EFFECT OF SUBSURFACE RADIAL DIFFERENTIAL ROTATION ON FLUX-TRANSPORT SOLAR DYNAMOS

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ABSTRACT

A near-surface radial gradient of rotation was recently inferred and quantified from MDI f-modes observations by Corbard & Thompson (2001). We show, from our preliminary simulation of the large-scale solar magnetic field, by using a flux-transport type dynamo, that despite being strong, this gradient plays only a small role compared to the tachocline in shearing the poloidal fields to produce toroidal fields. This happens primarily because the turbulent diffusivity near the surface wins in the competition of generation versus decay of the magnetic fields unless an abnormally high α-effect is considered there. This supports the results of previous flux-transport as well as interface and overshoot layer dynamo models that the major toroidal fields of the Sun are generated in the tachocline.

Key words: Sun; Dynamo; Rotation.

1. INTRODUCTION

Recent analysis of MDI observations of surface gravity waves from SoHO, by Corbard & Thompson (2001) has quantified the strong radial shear that resides in a thin layer just below the solar surface for depths down to about 15Mm. Motivated by these observations (reviewed in Section 2), we build new analytical models of the rotation rate in the convection zone (Section 3) and we investigate the role of this subsurface radial differential rotation in global solar dynamos (Section 4).

Flux-transport dynamos have recently been successful in reproducing many large-scale solar cycle features, including a difficult one, the phase relationship between the toroidal and poloidal components. These models can also produce the correct dynamo cycle period when the amplitude of the meridional flow that is observed near the surface is used. These models invoked solar-like differential rotation that contains a strong radial gradient in the tachocline. The strong toroidal fields are generated in the tachocline in such models. The surface eruption of these fields as bipolar spots is believed to be due to their buoyant rise through the convection zone.

The recent discovery of strong radial shear at the surface raises the questions: (i) where is the solar dynamo? (ii) Is it operating primarily near the surface? (iii) Do the toroidal fields manifest directly after they are generated there? (iv) Are the poloidal fields also generated there by the decay of active regions? (v) Is there no longer a need for flux-storage in the subadiabatically stratified overshoot layer and radiative zone? To answer these questions, we simulate a kinematic flux-transport dynamo incorporating this newly observed solar rotation profile.

To match the model output with the observed 'butterfly' and 'reverse butterfly' patterns, we produce a theoretical time-latitude diagram respectively for the solar toroidal and poloidal fields. Furthermore, we compare the field strengths produced in the tachocline and at the surface to see which radial shear (in the tachocline or near the surface) plays the more important role in the operation of the large-scale solar dynamo.

2. OBSERVATION OF THE SUBSURFACE SHEAR

In order to infer the radial gradient of angular velocity close to the surface, Corbard & Thompson (2001) used the small, but significant, radial dependance of the f modes with degrees between l=117 and l=300 observed by MDI between May 1996 and April 2001 (Schou, 1999). The outward gradient is found to be negative with a value of about 400nHz/R up to 30° latitude, it decreases to a small value above 30° and approaches zero at about 50° latitude(Fig. 1, dashed line).

At higher latitudes the gradient may reverse its sign: if so, this reversal takes place in a thin layer extend-

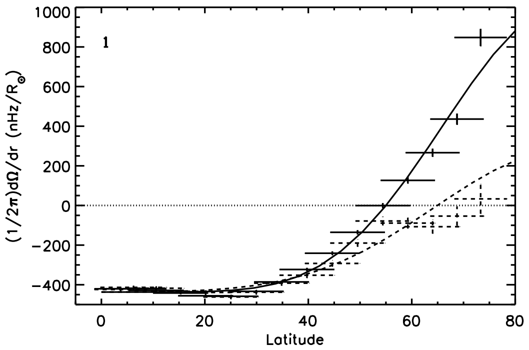

Figure 1. Average radial gradient of angular velocity as a function of latitude. **Full line:** *for depths down to about 15Mm (modes $160 \leq l \leq 250$).* **Dashed line:** *for depths down to about 5 Mm (modes $250 \leq l \leq 300$)*

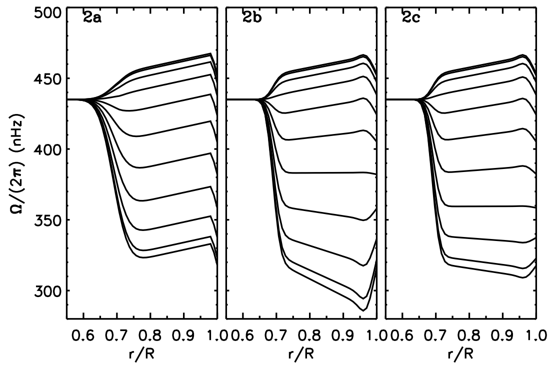

Figure 2. Profile of the solar angular velocity between 0.55R and the surface at different latitudes from equator (top) to pole (bottom), every 10 degrees. **a)** *The original model of Kosovichev (1996),* **b)** *model with the surface radial gradient inferred from Fig. 1 (full line),* **c)** *model with the surface radial gradient inferred from Fig. 1 (dashed line)*

ing only 5 Mm beneath the visible surface as evidenced by the most superficial modes with degrees $l > 250$ but can reach up to $800\mathrm{nHz}/R$ at 75-degree latitude (Fig. 1, full line).

3. ANALYTICAL MODELS OF THE ROTATION RATE

It has been shown by various authors that the meridional circulation takes the primary role in transporting the magnetic flux in flux-transport type dynamos. If that is the case, the toroidal fields generated by the near-surface radial shear will be transported towards the pole. But a near-surface radial gradient at low latitude, in conjunction with a poloidal source-term (such as a Babcock-Leighton type source, or an α-effect) could produce another propagation speed – the classical dynamo speed, according to Parker-Yoshimura sign rule. This speed is expected to compete with the advective speed due to poleward flow and would either enhance or reduce the net dynamo wave speed at the surface. Consequently, the magnetic patterns will drift either poleward or equatorward depending on the relative strengths of the aforementioned two speeds. To test this hypothesis we have constructed two analytical solar rotation models based on the observations described in the previous section.

To investigate how the dynamo works in the presence of such a strong near-surface radial shear, we prescribe a differential rotation profile by using a simple analytical expression of the rotation rate as a function of latitude and radius in the convection zone and the tachocline.

The model used here is described as follow (from the core to the surface): The rotation rate is taken constant in the radiative interior $[\mathbf{\Omega_0}]$, then there is the tachocline which has the same location $[\mathbf{r_{tac}}]$ and width $[\omega_{\mathbf{tac}}]$ at all latitudes and such that, at the top of the tachocline $[\mathbf{r_{cz}}]$ the rotation rate is given by Ω_{cz}, $[\mathbf{a_2}, \mathbf{a_4}]$ following:

$$\Omega(r_{cz}, \theta) = \Omega_{cz} + a_2\mu^2 + a_4\mu^4. \qquad (1)$$

Assuming no net torque across the tachocline we have:

$$\Omega_{cz} = \Omega_0 - \frac{a_2}{5} - \frac{3a_4}{35} \qquad (2)$$

The differential rotation coefficients (i.e. a_2 & a_4) are chosen to be the same at the surface ($r = R$) as at $r = r_{cz}$ (top of the tachocline) but the equatorial rate at the surface is given by $[\mathbf{\Omega_{eq}}]$ which may differ from Ω_{cz}.

We assume that there is a known gradient below the surface down to a radius $[\mathbf{r_s}]$. This depth is independant of latitude but the gradient depends on the latitude through $[\beta_\mathbf{0}, \beta_\mathbf{3}, \beta_\mathbf{6}]$:

$$\beta(\mu) = \beta_0 + \beta_3\mu^3 + \beta_6\mu^6 \qquad (3)$$

A (small) radial gradient is added in the convection zone in order to link the rotation rates at r_s and r_{cz}. Therefore (a_2, a_4) describe the differential rotation at r_{cz} and r_s but the differential rotation is varying slightly in between because of the gradients introduced.

The transition between different gradients are modeled by error functions centered at r_{tac}, r_{cz} and r_s and widths ω_{tac}, $[\omega_{\mathbf{cz}}, \omega_{\mathbf{s}}]$:

$$\phi_x(r) = 0.5(1 + erf(2(r - r_x)/\omega_x)), \qquad (4)$$

where x stands for tac, cz or s.

This model includes 13 parameters (see brackets in the description above) and can be written as:

$$\Omega(r,\mu) = A_1(r,\mu) + \phi_{tac}(r)(\Omega_{cz} - \Omega_0 + a_2\mu^2 + a_4\mu^4), \qquad (5)$$

with:

$$\begin{cases} A_1(r,\mu) & = \Omega_0 + \phi_{cz}\left\{\alpha(\mu)(r-r_{cz})\right\} + \phi_s(r)\times \\ & \quad \left\{\Omega_{eq} - \Omega_{cz} - \beta(\mu)(r-R) - \alpha(\mu)(r-r_{cz})\right\} \\ \alpha(\mu) & = \left\{\Omega_{eq} - \Omega_{cz} + \beta(\mu)(R-r_s)\right\}/(r_s - r_{cz}). \end{cases}$$

This form can be related to the original formulation proposed by Kosovichev (1996) (cf. Fig 2a) where the parameters were fixed by fitting the tachocline parameters from BBSO observations and assuming conservation of the angular momentum in the supergranulation layer.

In the two models proposed here we keep the parameters obtained by Kosovichev (1996) except for:

(i) the width of the tachocline, which is taken thinner according to more recent determinations (see the review from Corbard et al., 2001) ($\omega_{tac} = 0.05R$);

(ii) the position of the near-surface maximum, which is arbitrarily taken at the base of the layer sensed by the observed f modes ($r_s = 0.97R$);

(iii) the surface radial gradient, which is obtained by fitting the two cases presented in the previous section with Eq. (3) (in nHz/R):

- (a) β_0=437, β_3=-214, β_6=-503 (cf. Fig 2b)
- (b) β_0=437, β_3=0, β_6=-1445 (cf. Fig 2c)

(iv) the widths of the transitions at r_{cz} and r_s which are taken to be nonzero in order to avoid discontinuities in the derivative ($\omega_{cz} = \omega_s = 0.05R$);

4. WHERE IS THE SOLAR DYNAMO?

To focus primarily on the question of whether the major toroidal fields are generated near the surface or at the tachocline, we select the framework of a Babcock-Leighton type flux-transport dynamo. The reason is that the poloidal fields in Babcock-Leighton models are also generated near the surface, and therefore they can be available immediately after they are born for further generation of toroidal fields by the action of strong, surface radial shear. In a future paper, we shall explore how the poloidal fields generated at the tachocline can contribute, after they are transported to the surface, to the generation of the toroidal fields by this near-surface radial shear. To compute the dynamo, we prescribe a solar rotation profile parametrized from the recent observation of Corbard & Thompson (2001) (see Fig 2b). We keep the other ingredients – the meridional flow pattern, a Babcock-Leighton type poloidal source term that depends on latitude and magnetic field strength, and a depth-dependent diffusivity profile – as in Dikpati & Charbonneau (1999). Applying similar boundary conditions and employing the same numerical technique, we solve the dynamo equations 3a and 3b of Dikpati & Charbonneau (1999). We extract the solutions for the evolution of toroidal and poloidal fields at the required depths, and plot in time-latitude diagrams.

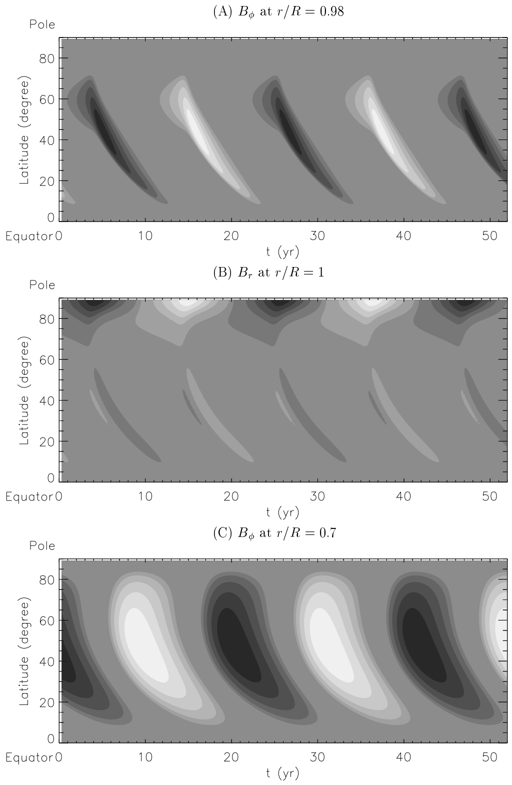

Figure 3. Time-latitude diagrams for the (A) toroidal field near the surface, (B) surface radial field, (C) toroidal field at the tachocline, obtained from a kinematic flux transport dynamo operating with surface radial shear. In (A), white shade represents 1.7 kGauss positive toroidal field, whereas in (C) the same shade represents 68 kGauss field. The dark shades represents negative fields (negative toroidal fields are pointing out of the plane of paper and negative radial fields are directed upward).

Figure 3 presents three such diagrams to show the evolutionary pattern of dynamo generated magnetic field components: toroidal fields close to the surface (at $0.98R$) (see top frame), surface radial fields (see middle frame) and finally the toroidal fields at the convection zone base (bottom frame). Note that the toroidal field at the top boundary is zero, therefore we extract the solution at $r = 0.98R$, at the center of the thin radial shear layer near the surface. The field strength (68 kGauss) generated at the convection zone base is about 40 times the field (1.7 kGauss) generated at $0.98R$. To make all the time-latitude diagrams visible, the contour shadings have been constructed in such a way that the same pure-white shade implies 1.7 kGauss field in the top frame, but 68 kGauss in the bottom frame.

The equatorward migration of the near-surface toroidal fields appears intriguing at first sight. What causes the equatorward migration of the near-surface toroidal fields, particularly when the advection is poleward there? Is it the classical speed determined by the product of a low-latitude negative shear and a positive α-effect near the surface? Most likely what we are seeing are the diffused parts of the tachocline-generated toroidal fields which are drifting equatorward by meridional flow there. To find a definite answer for the cause of this equatorward migration, we need to perform further studies. That will help determine the parameter values for which the classical dynamo wave speed near the surface can dominate over the advective speed.

Table 1 below presents the field strength generated at the convection zone base and near the surface when the surface radial shear is present and also when it is absent. We find that the presence of surface radial shear hardly contributes to the toroidal fields at $0.98R$, increasing the strength by 0.5 kGauss only. The toroidal fields generated in the shear layer located at or below the subadiabatically stratified convection zone base undergo much less decay than those generated near the surface. In this calculation, our choice of diffusion coefficient is $3\times10^{11}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$ in the main bulk of the convection zone whereas that at the tachocline is $3\times10^{8}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$. We ensure, by running a case with constant diffusivity ($10^{11}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$) throughout our computation domain ($0.6R$ to $1R$ in the northern meridional cut) that strengths of the toroidal fields generated at the tachocline and near the surface are comparable (see the third row in the Table 1).

Table 1. Dynamo-generated field strengths at different depths

$\frac{\eta_T}{\eta_{core}}$	$\frac{\partial\Omega}{\partial r}\|_{r=0.98R}$	$B_\phi\|_{r=0.98R}$	$B_\phi\|_{r=0.7R}$
1000	present	1.7 kGauss	68 kGauss
1000	absent	1.2 kGauss	68 kGauss
1	present	5.7 kGauss	28 kGauss

A toroidal field of strength $\sim 1-2$ kGauss can produce 3 kGauss sunspots if some mechanism, such as convective collapse, works there to concentrate the flux. However, the polarities of the leader and follower spots, produced from these near-surface toroidal fields, would appear with a phase completely opposite to that observed. The top two frames of Figure 3 reveal that B_r is in approximately $90°$ phase lead with respect to the $B_\phi|_{r=0.98R}$ rather than an observed $90°$ phase lag. On the other hand, if the spots are generated from the buoyantly rising toroidal fluxtubes from the base of the convection zone, they would appear with a correct phase with respect to their vector counterparts at the surface. Clearly the polar fields (in Figure 3B) are seen to reverse their sign from positive (white) to negative (black) when the subsurface toroidal field (in Figure 3C) is already negative (black).

5. CONCLUSIONS

Although the recent discovery of surface radial shear should intrigue the dynamo theorists to revisit an old question regarding the location of solar dynamo, our preliminary simulation of a kinematic flux-transport dynamo operating in the presence of this surface radial shear suggests that the strongest toroidal fields in the Sun are generated at the convection zone base, and manifest at the surface as bipolar spots due to their buoyant rise through the convection zone.

While the possibility of flux-concentration by convective collapse near the surface, and hence the 3 kGauss spot formation even from $\sim 1-2$ kGauss field cannot be ruled out, the polarities of the spots formed from this near-surface toroidal field would violate the observed relation between their polarity rule and the sign of the polar fields.

The presence of the surface radial shear cannot contribute much to the toroidal field generation there, because the large decay due to turbulent diffusion dominates over the field amplification.

The near-surface toroidal fields are not weak in this model, because the poloidal fields themselves are not weak ($\sim 100-200$ Gauss). Turbulent diffusion of $10^{11-12}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$ is not enough to reduce them. One remedy for this problem could be that a much higher supergranular diffusion ($10^{13-14}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$) near the surface destroys much of the poloidal flux before the polar concentration takes place. A forthcoming paper will explore the possibility of producing a solar-like polar field through a Babcock-Leighton source.

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