left(\kappa \nabla_X^2 + D_{ij} \partial_{X_i} \partial_{X_j} \right) \left\langle \Theta^{(0)} \right\rangle$$

Turbulent Diffusivity

$$\boxed{\begin{aligned} D_{ij} &= -\frac{1}{2}\left[\left\langle u_i \chi_j \right\rangle + \left\langle u_j \chi_i \right\rangle \right] \\ &= \kappa \left\langle \partial_l \chi_i \partial_i \chi_l \right\rangle > 0 \end{aligned}}$$

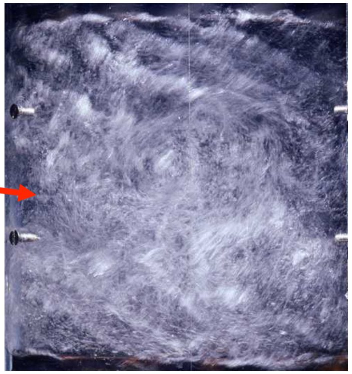
Frisch in « Lecture notes on turbulence » World Scientific, 1989

Two ways to cut the scale space

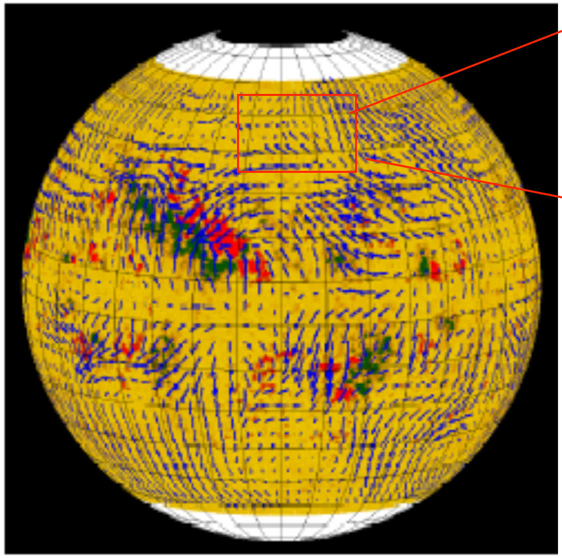


Mean Flow

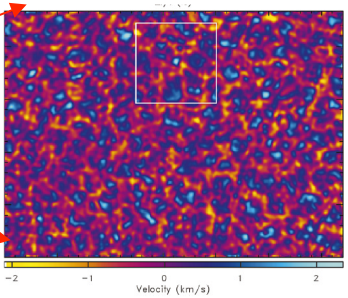
Fluctuations



RANS:
You keep the mean Flow
Parametrize fluctuations



Large scale



Small scales

LES:
You keep the large scale
Parametrize the small

Mathematical translation

$$\partial_t u_i + u_j \nabla_j u_i = -\nabla_i p + \frac{1}{\mathrm{Re}} \Delta u_i + f_i$$

$$u = \overline{u} + u'$$

- Spatial filter for LES
- Ensemble average for RANS

$$\partial_t \overline{u}_i + \overline{u}_j \nabla_j \overline{u}_i = -\nabla_i \overline{p} + \frac{1}{\mathrm{Re}} \Delta \overline{u}_i + \overline{f}_i - \nabla_j \tau_{ij}$$

Reynolds stress

$$\tau_{ij} = \overline{\overline{u}_i \overline{u}_j} - \overline{u}_i \overline{u}_j + \overline{\overline{u}_i u'_j} + \overline{u'_i \overline{u}_j} + \overline{u'_i u'_j}$$

LES

$$\tau_{ij} = +\overline{u'_i u'_j}$$

RANS

Parametrization: LES
Filters

$$\boxed{\overline{f(x)} = \int f(x')G(x-x')dx' \Leftrightarrow \hat{\bar{f}}(k) = \hat{f}(k)\hat{G}(k)}$$

Filter	Function	Fourier transform
Cut-off spectral	$\Pi_{i=1}^{3}\frac{\sin\left[k_c\left(x_i - x'_i\right)\right]}{\pi\left(x_i - x'_i\right)}$	$\begin{cases}1 & if\ \lvert k_i\rvert \le k_c\\ 0 & otherwise\end{cases}$
Gaussian	$\left(\frac{6}{\pi\Delta^2}\right)^{3/2}\exp\left(-\frac{6\lvert x-x'\rvert}{\Delta^2}\right)$	$\exp\left(-\frac{\Delta^2k^2}{24}\right)$
Top-hat	$\begin{cases}1/\Delta^3 & if\ \lvert x_i - x'_i\rvert \le \frac{1}{2}\Delta\\ 0 & otherwise\end{cases}$	$\Pi_{i=1}^{3}\frac{\sin\left[\frac{1}{2}\Delta k_i\right]}{\frac{1}{2}\Delta k_i}$

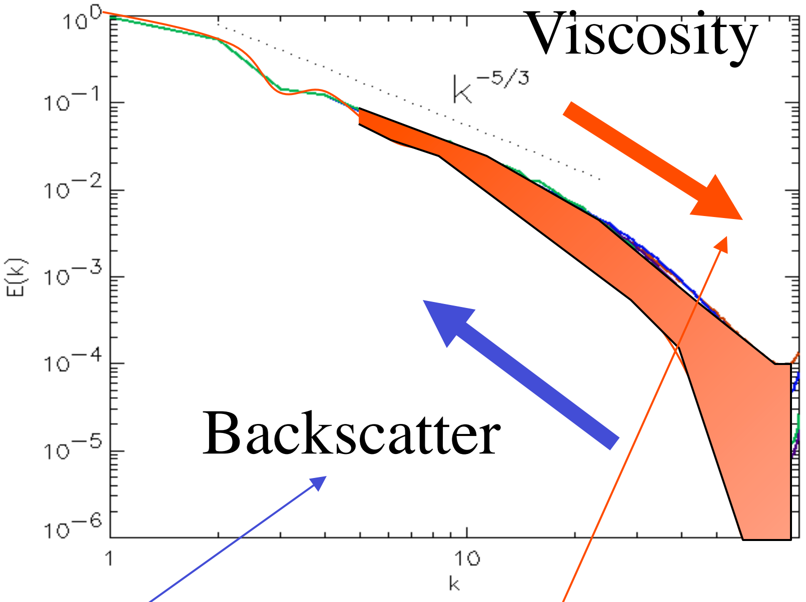
Parametrization: LES

Filters:

$$\overline{f} = \int f(x')G(x-x')dx'$$

$$\overline{\overline{f}} \neq \overline{f}$$

🟧 Filtered part



$$\tau_{ij} = \overbrace{\overline{\overline{u}_i\overline{u}_j} - \overline{u}_i\overline{u}_j + \overline{\overline{u}_i u'_j} + \overline{u'_i\overline{u}_j}} + \overline{u'_i u'_j}$$

Issue: Reynolds stress parametrization

Parametrization: LES Global models

Smagorinski	Spectral
$\tau_{ij} = \frac{1}{3}\tau_{kk}\delta_{ij} - 2\nu_T \overline{S_{ij}}$	$i k_j \hat{\tau}_{ij} = \nu_T(k) k^2 \hat{\bar{u}}_i$
$\nu_T = \left(C_s \Delta^2\right)\left(2\overline{S_{pq} S_{pq}}\right)^{1/2}$	$\nu_T(k) = \sqrt{\frac{E(k_c, t)}{Ko^3 k_c}}\left[0.441 + 15.2\exp\left(-\frac{3.03 k_c}{k}\right)\right]$
$\overline{S_{ij}} = \frac{1}{2}\left(\partial_i \overline{u_j} + \partial_j \overline{u_i}\right)$	$\frac{1}{2}\overline{u^2} = \int_0^\infty E(k)\, dk$
$C_s = 0.1 - 0.2$	$Ko = 1.3 - 1.8$

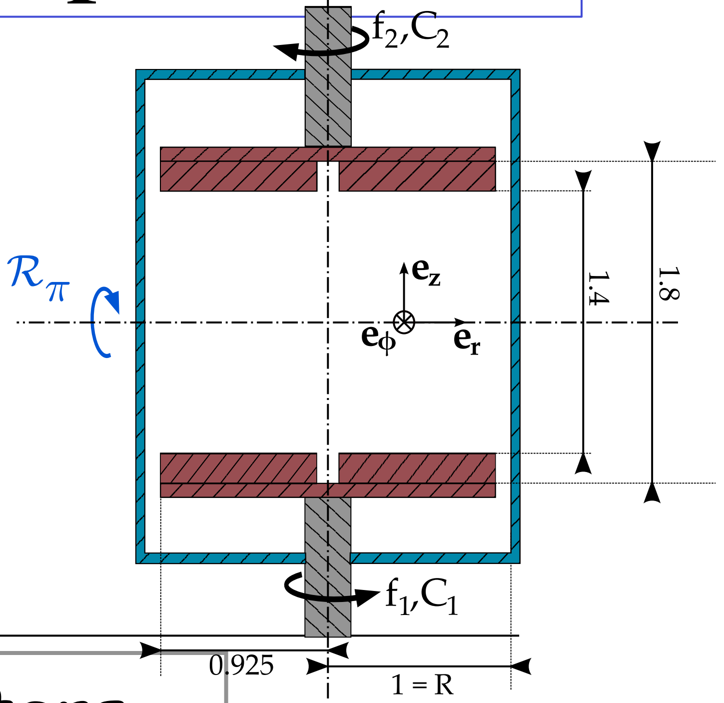
Lesieur, « Turbulence in Fluids », Kluwer, 1990

Parametrization: LES Gradient Model

$$\tau_{ij} = C\left(\Delta x\right)^2 \overline{S_{ik}}\,\overline{S_{jk}}$$

C= adjustable constant

Test via laboratory experiment



Control parameters

Reynolds number =
Turbulence

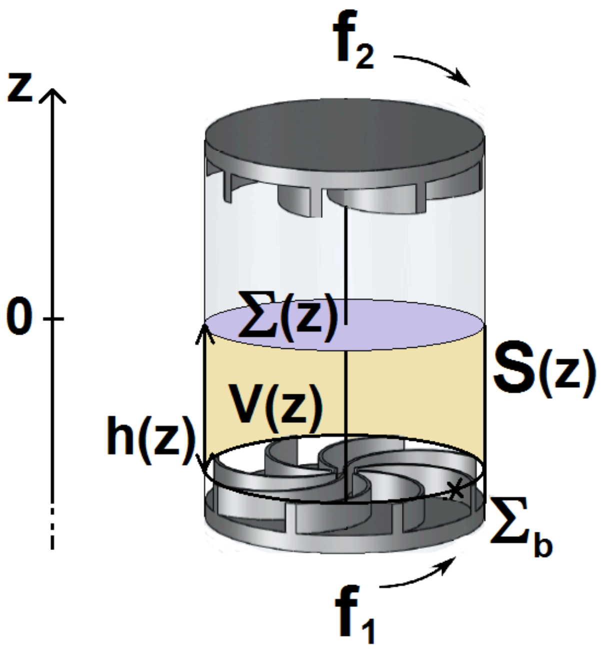
$$\boxed{Re = \frac{2\pi f R^2}{\nu}}$$

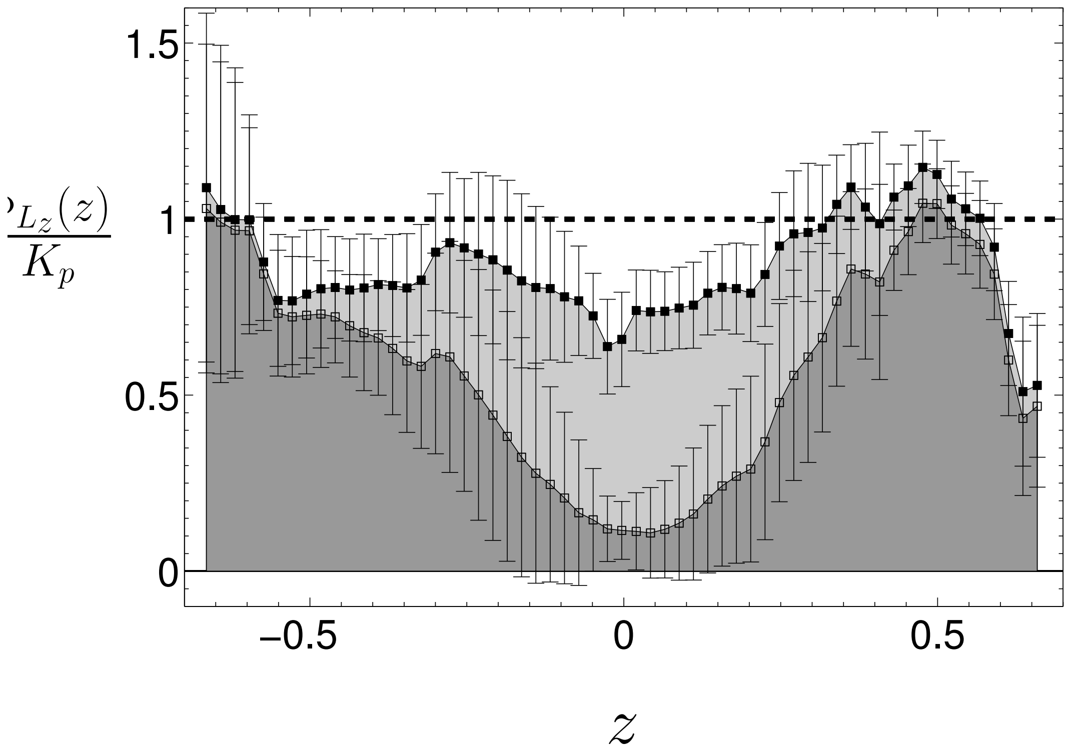
$$f = \frac{f_1 + f_2}{2}$$

Rotation number =
Asymmetry

$$\vartheta = \frac{f_1 - f_2}{f_1 + f_2}$$

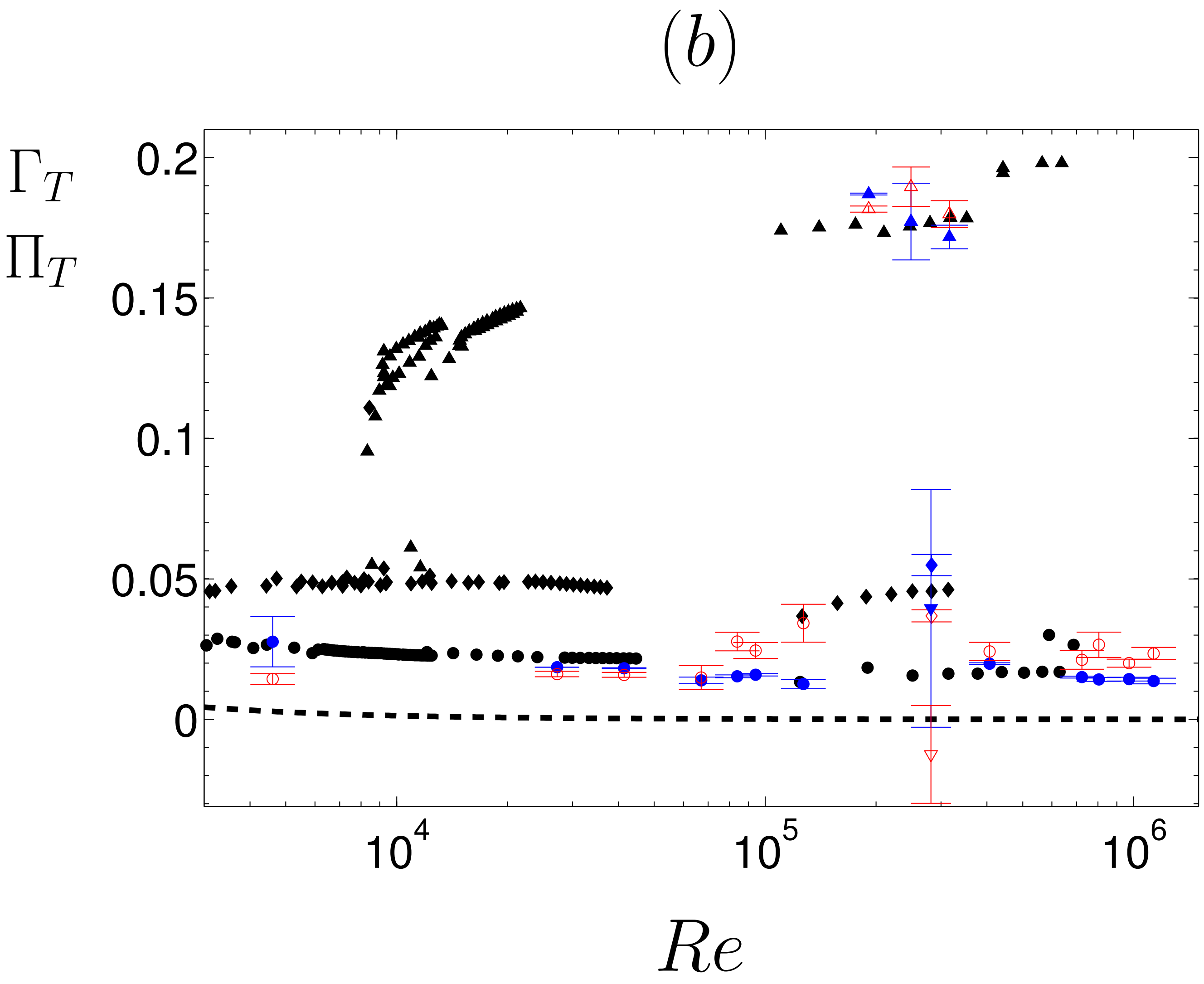
Calibration of C using angular momentum conservation

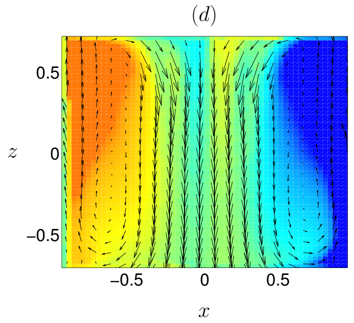


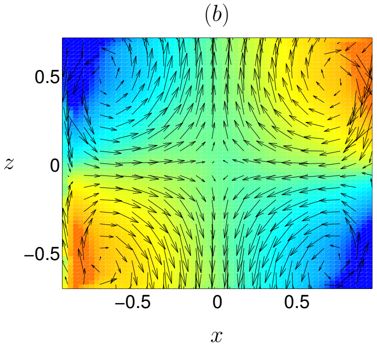


Kuzzay, Faranda and B. Dubrulle , Phys Fluids (2015)

Check LES vs Experiment







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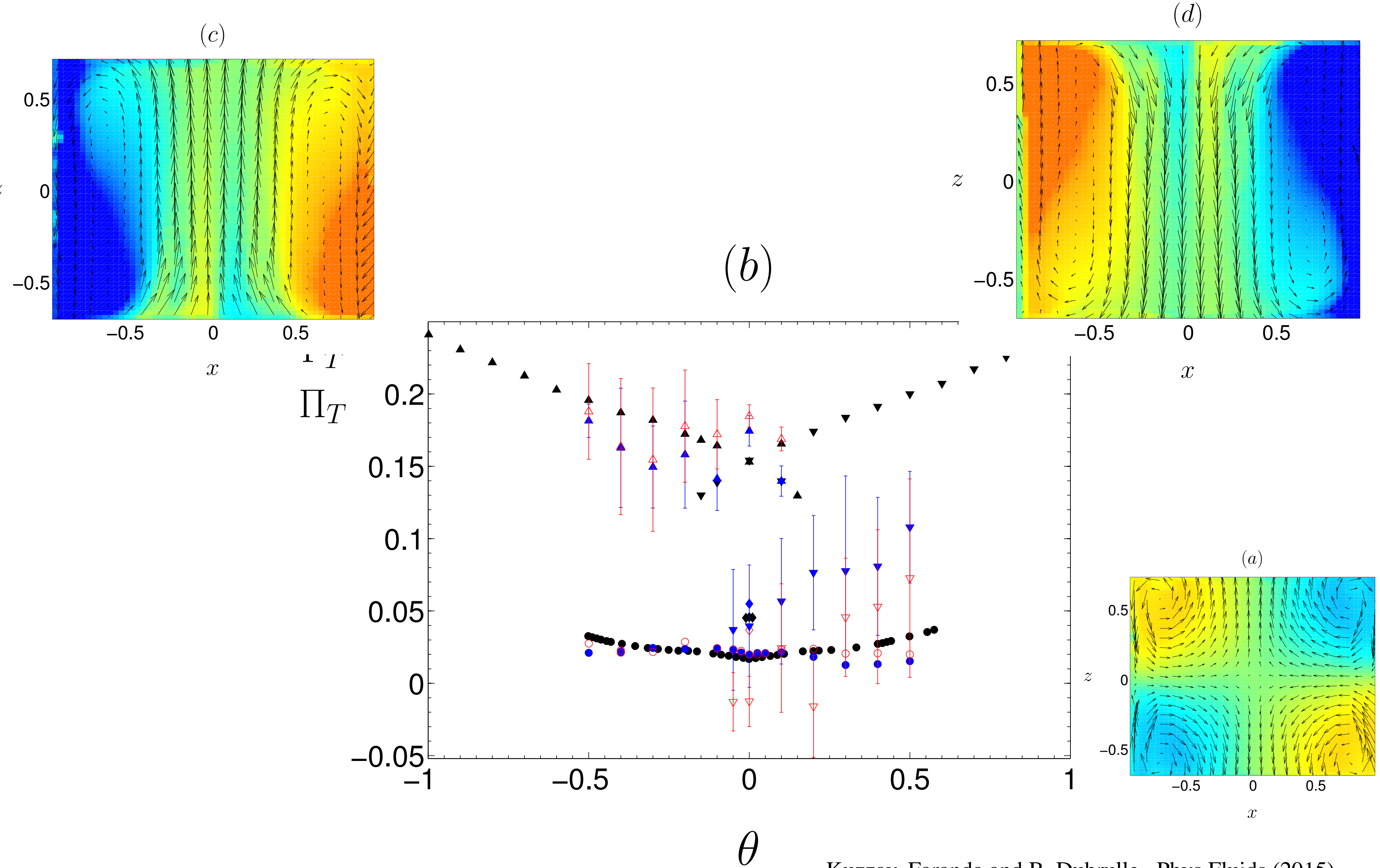


Kuzzay, Faranda and B. Dubrulle , Phys Fluids (2015)

Why planes do flight…

$$\frac{1}{2}\partial_t \left\langle \left(\delta u_\ell\right)^2 \right\rangle + \varepsilon = \nabla_\ell(\nu_T \nabla_\ell \left\langle \left(\delta u_\ell\right)^2 \right\rangle) + \nu \Delta_\ell \left\langle \left(\delta u_\ell\right)^2 \right\rangle$$

Kolmogorov Idea: $\nu_T = F\left(\varepsilon, \ell, \delta u_\ell^2\right)$

On dimensional ground $\nu_T = \varepsilon^\alpha \left(\delta u_\ell^2\right)^{(1-3\alpha)/2} \ell^{1+\alpha}$

Mixing length,smagorinski: $\alpha = 1/3$

Gradient Model: $\alpha = 0$

Why planes do flight…

One scale integration:

$$\varepsilon \frac{\ell}{d} = \left(\nu_T + \nu\right) \nabla_\ell \left\langle \left(\delta u_\ell\right)^2 \right\rangle$$

Two scale integration:

$$\frac{\left(\varepsilon \ell\right)^{1-\alpha}}{d\left(1-\alpha\right)} = K \left\langle \left(\delta u_\ell\right)^2 \right\rangle^{3(1-\alpha)/2}$$

So except if you take alpha=1, you get Kolmogorov spectrum!!!

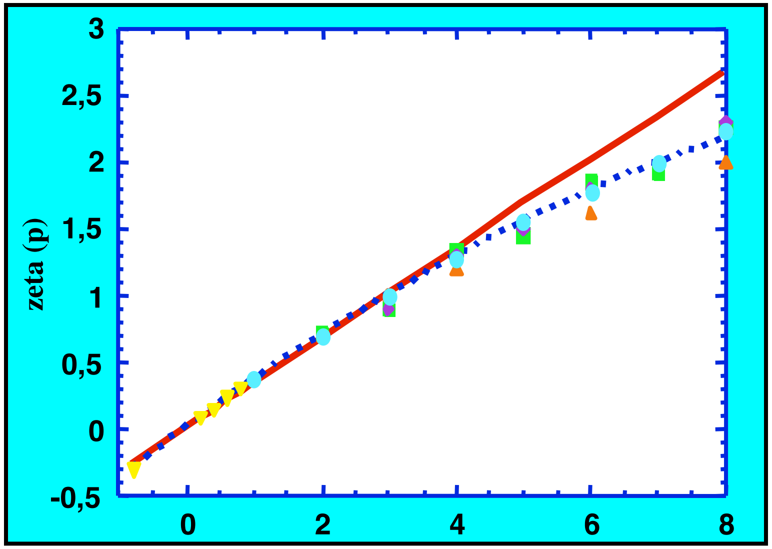
But burners do not burn….

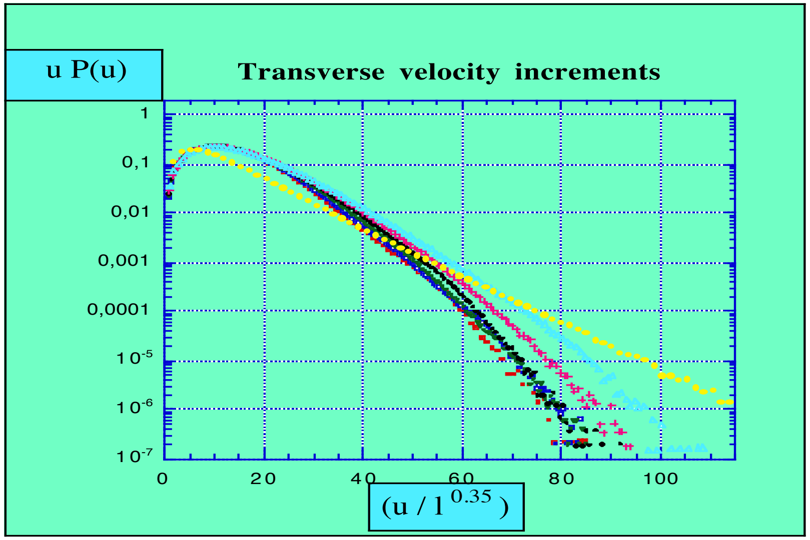
Kolmogorov self-similarity

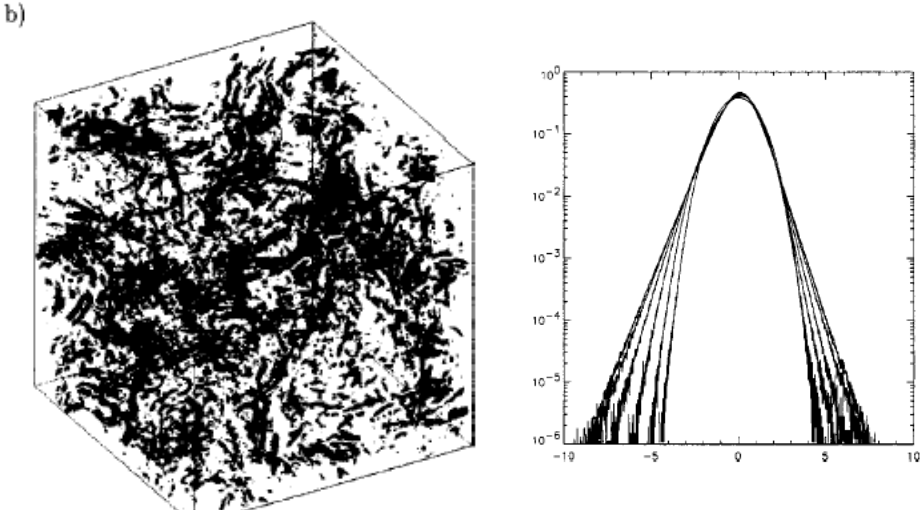
$$\delta u_l = u(x+l) - u(x) \propto l^{1/3}$$

$$\left\langle \delta u_l^n \right\rangle \propto l^{n/3}$$

$$\delta u P(\delta u, l) = F\left(\frac{\delta u}{l^{1/3}}\right)$$







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