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ASTRONOMY AND ASTROPHYSICS

# Complex dynamical behaviour of the asteroid 2335 James associated with the secular resonances $\nu_5$ and $\nu_{16}$: numerical studies and theoretical interpretation

**Ch. Froeschlé**[1], **A. Morbidelli**[2], **and H. Scholl**[1]

[1] Observatoire de la Côte d'Azur, Laboratoire G.D. Cassini, CNRS URA 1362, B.P. 139, F-06003 Nice Cedex, France
[2] Department of Mathematics, Facultés Universitaires ND de la Paix, Rempart de la Vierge 8, B-5000 Namur, Belgium

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**Abstract.** A forwards integration in the frame of the Sun–Jupiter–Saturn model over 1 Myr shows that the asteroid 2335 James is located in the secular resonance $\nu_{16}$. James is a $\omega$-librator during the whole period. A complex orbital evolution appears in a backwards integration over 5 Myr: James is temporarily located also in the $\nu_5$ secular resonance. The argument of perihelion $\omega$ alternates between circulation and libration. A theoretical explanation for this complex behaviour is provided. In a Sun–Mars–Jupiter–Saturn model, close encounters of James with Mars occur without ejecting James from the $\nu_{16}$ resonance region. The integration of a cloud of test objects surrounding the orbit of James confirms the chaotic nature of this resonance region as indicated by theory.

**Key words:** asteroids – celestial mechanics

## 1. Introduction

The three dominant resonances $\nu_5$, $\nu_6$ and $\nu_{16}$ cross the asteroid belt and play an important role in the orbital evolution of asteroids (e.g. Scholl et al. 1989). An asteroid is located in a secular resonance if its orbital precessional frequency $\dot{\varpi}$ or $\dot{\Omega}$ is equal to one of the eigenfrequencies of the system of planetary orbits in the frame of secular perturbation theory; $\varpi$ and $\Omega$ designate the longitude of perihelion and the longitude of ascending node, respectively. In the case of the secular resonance $\nu_{16}$, the average rate of the longitude of node of an asteroid is nearly equal to that of Jupiter; a strong increase in inclination of the asteroid orbit may be expected. At the resonances $\nu_5$ and $\nu_6$, where the mean rate of an asteroid's longitude of perihelion is nearly equal to the mean rate of the respective longitudes of perihelion of Jupiter and Saturn, large increases in eccentricity may occur for a secular resonant orbit.

Williams (1973a, b), Wetherill & Williams (1979), and Wetherill (1979) suggested the resonances $\nu_5$ and $\nu_6$ as sources of meteorites. Furthermore, Wetherill (1979) suggested that the resonance $\nu_{16}$ might be expected to cause large amplitude oscillations in inclination, and could be of importance to understand the origin of highly inclined Apollo-Amor asteroids. In a series of articles Froeschlé & Scholl (1986), Scholl & Froeschlé (1986) investigated by purely numerical integrations over $10^6$ years the orbital evolution of *test* asteroids located in or near these three secular resonances. The quantitative results are in agreement with those obtained by Williams. Asteroids with semimajor axes smaller than 2.6 AU located in the $\nu_5$ resonance were found to become Mars-crossers. Asteroids located in the $\nu_6$ resonance with a semimajor axis of 2.06 AU became Earth-crossers and even Venus-crossers. We showed that the secular resonance $\nu_{16}$ may cause large increases in inclination up to 20°. In particular, an asteroid entering the $\nu_{16}$ resonance at low inclination may pump up its inclination to a value of $\sim 23°$ and keep this inclination when leaving the $\nu_{16}$ resonance.

The orbital evolution of *known* asteroids situated in or near the $\nu_6$ and $\nu_5$ resonances was studied by Froeschlé & Scholl (1987), Scholl & Froeschlé (1990).

Asteroids located in the $\nu_6$ resonance and with a semimajor axis exceeding 2.6 AU do not become Earth-crossers. It seems also that the $\nu_5$ region at semimajor axes larger than 2.6 AU is not a good candidate source for Mars- and Earth-crossers. However, as suggested by Wetherill (1987), we found that the $\nu_5$ resonance region near 2.2 AU is a good candidate source for highly inclined ($>30°$) Apollo asteroids and highly inclined meteorites. The orbital evolution of (1580) Betulia with a semimajor axis of 2.2 AU investigated over 5 Myr (backwards) shows that the orbits of objects entering this $\nu_5$ resonance region with an inclination near 20° and an eccentricity of the order of 0.3 may be transformed into highly inclined orbits (up to 70°) and with a large eccentricity near 0.7. Such orbits become planet-crossing.

It remained to investigate the orbital evolutions of *known* asteroids located in the $\nu_{16}$ secular resonance. A major aspect of this investigation is to understand the role of this secular resonance for the delivery of planet-crossing objects as was done for the $\nu_6$ and $\nu_5$ secular resonances. According to Williams' (1989) list, only one asteroid, named (2335) James, is suggested to be located in the $\nu_{16}$ secular resonance. James is located at the interior edge of the main belt with a semimajor axis of 2.12 AU.

The present inclination of James is about 36°, which is a comparatively high value as compared to the mean inclination of $\simeq 14°$ of all asteroids. Also its eccentricity of 0.36 is large as compared to the mean value of 0.15 for the eccentricities of all asteroids. Since the perihelion distance of 1.36 AU is smaller than the semimajor axis of Mars (1.52 AU), James can be considered

*Send offprint requests to*: Ch. Froeschlé as a Mars-crosser. However, due to its high inclination, close encounters with Mars are not possible at present. What is the origin of the comparatively high inclination and eccentricity of James? We try to contribute to answer this question in the present paper.

In Sect. 2 we present the results of a forwards integration over 1 Myr and of a backwards integration over 5 Myr in the frame of the Sun-Jupiter-Saturn model. In the backwards integration we tried to find out whether or not James was recently locked into the $\nu_{16}$ secular resonance. We found a complex orbital evolution for James: this asteroid appears to be located temporarily in the two secular resonances $\nu_5$ and $\nu_{16}$ simultaneously. A theoretical explanation for the complex orbital evolution indicating chaotic behaviour is presented in Sect. 3. Of course, since James appears to be on a chaotic orbit, the orbital evolution of James shown in Sect. 2 does not necessarily represent its real orbital evolution. In order to estimate the possible orbital evolutions, we integrated a cloud of 6 objects surrounding James' orbit over 1 Myr backwards in the Sun-Jupiter-Saturn-Mars model. The results presented in Sect. 4 yield qualitatively similar orbital evolutions as compared to the one shown in Sect. 2. The various orbital evolutions differ in particular with respect for timescales.

## 2. Numerical results on the orbital evolution of 2335 James

### *2.1. Forwards integration over 1 Myr*

Like in the previous papers, only Jupiter and Saturn were the perturbing planets, i.e. the numerical integrations were carried out in the frame of the four body problem Sun–Jupiter–Saturn –asteroid. We used the N-Body program of Schubart & Stumpff (1966), which is a pure predictor method of Adam's type with fixed step size. The integration period is about 1 Myr. A step size of four days was adopted. The starting orbital osculating elements were taken from a list compiled by L. Schmadel (Table 1).

As in Froeschlé & Scholl (1987), we apply the two following criteria to find $\nu_{16}$ secular resonance cases:

— The time average of the precession rate $\langle\dot{\Omega}\rangle$ is nearly equal to $\dot{\nu}_{16}$.

— The secular argument $\Omega-\Omega_J$ (where the subscript J denotes Jupiter) librates either around 0° or 180°. When Jupiter and Saturn are the only perturbing planets, the eigenfrequency $\dot{\nu}_{16}$ is practically equal to $\langle\dot{\Omega}_J\rangle$ and also to $\langle\dot{\Omega}_S\rangle$, since the ascending nodes of Jupiter's and Saturn's orbits are always opposite (see Charlier 1902).

Averaging the derivatives $\dot{\Omega}_J=d\Omega_J/dt$ and $\dot{\Omega}_S=d\Omega_S/dt$ over the whole integration period, we found:

$\langle\dot{\Omega}_J\rangle=\langle\dot{\Omega}_S\rangle=-26''.02\ \mathrm{yr}^{-1}$.

For the asteroid James we obtain an averaged value of $\langle\dot{\Omega}\rangle=-26''.04\ \mathrm{yr}^{-1}$. Moreover, the resonance argument $\Omega-\Omega_J$ librates about 0° as shown in Fig. 1. Moreover, in Fig. 2 one clearly sees the banana shape of the libration in polar coordinates inclination, $\Omega-\Omega_J$. The libration is clockwise.

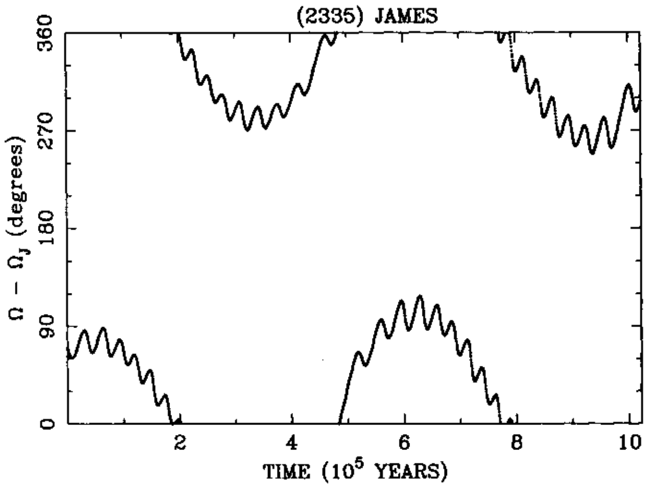


**Fig. 1.** The time evolution of the resonance argument $\Omega-\Omega_J$ of (2335) James in the Sun-Jupiter-Saturn model; forwards integration over 1 Myr

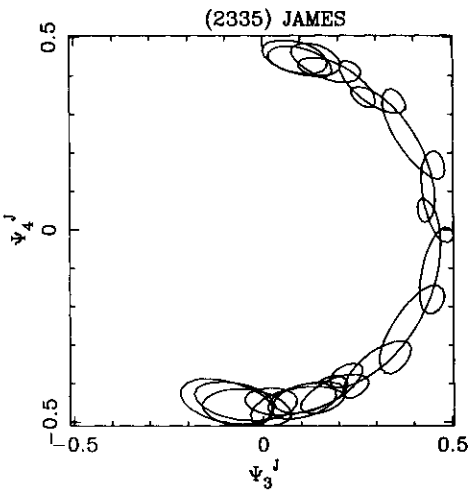


**Fig. 2.** Orbit of (2335) James in a polar diagram. The axes are defined by $\Psi_3^J=[2(1-e^2)^{1/2}(1-\cos i)]^{1/2}\cos(\Omega-\Omega_J)$, $\Psi_4^J=[2(1-e^2)^{1/2}(1-\cos i)]^{1/2}\sin(\Omega-\Omega_J)$. Same model as in Fig. 1, forwards integration over 7 $10^5$ yr

**Table 1.** Starting values of (2335) James

Epoch 2445200,5 J.D. Ecliptic 1950,0. Equinox 1950,0.
The elements were provided by L. Schmadel, Astron. Rechen-Inst.

| $\omega$ | $\Omega$ | $i$ | $e$ | $a$ (AU) | $M$ (mean anomaly) |
|---|---|---|---|---|---|
| 70°49311 | 19°55953 | 36°34100 | 0.359263 | 2.1229533 | 154°47157 |

It is very interesting to note that test asteroids with inclinations not exceeding 10° were found to librate only around 180° (Scholl & Froeschlé 1986). In these cases, the banana shaped libration is counter-clockwise.

According to Fig. 3, James' argument of perihelion $\omega$ is librating about 90° in accordance with Vashkov'yak (1986). This is the first known *ω-librating* asteroid located in a secular resonance. The libration of $\omega$ is associated with the short periodic variations of both the critical argument $\Omega-\Omega_J$ (Fig. 1) and of the inclination (Fig. 4). We note a large variation in inclination in the range $34° \leq i \leq 46°$ which is due to two phenomena. Firstly, a periodic variation (period of $\simeq 32.000$ yr) associated with the libration of $\omega$ with amplitudes between 4° and 8°. Secondly, a long periodic variation with a period of $\simeq 6\,10^5$ yr and an amplitude of $\simeq 3°$ associated with the libration of the critical argument $\Omega-\Omega_J$. The amplitude of $\simeq 3°$ is obtained by averaging over the short periods. Hence, the main contribution to the inclination variations is due to the $\omega$-libration.

An analogous behaviour only with respect to periods can be seen for the eccentricity in Fig. 5 whereas the eccentricity variations associated with the two periods have the same amplitude of about 0.2. The total eccentricity variation is in the range $0.05 < e < 0.52$. This makes James a temporary Mars-crosser. Therefore, it is natural to ask wether or not James was recently captured in the $\nu_{16}$ secular resonance.

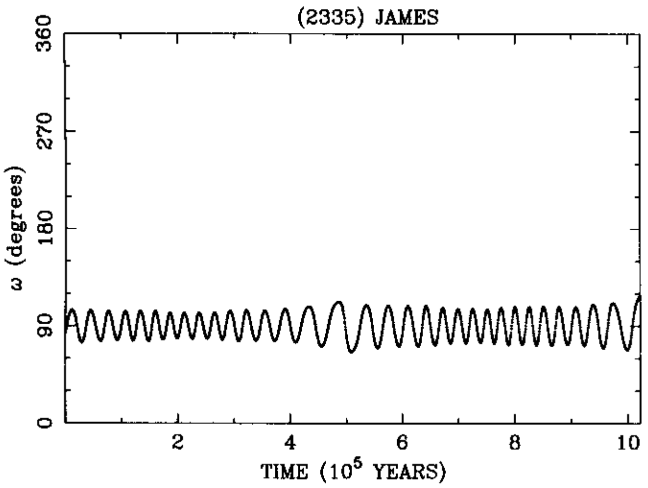


**Fig. 3.** The time evolution of the argument of perihelion $\omega$ of (2335) James; same model and same integration period as in Fig. 1

### 2.2. *Backwards integration over 5 Myr*

In a backwards integration with slightly different starting values (Table 2, Ephemerides of Minor Planets, Leningrad 1986) we computed the orbit of James in the same model Sun–Jupiter–Saturn. The secular argument $\Omega-\Omega_J$ (Fig. 6) librates over all the period of integration time, which indicates that James was in the secular resonance $\nu_{16}$ since 5.2 Myr ago. We note a particular behaviour of the libration. Between $t=0$ until $-4\,10^6$ yr, the libration of $\Omega-\Omega_J$ is about zero, then suddenly there is a turn over and the resonant argument librates around 180°. This is the

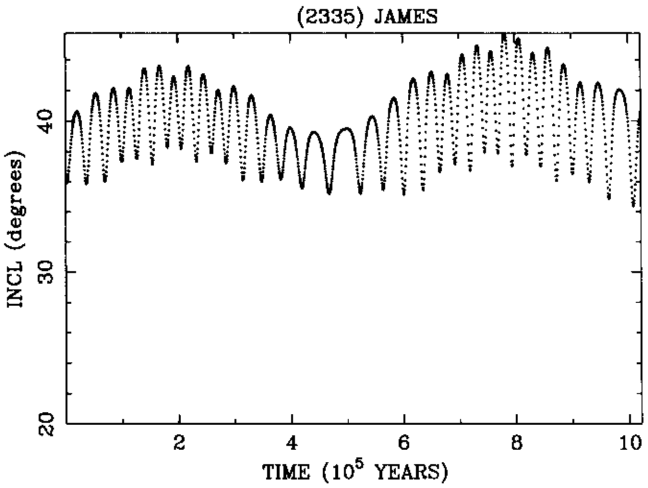


**Fig. 4.** Inclination $i$ vs. time of (2335) James; same model and same integration period as in Fig. 1

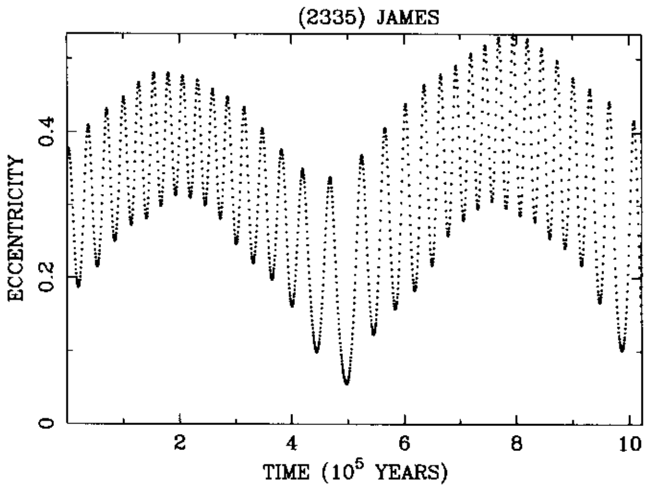


**Fig. 5.** Eccentricity $e$ vs. time of (2335) James; same model and same integration period as in Fig. 1

**Table 2.** Starting values of (2335) James

Epoch 2446600,5 J.D. Ecliptic 1950,0. Equinox 1950,0.
The elements were taken from the Ephemerides of Minor Planets 1986.

| $\omega$ | $\Omega$ | $i$ | $e$ | $a$ (AU) | $M$ (mean anomaly) |
|---|---|---|---|---|---|
| 79°43256 | 19°52309 | 36°33140 | 0.3595230 | 2.1234679 | 240°62617 |

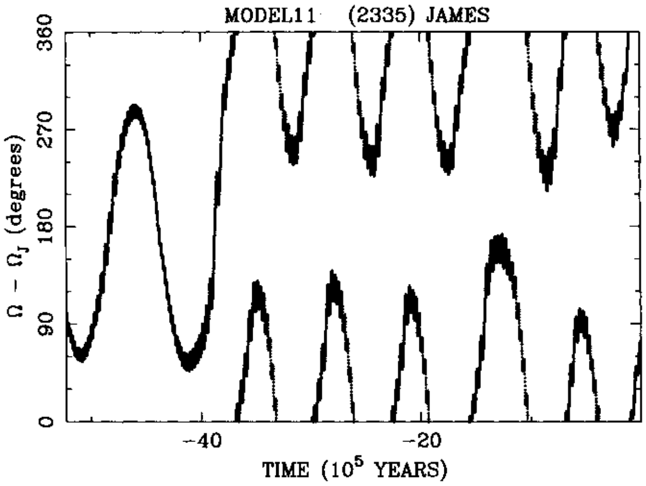


**Fig. 6.** The same evolution of the resonance argument $\Omega - \Omega_J$ of (2335) James; same model as in Fig. 1, backwards integration over $\sim 5\ 10^6$ yr

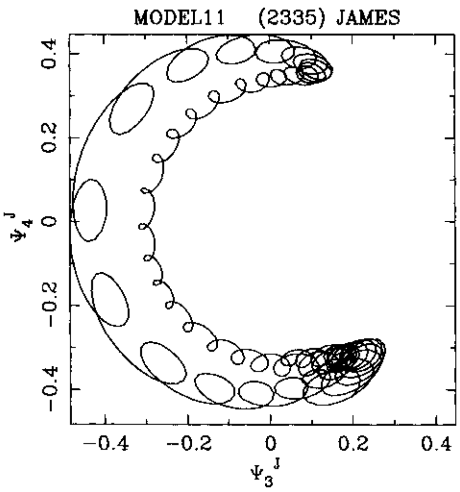


**Fig. 7.** Orbit of (2335) James in a polar diagram $\Psi_3^J, \Psi_4^J$; backwards integration between $-5\ 10^6$ to $-4\ 10^6$ yr

first time that such a phenomenon is found. As one can see in Fig. 7, also the libration about 180° has a banana shape. Contrary to the libration about 0°, the libration about 180° is counter-clockwise.

The inclination shows two different modes (Fig. 8). The first mode ($0 < t < -4$ Myr) is characterized by a variation of $i$ about 40° while $i$ oscillates about 31° in the second mode ($-4 < t < -5$ Myr) with a comparatively large amplitude. The change of the mode occurs ($t = -4$ Myr) when the critical argument changes libration center. The change between the two modes has the drastic effect of increasing the inclination from a minimum of 22° to a maximum of 46°.

The eccentricity shows an equivalent change between two modes (Fig. 9). In the first mode, the eccentricity varies between 0.01 and 0.55 while it varies between 0.03 and 0.4 only in the second mode.

The $\omega$-libration about 90° found in the forwards integration is not maintained over all the backwards integration (Fig. 10).

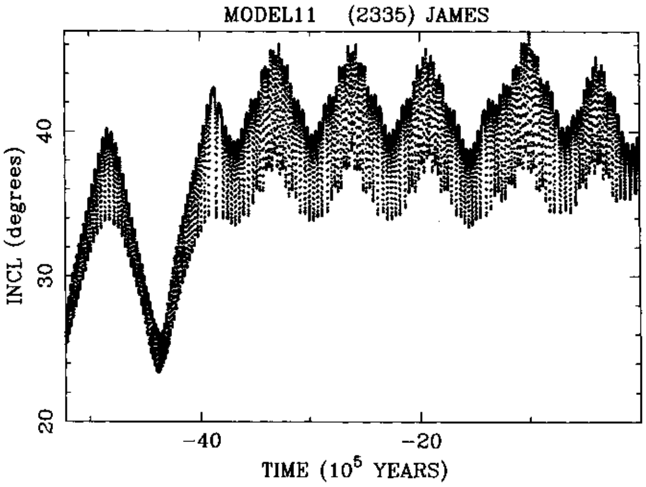


**Fig. 8.** Same as Fig. 4, backwards integration over $\sim 5\ 10^6$ yr

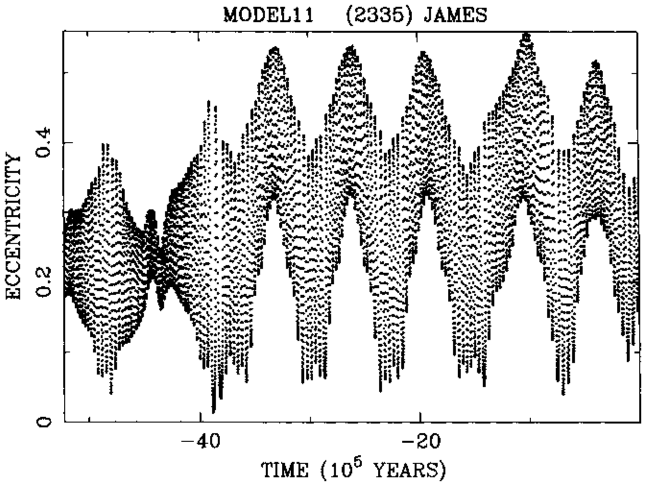


**Fig. 9.** Same as Fig. 5, backwards integration over $\sim 5\ 10^6$ yr

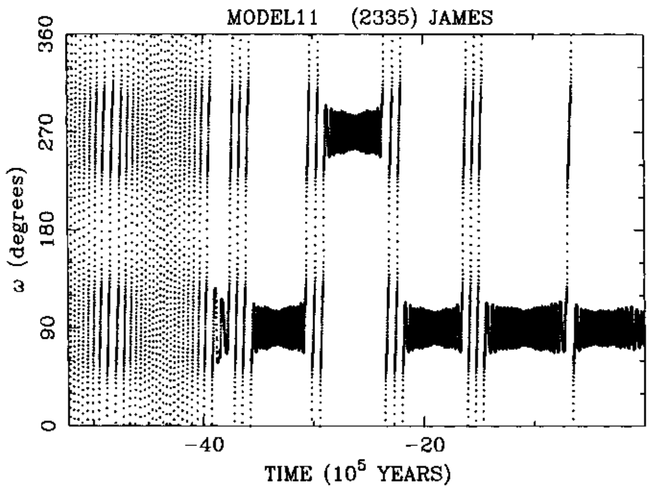


**Fig. 10.** Same as Fig. 3, backwards integration over $\sim 5\ 10^6$ yr

Alternations between libration about 90°, circulation and libration about 270° occur. Such alternations were also found by Vashkov'yak (1986). It is not surprising that the instants of time for alternation differ between our calculations and Vashkov'yak since such alternations suggest chaotic behaviour (see below). We find that $\omega$ is always circulating for $t \leq -40\ 10^5$ yr.

The examination of the $\nu_5$ resonance argument $\varpi - \varpi_J$ yielded a very surprising result: James is a temporary $\nu_5$ secular resonant case (Fig. 11) before $-4$ Myr. We described the orbital evolution from $-5$ Myr to $-4$ Myr. At $-5$ Myr, James librates in the $\nu_5$ resonance about 0° (1 libration). After two inner circulations indicated by the negative derivative of the critical argument (Morbidelli & Henrard 1991), James librates again around 0° in the $\nu_5$ resonance. Then it becomes an outer circulator (the derivative is positive). After one outer circulation it again librates about 0° and finally leaves the $\nu_5$ secular resonance for the remaining integration time. James crossed the resonance $\nu_5$ in the sense that $\varpi - \varpi_J$ had negative derivative at the beginning (direction $-5$ Myr to $-4$ Myr) and positive derivative at the end.

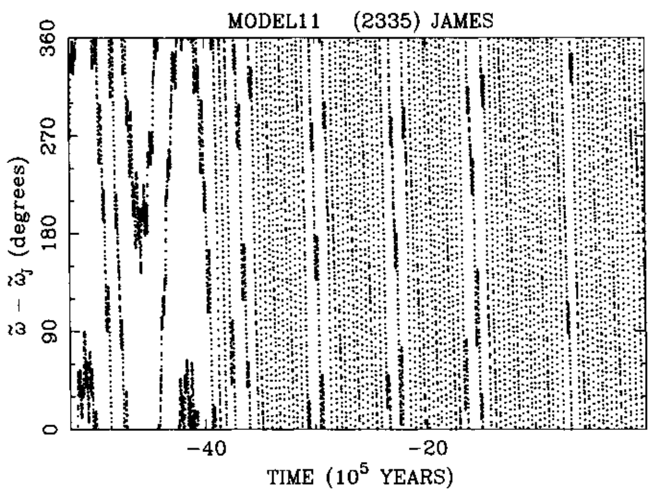


**Fig. 11.** The time evolution of the resonance argument $\varpi - \varpi_J$ of (2335) James; same model and same integration period as in Fig. 6

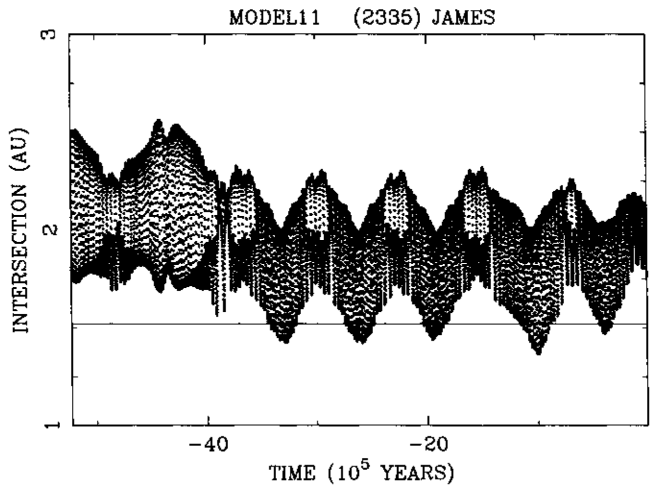


**Fig. 12.** The time evolution of the intersection between James' orbit and the ecliptic. James suffers close encounters with Mars. Same model and same integration period as in Fig. 6

We would like to point out that during these alternations between different modes of the $\nu_5$ resonance, James is always librating in the $\nu_{16}$ resonance.

Is James a planet crosser? Since James has a high inclination, the relevant variable, which shows the possible crossings of James with any planet, is the heliocentric distance at low latitude in an ecliptical coordinate system. We plotted (Fig. 12) at each instant of time the two intersection points of James with the ecliptic. It is clear that after leaving the $\nu_5$ secular resonance, i.e. when the eccentricity suffers large oscillations between 0.05 and 0.45, James becomes a Mars crosser. Then, it will be interesting to include Mars in our integration in order to study more precisely the effects of close approaches with Mars on the orbital evolution of James. As suggested by Wetherill & Williams (1979), the coupled effects of the large $e$ and $i$ variations and the ability of Mars to knock material in and out the secular resonances during close approaches may produce a diffusion of orbits which gives rise to Earth crossing orbits. In Sect. 4, we present the results of orbital evolution of asteroids, taking into account the Mars perturbations.

## 3. A theoretical interpretation of the orbital evolution of James

Our aim is to give a theoretical explanation of the complex and charming behaviours of the asteroid James. Our tool is the theory of the secular resonant dynamics, developed by Morbidelli & Henrard (1991a and 1991b). In order to allow an easier comprehension of this section, we briefly summarize in the following the most important results described in their papers.

The Hamiltonian of the problem is that of a massless asteroid perturbed by the planets (Jupiter and Saturn for simplicity), whose orbits are assumed to vary following a linear law according to Bretagnon (1974). The global size of the perturbation is of the order of the ratio between the mass of Jupiter and that of the Sun ($\sim 10^{-3}$). A "small parameter" $\varepsilon$ is defined, such that $\varepsilon^2 = 10^{-3}$. The Hamiltonian is then expanded in Taylor series with respect to the Poincaré variables of the planets, which are all of order $\varepsilon$ due to the fact that the inclinations and the eccentricities of the planets remain always small during their quasi-periodic variation. In this way, the perturbation is expanded into the convergent series in $\varepsilon$

$$\sum_{m \geq 0} \varepsilon^{2+m} K_m, \tag{1}$$

where $m$ designates the degree of the functions $K_m$ considered as polynomials in the variables of the planets. Here the "small parameter" $\varepsilon$ does not have to be identified with the mass of Jupiter. It is a small number which allows to separate the Hamiltonian in terms of different size, suitable to be studied with a perturbative approach. The masses of the planets give a contribution of order $\varepsilon^2$, while each term in $e'$ or $i'$ (eccentricity and inclination of a planet) gives a contribution of order $\varepsilon$. So, for example, in formula (1) $\varepsilon^2 K_0$ turns out to be the perturbation (linear term in the masses) given by the planets assumed on coplanar circular orbits, while $\varepsilon^3 K_1$ is the linear expansion of the perturbation (again, linear term in the masses) with respect to the eccentricities and the inclinations of the planets. We stress that no expansion is performed with respect to the variables of the asteroid.

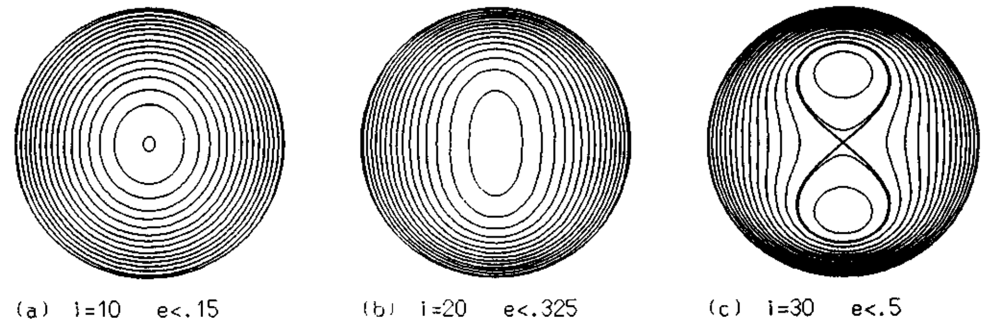


**Fig. 13.** Secular motion of the asteroid described by $\bar{K}_0$; each picture represents a surface $\sqrt{a}(1-\sqrt{1-e^2}\cos i)=\text{const.}$ in polar coordinates $e$ and $\omega$; below each picture are reported the inclination of the circular orbit and the maximal eccentricity of the orbits on the plane

In the hypothesis that the mean motions of the asteroid and of the planets are non-resonant, the perturbation (1) is then normalized with respect to the mean longitudes, which are the only fast angles of the problem. If one considers the Hamiltonian up to order 3 in $\varepsilon$, this is equivalent to averaging (1) over the mean longitudes, thus getting the model:

$$\varepsilon^2\overline{K_0}+\varepsilon^3\overline{K_1}. \tag{2}$$

The bars over each term designate that the functions have been averaged over the fast angles, so that they describe now the secular motion of the asteroid. The semimajor axis does not show variations along this secular motion (the mean longitude has been eliminated), so that it will be assumed constant hereafter.

The Hamiltonian $\bar{K}_0$ has been studied in detail by Kozai (1962). It is integrable, as it is a two-degrees of freedom Hamiltonian with two constants of motion, which are $\sqrt{a}(1-\sqrt{1-e^2}\cos i)$ and $\bar{K}_0$ itself. The three pictures reported in Fig. 13 show the motion described by $\bar{K}_0$ in polar coordinates $e$, $\omega$ (eccentricity, argument of perihelion). Each picture is drawn on a different surface $\sqrt{a}(1-\sqrt{1-e^2}\cos i)=\text{const.}$ Below each picture the inclination of the circular orbit and the limit eccentricity of the planar orbit on the surface are indicated. On each of these three surfaces the motion is found as a level curve for $\bar{K}_0$. These pictures have been plotted for simplicity by taking for $\bar{K}_0$ the first terms reported in Yuasa (1973), namely up to degree 4 in the eccentricity and inclination of the asteroid, so that they must be considered as qualitative. However they will reproduce all the essential topological features of the "real" dynamics, namely the dynamics given by $\bar{K}_0$ not truncated in the eccentricity or inclination of the asteroid (which may be large). As one sees, when the inclination is low (Fig. 13, part a), $\omega$ is a circulating variable, and $e$ and $\omega$ evolve along quasi-circular trajectories. When the inclination becomes higher (Fig. 13, part b), these trajectories are no longer circular, although $\omega$ is still a circulating angle. The orbit with $e=0$ is a stable equilibrium point for $\bar{K}_0$. If the inclination is greater than some critical value (Fig. 13, part c), the orbit with $e=0$ becomes unstable and two stable equilibrium points appear at $\omega=90°$ and $\omega=270°$ (as the pictures are symmetric), surrounded by a separatrix: we will call it "Kozai separatrix". Inside of the separatrix $\omega$ is a librating angle, while outside it circulates. According to Kozai (1962, who considers only the perturbation of Jupiter), for a semimajor axis equal to 2.12 AU (the value of James), the orbit with $e=0$ passes from stability to instability at $i=34.704°$. Moreover, for each value of the semimajor axis, the equilibrium points at $\omega=90°$ form a family in the variables $e$, $i$. In Fig. 14 we show its location for $a=2.12$ AU considering also the contribution given by Saturn.

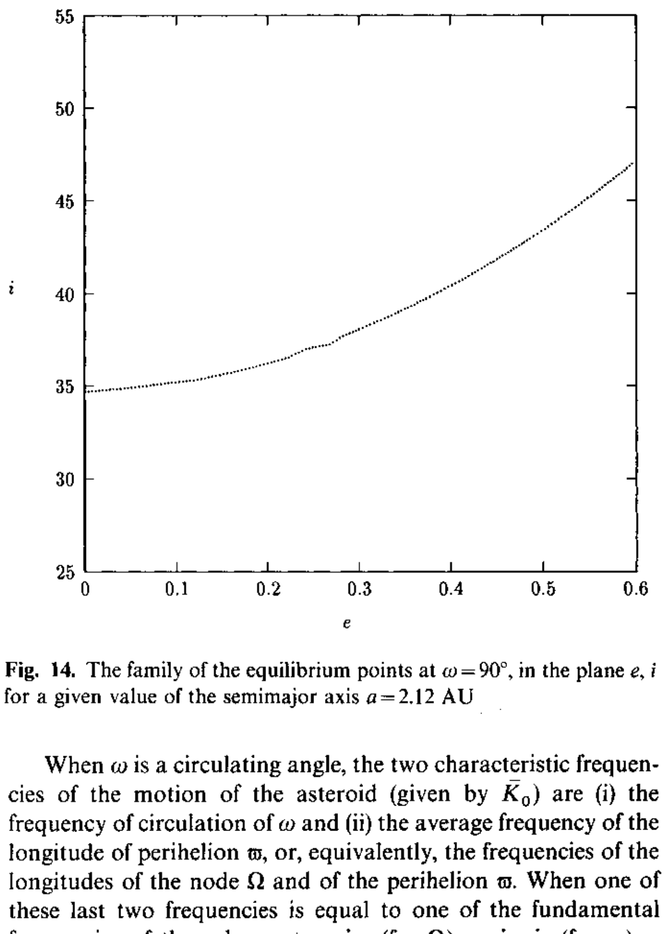


**Fig. 14.** The family of the equilibrium points at $\omega=90°$, in the plane $e$, $i$ for a given value of the semimajor axis $a=2.12$ AU

When $\omega$ is a circulating angle, the two characteristic frequencies of the motion of the asteroid (given by $\bar{K}_0$) are (i) the frequency of circulation of $\omega$ and (ii) the average frequency of the longitude of perihelion $\varpi$, or, equivalently, the frequencies of the longitudes of the node $\Omega$ and of the perihelion $\varpi$. When one of these last two frequencies is equal to one of the fundamental frequencies of the solar system $\dot{\nu}_{16}$ (for $\Omega$) or $\dot{\nu}_5$, $\dot{\nu}_6$ (for $\varpi$), a secular resonance occurs. The location of the secular resonances has been computed by Williams & Faulkner (1981) and confirmed by Morbidelli & Henrard (1991a). Their works show that an overlapping of the $\nu_{16}$ and $\nu_5$ resonances is possible ($\mathrm{d}\Omega/\mathrm{d}t=\dot{\nu}_{16}$ and $\mathrm{d}\varpi/\mathrm{d}t=\dot{\nu}_5$).

Conversely, when $\omega$ is a librating angle, the two characteristic frequencies of the asteroid become (i) the frequency of libration of $\omega$ and (ii) the average frequency of the longitude of perihelion (evidently equal to that of the node). So, in this case, the asteroid can no longer be in the $\nu_{16}$ and in the $\nu_5$ resonances at the same time. Morbidelli & Henrard showed that only the $\nu_{16}$ resonance is possible. The location of the centers of libration of $\omega$ where $\mathrm{d}\Omega/\mathrm{d}t=\dot{\nu}_{16}$ is reported in Fig. 15 with respect to $a$, $i$ and $e$.

We come now to the analysis of Fig. 10 where the evolution of the argument of perihelion $\omega$ of James is shown. $\omega$ is a circulating

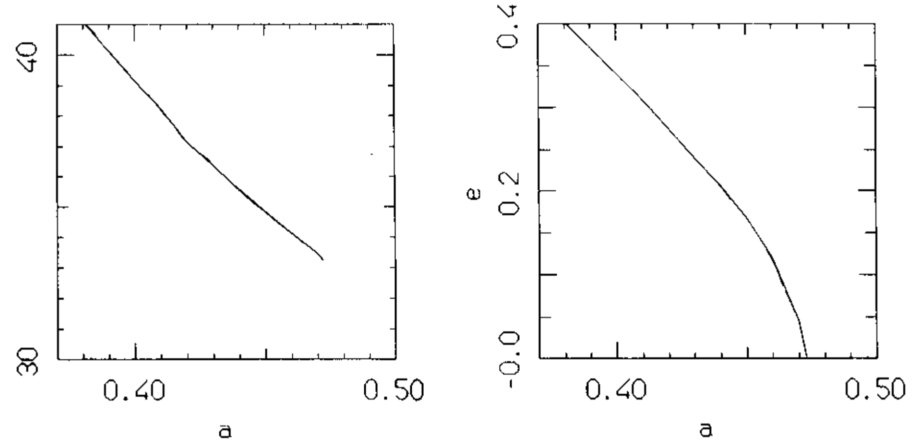


**Fig. 15.** The family of the equilibrium points of $\omega$ with the property that the frequency of the node $\Omega$ is equal to $\dot{\nu}_{16}$. It is represented on the left as a function of $a$ and $i$, on the right as a function of $a$ and $e$

angle up to $t=-40\,10^5$ yr, then it starts to librate around 90°. This phenomenon must be related to a transition through the Kozai separatrix. From $-40\,10^5$ to $-8\,10^5$ yr, the libration is interrupted by short temporary periods of circulation. These correspond to transitions through the Kozai separatrix from the inner to the outer region and vice-versa.

The passage of $\omega$ from circulation to libration at $t=-40\,10^5$ yr has the following main dramatic effects:

1. James leaves the $\nu_5$ resonance and $\varpi-\varpi_J$ becomes a "fast" angle (Fig. 11);

2. James is still in the $\nu_{16}$ resonance, but the critical angle $\Omega-\Omega_J$ starts to librate around 0° (instead of 180°) (Fig. 6);

3. The libration in the variables $i$, $\Omega-\Omega_J$ changes from counter clockwise to clockwise (Figs. 6 and 8).

The first of these effects is explained above.

We come now to the explanation of points 2) and 3). A description of the resonant dynamics can be given starting from model (2). This has been done by Morbidelli & Henrard who have studied, with different techniques, the cases where $\omega$ circulates and librates. In case of circulating $\omega$, the critical angle $\Omega-\Omega_J$ can have two main equilibrium points: a stable one at 180° and an unstable at 0°. The libration around the stable equilibrium is counter clockwise. Indeed, the equilibrium points can be approximated locally by harmonic oscillators of the kind

$$\frac{1}{2}aP^2+\frac{1}{2}b\cos\sigma_0\sigma^2, \tag{3}$$

where $\sigma=\Omega-\Omega_J+180°$ and $P$ is the action conjugated to $\sigma$ in the Hamiltonian problem (2). $\sigma_0$ is the value of $\sigma$ at the equilibrium point, thus being 0° or 180°. The coefficients $a$ and $b$ are given by the relations:

$$a=\frac{1}{T}\int_\gamma \frac{\partial^2\bar{K}_0}{\partial P^2}(i(t),e(t),\omega(t))\,\mathrm{d}t$$
$$b=\frac{1}{T}\int_\gamma -\bar{K}_1(i(t),e(t),\omega(t),\sigma=0)\,\mathrm{d}t \tag{4}$$

where $\gamma\equiv(i(t),e(t),w(t))$ is the trajectory described by $\bar{K}_0$, whose period is indicated by $T$.

In case of libration of $\omega$, the picture of the resonant dynamics still has two main equilibrium points, at $\Omega-\Omega_J=0°$ and $\Omega-\Omega_J=180°$. Locally these points can be represented by harmonic oscillators as in (3), but the coefficients $a$ and $b$ have now the more complicated form:

$$a=\frac{1}{2}\left[\frac{\partial^2\bar{K}_0}{\partial P^2}(i_0,e_0,\omega_0)-\left(\frac{\partial^2\bar{K}_0}{\partial Z^2}(i_0,e_0,\omega_0)\right)^{-1}\times\left(\frac{\partial^2\bar{K}_0}{\partial Z\partial P}(i_0,e_0,\omega_0)\right)^2\right]$$
$$b=\frac{1}{2}\left[-\bar{K}_1(i_0,e_0,\omega_0,\sigma=0)\cos\sigma-\varepsilon\left(\frac{\partial^2\bar{K}_0}{\partial\omega^2}(i_0,e_0,\omega_0)\right)^{-1}\times\left(\frac{\partial^2\bar{K}_1}{\partial\omega\partial\sigma}(i_0,e_0,\omega_0,\sigma_0)\right)^2\right], \tag{5}$$

where $(i_0,e_0,\omega_0)$ denote the equilibrium point $\bar{K}_0$ and $Z$ is the action conjugated to $\omega$.

Morbidelli & Henrard have verified that the coefficient $b$ given by (4) has the same sign as that computed by (5). Conversely the coefficient $a$ turns out to have different sign in the two cases. This implies that, when $\omega$ librates, the stable equilibrium point must be at $\Omega-\Omega_J=0°$ instead of 180°, and the sense of libration must change from counter clockwise to clockwise, as observed in the simulations of the motion of James.

The following feature pointed out in Sect. 2.1 concerning Fig. 5 can now also be understood: when $\omega$ is librating, the eccentricity shows a large secular variation associated with a libration of the critical argument $\Omega-\Omega_J$; this is not valid when $\omega$ is circulating. Indeed, the secular resonance $\nu_{16}$ causes a change of $\sqrt{a}(1-\sqrt{1-e^2}\cos i)$. This produces a change of the guiding center of libration of $\omega$ resulting in a large variation of eccentricity according to Fig. 14.

A quantitative theoretical analysis shows that, for $a=2.12$ AU, the equilibrium point of $\omega$ where $\mathrm{d}\Omega/\mathrm{d}t=\dot{\nu}_{16}$ is at $e=0.316$, $i=38.4381'$ (see Fig. 15). We look now at Figs. 5, 4, and 1, where the evolution of the eccentricity, the inclination, and of $\Omega-\Omega_J$ is reported as a function of time: the short periodic oscillations (period $\simeq 32{,}000$ yr) are related to the libration of $\omega$. So, the mean values of the short periodic oscillations of $e$ and $i$ give the values $e_0$, $i_0$ of the instantaneous center of libration. As one sees, the value of $e_0$, $i_0$ in coincidence with $\mathrm{d}(\Omega-\Omega_J)/\mathrm{d}t=0$ is in very good agreement with the theoretical computations reported above.

The period of the secular libration of $\Omega-\Omega_J$ is computed to be 575,000 yr at the equilibrium point of $\omega$. This is again in good agreement with Fig. 1.

We finally come to the problem of the origin of the transitions through the Kozai separatrix, which are related to the transitions of $\omega$ from the state of circulator to that of librator and vice-versa. The secular evolution of $e$ and $i$, forced by the term $\varepsilon^3 \bar{K}_1$ in (2) and which is particularly important due to the presence of the secular resonances, changes the value of $\sqrt{a}(1-\sqrt{1-e^2}\cos i)$, which is a constant of motion if $\bar{K}_0$ is considered alone. In particular, the area enclosed by the Kozai separatrix becomes greater when $\sqrt{a}(1-\sqrt{1-e^2}\cos i)$ increases, namely, for example, when the inclination increases with constant eccentricity (see Fig. 13). On the contrary, the theory of the adiabatic invariant (see Henrard 1991) states that $e$ and $i$ evolve in such a way that the area enclosed by the trajectory in the coordinates $e$, $\omega$ stays constant on the average. This fact may lead to transitions through the Kozai separatrix. Indeed, looking at figures 8 and 10 we see that the first transition of $\omega$ from circulator to librator ($t=-40\ 10^5$ yr) occurs when the inclination reaches a maximum during the secular resonant evolution. Conversely the following temporary transitions from librator to circulator occur when the inclination of James reaches a minimum.

Unlike Fig. 13, part c, however, we expect that the Kozai separatrix gives origin to a chaotic zone, where motion is unpredictable. This is due to the presence in the full problem of all the frequencies (fast and slow) which are neglected when the integrable approximation $\bar{K}_0$ is considered alone. The existence of such a chaotic zone could explain why the transitions through the separatrix do not occur at each time when the inclination is at maximum or at minimum.

It would be interesting, moreover, to extend the numerical simulation in the past and in the future in order to verify whether or not more transitions like that at $t=-40\ 10^5$ yr occur. In principle this should happen, although probably at irregular intervals.

A similar explanation was already proposed by Vashkov'yak (1986). He observed that the variation of the "energy" $\bar{K}_0$ (i.e. the value of $\bar{K}_0$ along the orbit) is large and long-periodic due to the close commensurability between $\langle\Omega\rangle$ and $\langle\Omega_J\rangle$. Moreover, he verified that $\omega$ passes from libration to circulation when such "energy" is greater than that of the instantaneous Kozai separatrix for a comparatively long time with respect to the typical period of libration/circulation of $\omega$.

As we have already pointed out, our numerical simulations show these transitions from librating to circulating mode at different times than the ones of Vashkov'yak. This is not surprising as the evident chaotic nature of the problem makes the results very sensitive to the initial conditions and to the features of the model adopted, so that they must be compared only from a qualitative point of view.

## 4. Orbital evolution of a cloud of test asteroids surrounding the orbit of James

According to the previous discussions about the orbital evolution of James, it appears that James is in a chaotic region. Therefore, the orbital evolution presented in Sect. 2 may not be the real one of James. In order to estimate possible orbital evolutions, we integrated a cloud of 6 objects surrounding James' orbit as well as James over 1 Myr backwards in the Sun–Mars–Jupiter–Saturn model. Mars was included since we found that James is a temporary Mars-crosser. Since close Mars encounters are possible, and integration scheme with variable step size had to be chosen. The integrations were performed with the DVDQ code of Krogh (1970). This code is based on a predictor-corrector scheme with variable step size and variable order with respect to differences.

The starting values of the osculating elements of James and of the six surrounding test asteroids are given in Table 3. For each test asteroid only one starting osculating element differs from those of James. The different close approaches of James with Mars can be seen in Fig. 16, where the evolution of the semimajor axis $a$ is given as a function of time $t$. Mars encounters result in discontinuities of the semimajor axis. These close approaches do not force James to leave the $\nu_{16}$ resonance (Fig. 17). The argument $\Omega-\Omega_J$ librates like in Fig. 6 around 0°. The oscillations in $e$ and $i$ are a somewhat larger, i.e., $0.01\le e\le 0.58$ instead of $0.05\le e\le 0.58$ and $34°\le i\le 48°$ instead of $33°\le i\le 46°$. As previously, the argument of perihelion librates around 90°, and circulates only a short period of time as in Fig. 10.

All the six cloud asteroids remain in the $\nu_{16}$ secular resonance during the whole integration period.

**Table 3.** Starting value of (2335) James and six test asteroids

Epoch 2447400,5 J.D. Eclipitc 1950,0. Equinox 1950,0.
The elements were taken from the Ephemerides of Minor Planets 1988.

| Body Name | $\omega$ | $\Omega$ | $i$ | $e$ | $a$ (AU) | $M$ (mean anomaly) |
|---|---|---|---|---|---|---|
| (2335) James | 79°.44290 | 19°.50678 | 36°.33800 | 0.3592595 | 2.1236516 | 135°.37585 |
| (2335) James $\omega$ | 79°.45290 | 19°.50678 | 36°.33800 | 0.3592595 | 2.1236516 | 135°.37585 |
| (2335) James $\Omega$ | 79°.44290 | 19°.50778 | 36°.33800 | 0.3592595 | 2.1236516 | 135°.37585 |
| (2335) James $i$ | 79°.44290 | 19°.50678 | 36°.34800 | 0.3592595 | 2.1236516 | 135°.37585 |
| (2335) James $M$ | 79°.44290 | 19°.50678 | 36°.33800 | 0.3592595 | 2.1236516 | 136°.37585 |
| (2335) James $e$ | 79°.44290 | 19°.50678 | 36°.33800 | 0.3602595 | 2.1236516 | 135°.37585 |
| (2335) James $a$ | 79°.44290 | 19°.50678 | 36°.33800 | 0.3592595 | 2.1246516 | 135°.37585 |

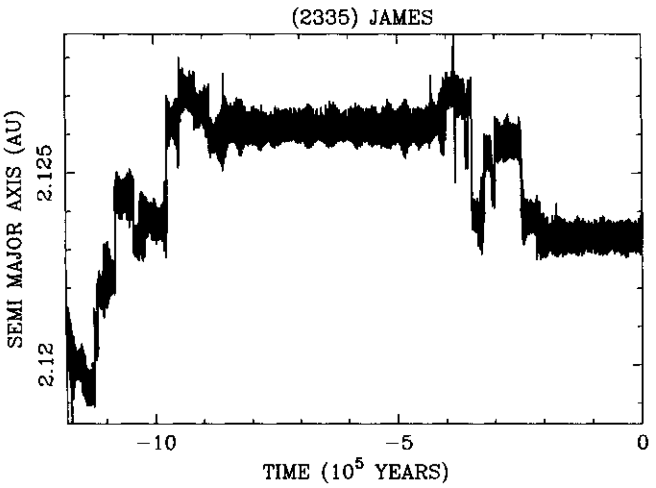


**Fig. 16.** Semimajor axis $a$ vs. time of (2335) James, in the Sun-Mars-Jupiter-Saturn model; backwards integration over 1 Myr

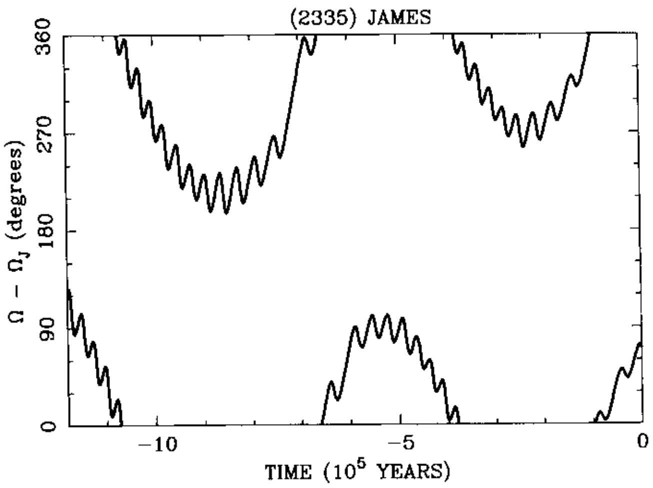


**Fig. 17.** The time evolution of the secular argument $\Omega - \Omega_J$ of (2335) James; same model and same integration period as in Fig. 16

All six asteroids alternate between $\omega$-libration and circulation at differing instants of time. Five of them remain for a comparatively long period in the $\omega$-circulation mode and, consequently, become also $\nu_5$ resonant cases. The critical variable of the $\nu_{16}$ resonance changes its center of libration from 0° to 180°. The exceptional asteroid (James $a$ in Table 3) makes only one circulation for $\omega$ and, therefore, remains too short in the circulation mode to show the same behaviour as the other five ones.

Five cloud asteroids show discontinuities in semimajor axis indicating close approaches with Mars. The only exceptional case is James $e$. None of the close approaches to Mars forces an asteroid to leave the secular resonance.

## 5. Conclusion

A forwards integration confirms, as found by Williams, that James is located in the secular resonance $\nu_{16}$; it will stay a resonant $\nu_{16}$ asteroid at least for 1 Myr. The secular argument $\Omega - \Omega_J$ oscillates around 0°. We found that James is an $\omega$-librator during all the integration period. Consequently, the eccentricity suffers large oscillations between 0.05 and 0.52, as well as the inclination which shows large variations $34' \leq i \leq 46°$.

A backwards integration over 5 Myr shows that James was located in both secular resonances $\nu_5$ and $\nu_{16}$ during a period of one million years. During this one million years period James' argument of perihelion $\omega$ is circulating, and the critical argument $\Omega - \Omega_J$ librates around 180°. When James becomes an $\omega$-librator (near $-4\ 10^6$ yr), James leaves the resonance $\nu_5$, and the secular argument $\Omega - \Omega_J$ starts to librate around 0° instead of 180°. This turn over of libration has an important effect: the inclination is pumped up. When $\Omega - \Omega_J$ librates around 180°, the inclination varies from 22° to 40°, while when $\Omega - \Omega_J$ librates around 0°, the inclination varies from 34° to 46°.

Backwards numerical integrations over 1 Myr of test asteroids located near James show – for most of them – the same orbital evolution as James. They are temporarily located in both secular resonances $\nu_5$ and $\nu_{16}$. Then due to a close approach with Mars they leave the resonance $\nu_5$. Associated with leaving the resonance $\nu_5$ we observed, as for James, the transition between circulation and libration of the argument of perihelion $\omega$, and the inversion of the libration center of $\Omega - \Omega_J$. We found also the two modes of inclination. In Froeschlé & Scholl (1988) we conjectured that the libration of $\Omega - \Omega_J$ around 0° is associated with large inclinations ($\geq 40°$), while libration around 180° is associated with low inclinations ($\leq 30°$). In this paper a theoretical explanation of the complex behaviour of James' orbital evolution is provided and confirms the conjecture of 1988.

It is also clearly shown that the region surrounding James is a chaotic region, several close approaches with Mars may happen during 1 Myr.

## References

Bretagnon P., 1974, A&A 30, 141
Charlier C.V.L., 1902, Die Mechanik des Himmels, Veit. Leipzig, p. 217
Froeschlé Ch., Scholl H., 1986, A&A 166, 326
Froeschlé Ch., Scholl H., 1987, A&A 179, 294
Froeschlé Ch., Scholl H., 1988, in: The Few Body Problem, ed. M.J. Valtonen, Kluwer, Dordrecht, p. 123
Henrard J., 1991, Dynamics reported (submitted)
Henrard J., 1990, (preprint)
Kozai Y., 1962, AJ 67, 591
Krogh F.T., 1970, JPL Technical Utilization Document No. CP-38
Morbidelli A., Henrard J., 1991a, Celest. Mech. (submitted)
Morbidelli A., Henrard J., 1991b, Celest. Mech. (submitted)
Nakai H., Kinoshita H., 1985, Celest. Mech. 36, 391
Poincaré H., 1892, Les Méthodes Nouvelles de la Mécanique Céleste, Gauthier-Villars, Paris
Scholl H., Froeschlé Ch., 1986, A&A 170, 138
Scholl H., Froeschlé Ch., Kinoshita H., Yoshikawa M., Williams J.G., 1989, in: Asteroids II, eds. R.P. Binzel, T. Gehrels, M. Shapley Mathews, Univ. of Arizona Press, Tucson
Scholl H., Froeschlé Ch., 1990, A&A 227, 255
Schubart J., Stumpff P., 1966, Veröffentl. Astron. Rechen-Inst. Heidelberg, 18
Vashkov'yak M.A., 1986, Kosm. Isseld, Tom. 24, Vyp. 3, 323

translated in english in Cosmic Res., 1986, Plenum Publ. Corp.
Wetherill G.W., 1979, Icarus 37, 96
Wetherill G.W., 1987, Philos. Trans. R. Soc. London, Ser. A 323, 323
Wetherill G.W., Williams J.G., 1987, J. Geophys. Res. 73, 635
Williams J.G., 1969, Ph.D. Thesis, University of California, Los Angeles
Williams J.G., 1973a, in: EOS. Trans. Am. Geophys. Union. 54, 233
Williams J.G., 1973b, BAAS 5, 363
Williams J.G., 1989, in: Asteroids II, eds. R.P. Binzel, T. Gehrels, M. Shapley Mathews, Univ. of Arizona Press, Tucson, p. 1034
Williams J.G., Faulkner J., 1981, Icarus 46, 390
Yuasa M., 1973, PASJ 25, 399