# Inversion of IRIS helioseismic data and the solar structure

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## 1. INTRODUCTION

Different methods for inverting the solar oscillations frequencies have been used to infer sound speed and density in the solar interior. The results obtained from the data provided by the various helioseismic experiments lead to the conclusion of an agreement between solar sound speed and the up to date solar model sound speed within $2.10^{-3}$. However a discrepancy appearing just below the convection zone is probably due to the lack of some additional mixing in the model. This feature is well established because many solar oscillation modes are observed which probe this region of the Sun. On the contrary, the structure of the solar core is not yet precisely obtained because few modes penetrate in this region and their frequencies are not easy to determine with great accuracy. It is the aim of the whole solar disc observations like IRIS network to obtain these frequencies. Here we present results on the internal sound speed of the sun obtained by inverting the data provided by these observations. The regularized least square method that we have used (Gonczi *et al* (1998)) is given in section 2. The results obtained with IRIS data are discussed in section 3.

## 2. THE RLS INVERSION METHOD

For each mode $i \equiv \{n, l\}$, $i = 1, N$, the relative differences $\delta\nu_i/\nu_i$ between solar frequencies and the frequencies $\nu_i$ of a solar model are expressed as:

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{R_\odot} K^i_{c_0} \frac{\delta c}{c_0} \mathrm{d}r + \int_0^{R_\odot} K^i_{\rho_0} \frac{\delta\rho}{\rho_0} \mathrm{d}r + \frac{F(\nu_i)}{Q_i} + \epsilon_i \tag{1}$$

Here $N$ is the number of observed frequencies, $\epsilon_i$ is the observational error, $F(\nu_i)/Q_i$ represents the contribution of the uncertainties on physics and structure of surface layers, $Q_i = E_i(\nu_i)/E_{l=0}(\nu_i)$ is a normalized energy of mode $i$, $\delta c/c_0$ and $\delta\rho/\rho_0$ are the relative differences in sound speed and density between the Sun and the model and $K^i_{c_0}$ and $K^i_{\rho_0}$ are the related kernels.

The two independent unknown functions $\delta c/c_0$ and $\delta\rho/\rho_0$ and the surface term $F(\nu_i)$ are developed respectively on piece-wise polynomials of second order in $r$ ($\psi_j(r)$) and Legendre polynomials of $\nu_i/\nu_{max}$ where $\nu_{max}$ is the frequency maximum of the considered set:

$$\frac{\delta c}{c_0} = \sum_{j=1}^{N_f} c_j^1 \psi_j(r) \;\; ; \;\; \frac{\delta \rho}{\rho_0} = \sum_{j=1}^{N_f} c_j^2 \psi_j(r) \;\; ; \;\; \frac{F(\nu_i)}{Q_i} = \sum_{p=1}^{N_s} \beta_p P_p^0(\nu_i/\nu_{max}) \qquad (2)$$

The coefficients $\mathbf{C} = \{c_j^1, c_j^2, \beta_p\}_{\substack{j=1,N_f \\ p=1,N_s}}$ are obtained by minimizing the quantity:

$$\mathbf{J}(\mathbf{C}) = (N - N_s - 2N_f)\chi_\nu^2 + \mu_c T_r = \chi^2 + \mu_c T_r \qquad (3)$$

with $\chi^2 = \sum_1^N r_i^2$

$$r_i = \frac{\frac{\delta\nu_i}{\nu_i} - \left[\int_0^{R_\odot} K_{c_0}^i \frac{\delta c}{c_0} \mathrm{d}r + \int_0^{R_\odot} K_{\rho_0}^i \frac{\delta \rho}{\rho_0} \mathrm{d}r + \frac{F(\nu_i)}{Q_i}\right]}{\sigma_i} \qquad (4)$$

and $T_r$ is a smoothing term:

$$T_r = \int_0^{R_\odot} f(r) \left[ \left( \frac{\mathrm{d}^k}{\mathrm{d}r^k} \frac{\delta c}{c_0} \right)^2 + \frac{\mu_\rho}{\mu_c} \left( \frac{\mathrm{d}^k}{\mathrm{d}r^k} \frac{\delta \rho}{\rho_0} \right)^2 \right] \mathrm{d}r \qquad (5)$$

$\sigma_i^2$ is the variance of observational errors assumed to be gaussian. We have taken $f(r) = 1$ for $k = 1$ and $f(r) = r^{-2}$ for $k = 2$. The solution of the inversion depends crucially on the choice of the different parameters which are to be fixed: the number $N_s$ of components in the surface term development, the trade-off parameter $\mu_c$, the ratio $\mu_\rho/\mu_c$, the number $N_f$ of the grid points and their distribution, here taken according to the density of turning points along the radius.

The number of grid points and the number of surface terms have been determined by looking at the condition number of the system of Euler equations derived from the minimization of $\mathbf{J}(\mathbf{C})$ (ratio of larger to smaller singular value of the system) and at the residuals $r_i$ which should be randomly distributed as a function of the frequency and turning points (Basu & Thompson (1996)). This property is tested by computing the number $N_{ri}$ of sign changes of the residuals when they are ordered in increasing frequencies or turning points radius and comparing this number to the expected value $N/2 \pm \sqrt{N}/2$ when the distribution is random. The factor $q = (2N_{ri} - N)/\sqrt{(N)}$ introduced in Gonczi *et al* (1998), $q_\nu$ (when ordered in frequencies) and $q_{rt}$ (when ordered in increasing turning points positions) must be less than 1.

The trade-off parameter $\mu_c$ is determined by plotting the so-called L-curve which gives the smoothing term $\log(T_r)$ as a function of $\chi_\nu^2$ at given $\mu_\rho/\mu_c$ ratio. We have to find a compromise between a good fit of the data, which means a small $\chi_\nu^2$, and a rather smooth physically acceptable solution, which means a small regularization term $\log(T_r)$.

In Gonczi *et al* (1998), two smoothing constraints, first and second derivative ($k = 1, 2$ in Equation 5) of the sound speed and density relative differences have been tested and it shows that in the solar core, the result below $r < 0.17$ is not reliable and depends strongly on the regularizing constraint.

## 3. INVERSION OF THE SOLAR STRUCTURE

We need to invert globally the frequencies of modes of all degrees which means that generally data from resolved observations with $l > 2$ or 3 are complemented with whole disc observations

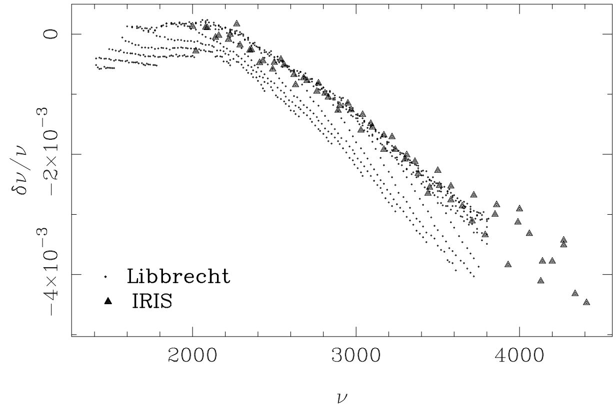


*Figure 1. Relative difference between the solar observed frequencies given by IRIS data for $l = 0$ to 3 and Libbrecht data for $l > 3$ and the theoretical frequencies of the standard solar model.*

providing more accurate frequencies for low degree modes $l < 3$ or 4. Low degree frequencies from IRIS observations are given by Gelly *et al* (1997) for the three years 1990, 1991, 1992. Due to the shift of frequencies with time induced by the solar cyle effect on the solar structure, we restrain our inversion to combination of IRIS data from 1990 with Libbrecht data obtained at Big Bear Solar Observatory for $l = 3$ to 60 at the same epoch. The relative difference between these frequencies and the frequencies of a theoretical standard model, here the model S of Christensen-Dalsgaard *et al* (1996) is shown in figure 1.

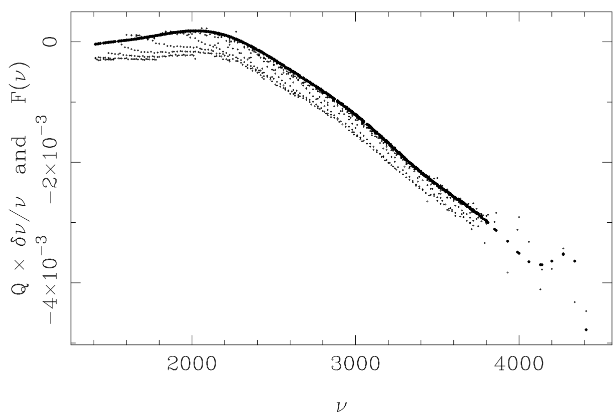


*Figure 2. Normalized relative frequency differences between the Sun and the solar model and the function $F(\nu)$.*

The inversion is carried according to Gonczi *et al* (1998) with first derivative regularization term. The function $F(\nu)$ is plotted in figure 2. It fits the behavior relatively to frequency of the normalized relative difference between observed and theoretical frequencies which is due to surface effects. A good determination of this function is important because it affects the solution for the sound speed in the solar core.

The difference between solar sound speed and the sound speed of the standard model obtained by inversion is plotted as a function of the radius in figure 3. The result is very similar to that obtained by inverting the GOLF/MDI 144 days data with a maximum of the difference below the convection zone at $r = 0.68R_\odot$, a negative value in the solar core from $r = 0$ to $0.3R_\odot$ and

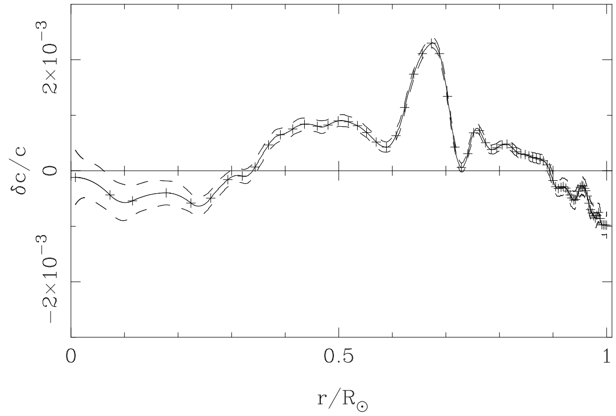


*Figure 3. Relative sound speed difference between the Sun and the solar model induced from IRIS data for $l = 0$ to 3 and Libbrecht data for $l = 4$ to 60. The dashed lines represent the uncertainty on the results of inversion arising from the errors in the data.*

a minimum around $0.2R_\odot$. We note that this minimum is smaller than for GOLF/MDI data (see Gonczi *et al* (1998)) but the results in this region are not reliable enough to be interpreted in terms of the evolution of the solar structure.

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