Diffraction model for the external occulter of the solar coronagraph ASPIICS

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Outline

1) Proba-3 mission and ASPIICS
2) Diffraction from external occulters
   a) How to compute diffraction
   b) Diffraction patterns for several occulters
3) Penumbra profile
4) Conclusion
Proba-3 mission and ASPIIICS
ESA Proba-3 mission

• In-orbit demonstration of precise **Formation Flying**
• Two spacecraft flying **150m apart**, controlled with a millimeter accuracy
ESA Proba-3 mission

• In-orbit demonstration of precise **Formation Flying**
• Two spacecraft flying **150m apart**, controlled with a millimeter accuracy
• The formation will be **co-aligned with the Sun** during the **6-hours** apogee phase
Solar coronagraph ASPIICS

• Associtation de Satellites Pour l’Imagerie et l’Interférométrie de la Couronne Solaire

• A 1,42m diameter occulting disk carried by the Occulter Spacecraft
  A 5cm Lyot-style coronagraph on the Coronagraph Spacecraft

• Observation of the K-corona
  - Findings on the heating process
  - Alven’s waves, dynamics of the plasma
  - Coronal Mass Ejections

Lamy, 2010
Renotte, 2015

ASPIICS in a nutshell
White light [540nm ; 570nm]
2,8 arcsec/pixel
High cadence
Coronagraphy and diffraction

- The **corona of the sun** is much fainter than the solar disk itself. Observation in white light requires **perfect eclipse conditions**.

\[
B_{\text{corona}} \approx 10^{-6} \text{ to } 10^{-9} B_{\text{sun}}
\]

Allen, 1997
Cox, 2000
Coronagraphy and diffraction

- The hybrid externally occulted Lyot solar coronagraph ASPIICS

![Diagram of coronagraphy and diffraction](image-url)
Coronagraphy and diffraction

- The hybrid externally occulted Lyot solar coronagraph ASPIICS
Coronagraphy and diffraction

- The hybrid externally occulted Lyot solar coronagraph ASPIICS
Coronagraphy and diffraction

- The hybrid externally occulted Lyot solar coronagraph ASPIICS
Diffraction by an external occulter
Diffraction by an external occulter

Point source at $\infty$

External occulter

Plane of the entrance aperture

Point source at $\infty$

\[ \Psi(z(x, y)) \]

\[ \Psi_0(x, y) \]

\[ N_f = \frac{R^2}{\lambda z} \approx 6400 \]

- $\lambda = 550\text{nm}$
- $R = 710\text{mm}$
- $z = 144,348\text{m}$

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Diffraction by an external occulter

• Major issue in solar coronagraphy: the Sun is an extended light source

• The diffraction pattern must be known over a large spatial extent ≠ stellar coronagraphy
## How to compute diffraction?

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<th>Method</th>
<th>Apodisation</th>
<th>No Axis-symmetry</th>
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<td>Boundary diffraction integral</td>
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</table>

- **Apodisation** indicates the method can handle apodization.
- **No Axis-symmetry** indicates the method cannot handle axis-symmetry.

**References**:
- Rougeot & Aime, 2018
- Aime, 2013
- Vanderbei, 2003
- Aime, 2013
- Cady, 2012
- Born & Wolf
How to compute diffraction

• Brute force 2D FFT

The occulter is padded in a 2D arrays

\[ \Psi_z(x, y) = \mathcal{F}^{-1} \left[ \mathcal{F} \left[ \Psi_0(x, y) \right] \times \exp(-i\pi \lambda z (u^2 + v^2)) \right] \]

Occulter \hspace{1cm} Fresnel filter

Condition from the Fresnel filter: \( \sigma > \sqrt{\frac{Kz}{\lambda}} \)

Consequence: **K of very large size**

In Rougeot & Aime 2018, we tried 156000 x 156000, not sufficient for petalized shape
How to compute diffraction

• **Maggi-Rubinowicz representation**

Requires a binary mask (1 or 0)

\[
\Psi_z(x, y) = -\Psi_z^{(d)}(x, y) \quad \text{in the geometrical shadow}
\]

\[
\Psi_z(x, y) = \Psi_0(x, y) - \Psi_z^{(d)}(x, y) \quad \text{otherwise}
\]

**Boundary diffraction integral**  
\[
\Psi_z^{(d)}(x, y) = \frac{1}{4\pi} \oint \text{\(W\) \(dl\)}
\]

Sampling of the occulter edge must be carefully chosen

*Cady, 2012  
Born & Wolf  
Rougeot & Aime, 2018*
Diffraction by an external occulter

- The **sharp-edged** occulting disk

Occulting ratio of 1.05 solar radius at \( z = 144 \text{m} \)
Diffraction by an external occulter

- The **sharp-edged** occulting disk

The bright spot of Arago (or Poisson... demonstrated by Fresnel)

Credit: Minerva.union.edu

Geometrical umbra
Diffraction by an external occulter

- The **sharp-edged** occulting disk
Diffraction by an external occulter

• The **apodized** occulting disk

Variable radial transmission
Diffraction by an external occulter

• The **apodized** occulting disk

\[ \Delta = 20\text{mm} \]
Diffraction by an external occulter

• The *serrated* (or petalized) occulter

In *stellar coronagraphy*, the reasoning starts from the ideal apodized occulter

\[ \tau_{apod}(r) = \int \tau_{petal}(r, \theta) d\theta \]

The petalized occulter is the **discrete substitute**

*Cady, 2006
Vanderbei et al., 2007*
Diffraction by an external occulter

• The **serrated** (or saw-toothed) occulter

In **solar coronagraphy**, the reasoning is well different!

The diffraction occurs perpendicularly to the edge.
A toothed disc rejects the light outside the central region

Boivin (1978) predicted the radius of the dark inner region of the diffraction pattern based on geometrical considerations.
**Diffraction by an external occulter**

- The **serrated** (or saw-toothed) occulter

\[ N_t = 1024 \text{ ; } \Delta = 20\text{mm} \]

\[ R = 710\text{mm} \]

\[ N_t \text{ teeth} \]

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Diffraction by an external occulter

- The **serrated** (or saw-toothed) occulter
Diffraction by an external occulter

• The **serrated** (or saw-toothed) occulter
Diffraction by an external occulter

- The *serrated* (or saw-toothed) occulter

![Graph](image-url)
Diffraction by an external occulter

- The **serrated** (or saw-toothed) occulter

![Diagram showing diffraction patterns and labeled regions A and B.](image)
Diffraction by an external occulter

- The **serrated** (or saw-toothed) occulter
Diffraction by an external occulter

- We numerically verified the geometrical predictions of Boivin (1978)

![Graph showing diffraction by an external occulter](image)

Size of teeth ↗

Boivin radius $B$ (mm)

Number of teeth $N$

$\Delta = 5mm$

$\Delta = 10mm$

$\Delta = 20mm$

$\Delta = 30mm$

Rougeot & Aime, 2018
Diffraction by an external occulter

• The **serrated** (or saw-toothed) occulter
Penumbra profile
• **Convolution** of the diffraction pattern $|\Psi_z(x,y)|^2$ with the solar disk Centre-to-limb darkening function

$R_{\text{sun}} = 16,2'$

$z = 144,348 \text{m}$

Umbra of 38mm
Penumbra profiles

- The **sharp-edged** occulting disk

\[ I_z(r = 0) \approx 10^{-4} I_{\text{sun}} \]
Penumbra profiles

• The *serrated* (or saw-toothed) occulter

\[ \Delta = 20 \text{mm} \]

\[ N_t = 464 \]

- \( N_t \) increases
- \( \Delta \) increases

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Penumbra profiles

- The **serrated** (or saw-toothed) occulter

Integrated illumination over the pupil, normalized to the sharp-edged disk case:

\[
\frac{\int_{\text{pupil}} I_{\text{serrated}} \, ds}{\int_{\text{pupil}} I_{\text{sharp}} \, ds}
\]

Boivin radius \((N_t, \Delta) > r_{\text{sun}} = 671\text{mm}\)
Conclusion
Conclusion

• Model of diffraction for external occulters in solar coronagraphy
  - Limits of Fresnel diffraction integrals using 2D FFT
  - Maggi-Rubinowicz representation

• Assessment of the theoretical performance of serrated occulters

• References
Other works

• Propagation of the diffracted wavefronts inside the Lyot-style coronagraph
  - end-to-end performance in straylight rejection
  - impacts of the size of the internal occulter and the Lyot stop
  - PSF in the vignetting zone

• On-going/future works:
  - optical aberrations of the optics
  - effects of surface roughness scattering
Questions?

Thank you for your attention!
Propagation inside the coronagraph
Propagations inside the coronagraph

- The hybrid externally occulted Lyot solar coronagraph ASPIICS
Propagating inside the coronagraph

- Propagation of the diffracted wavefront from one plane to the next one:
  - Fourier optics formalism
  - Ideal optics
  - Perfect axis-symmetric geometry

- Numerical implementation: successive FFT with arrays of large size

- Objective:
  - Estimate the level and spatial distribution of the residual diffracted sunlight
  - Address the rejection performance of the coronagraph
Propagation inside the coronagraph

- Intensity in plane O’, where the internal occulter is set
Propagation inside the coronagraph

- Intensity in plane O’, where the internal occulter is set

![Graph showing intensity variations with and without external occulter](Rougeot et al., 2017)

With external occulter
Without external occulter
Solar disk image (out-of-focused)
Propagation inside the coronagraph

- Intensity in plane O’, where the internal occulter is set

![Graph showing intensity with and without external occulter]

- With external occulter
- Without external occulter

Solar disk image (out-of-focused)
Propagation inside the coronagraph

- Intensity in plane C, where the Lyot stop is set

Without external occulter

With external occulter
Propagation inside the coronagraph

- Intensity in plane C, where the Lyot stop is set

![Graph showing intensity comparison with and without external occulter](Image)

- With external occulter
- Without external occulter (Lyot coronagraph)
Propagation inside the coronagraph

- Intensity in plane C, where the Lyot stop is set

![Graph showing intensity distribution with and without external occulter](image-url)
Propagation inside the coronagraph

• Intensity in plane D, final focal plane with the detector

With external occulter
Propagation inside the coronagraph

- Intensity in plane D, final focal plane with the detector

![Graph showing intensity in different scenarios]

- No occulter and stop
  - Solar disk image
- Just the external occulter
  - No internal occulter
  - No Lyot stop
- Without external occulter
  - (Lyot coronagraph)
- With external/ internal occulators and Lyot stop

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Propagation inside the coronagraph

- Impact of **sizing** the internal occulter and the Lyot stop
- Intensity on plane D, the final focal plane
Propagation inside the coronagraph

- Impact of sizing the internal occulter and the Lyot stop

Residual diffracted sunlight @ $1.3R_\odot$

Better rejection

Closer to solar edge
Propagation inside the coronagraph

- PSF in the **vignetted zone**

![Image of PSF plots in vignette zones](image-url)
Annex on diffraction formulations
2D FFT technique

The **Fresnel filter** $\exp(\imath \pi \lambda z u^2)$ has its phase varying as $u^2$

At the edge of the array, $u_c = 1/2\sigma$

We impose that the (maximum) phase variation at the edge of the array is $<\pi$

$$\sigma > \sqrt{\frac{\lambda z}{K}}$$

Consequence: $\sigma \downarrow \implies K \uparrow$
2D FFT technique

Very sensitive to numerical sampling: impact of the size of the array
2D FFT technique

Very sensitive to numerical sampling: impact of sampling

Sampling too small regarding Fresnel filter’s condition
Sampling meeting the condition
Sampling too large to correctly sample the occulter
The Hankel transformation

Fourier wave optics formalism
Fresnel free-space propagation
Axis-symmetric (apodized) occulter

\[ \Psi_z(x, y) = (1 - f(r)) \ast \frac{1}{i\lambda z} \exp \left( \frac{i\pi}{\lambda z} (x^2 + y^2) \right) \]

\[ \Psi_z(r) = \frac{\varphi_z(r)}{i\lambda z} \int_0^R 2\pi\rho \times f(\rho) \times \exp \left( \frac{i\pi \rho^2}{\lambda z} \right) \times J_0 \left( \frac{2\pi \rho r}{\lambda z} \right) d\rho \]

Lommel series – decomposition into series (Aime, 2013)
The Hankel transformation

**Fresnel free-space propagation**

Axis-symmetric (apodized) occulter

\[
\Psi_z(x, y) = (1 - f(r)) \ast \frac{1}{i\lambda z} \exp\left(\frac{i\pi}{\lambda z} (x^2 + y^2)\right)
\]

**Radial apodization**

\[
\Psi_z(r) = \frac{\varphi_z(r)}{i\lambda z} \int_0^R 2\pi \rho \times f(\rho) \times \exp\left(\frac{i\pi \rho^2}{\lambda z}\right) \times J_0\left(\frac{2\pi \rho r}{\lambda z}\right) \, d\rho
\]

Lommel series: decomposition into series (Aime, 2013)
Vanderbei et al. approach

Based on Fresnel diffraction theory
For serrated or petal-shaped occulter, i.e. a periodic pattern by rotation

\[
\Psi_z(r, \theta) = \Psi_z^{\text{apod}}(r) + \sum_{j=1}^{\infty} f_1(j, N_t) \times \int_0^{R+\Delta} f_2(j, \rho) \times J_{jN_t} \left( \frac{2\pi r \rho}{\lambda_z} \right) \rho d\rho
\]

In stellar coronagraphy:
\( N_t \approx 20 \), and very small working angles: \( j=1 \) dominates

In solar coronagraphy:
\( N_t \approx 100 - 1000 \), and large region (671mm): the computation is very heavy
Penumbra and Boivin radii
Penumbra for serrated occulters

Convolution of the diffraction intensity $|\Psi_z(x, y)|^2$ with the solar stenope image includes limb darkening function.

$R_{\text{sun}} = 16.2'$

Penumbra: $\int \text{Diffraction} \times \text{Solar image}$
Penumbra for serrated occulters

Diffraction pattern
Solar image

$I(x = 0)$
Penumbra for serrated occulters

\[ I(x_1) \approx I(0) \]

Boivin’s radius

Diffraction pattern

Solar image

\[ R = 710 \text{mm} \]
Penumbra for serrated occulters

\[ I(x_2) > I(0) \]

Boivin’s radius

Intensity

R=710mm

X-axis

\[ I(x_2) > I(0) \]
Penumbra for serrated occulters

\[ I(x = 0) \downarrow \]

Boivin’s radius

Diffraction pattern

Solar image

\[ R_{\text{sun}} = 671 \text{mm} \]
Penumbra for serrated occulters

We can predict the penumbra depth for serrated occulters:

The deepest umbra is achieved when:

$$\text{Boivin radius} (N_t, \Delta) > r_{\text{sun}}$$

The second parameter is the intensity level of the diffraction pattern

$\rightarrow$ Large number of teeth preferred!