THE ASTRONOMICAL JOURNAL VOLUME 111, NUMBER 6 JUNE 1996

THE KIRKWOOD GAP AT THE 2/1 COMMENSURABILITY WITH JUPITER: NEW NUMERICAL RESULTS

ALESSANDRO MORBIDELLI
C.N.R.S., Observatoire de la Côte d'Azur, B.P. 229, 06304 Nice Cedex 4, France

Received 1995 November 21; revised 1996 March 13

ABSTRACT

In this paper it is pointed out that the 2/1 resonance with Jupiter is not completely empty of asteroids. However, among the ten bodies presently in the 2/1 resonance with $e<0.5$, only four have a quite regular orbit and moderate eccentricity and inclination. The integration of the evolution of the orbits of 24 fictitious particles over 1 Gyr shows that in the 2/1 resonance all orbits diffuse chaotically. In some parts of the resonance, however, the diffusion speed is very slow, so that some bodies can survive at least 1 Gyr without becoming Mars crosser. From the global chart of diffusion, some hypotheses are discussed about the origin of the four "regular" asteroids, as well as on the depletion of the Themis family members in the 2/1 resonance. © *1996 American Astronomical Society.*

1. INTRODUCTION

It is well known, since the original work by Kirkwood (1866), that the distribution of bodies in the asteroid belt shows evident gaps associated with the main mean motion commensurabilities with Jupiter. Nowadays, Celestial Mechanics has provided good explanations for almost all of these gaps. In the case of the 4/1, 3/1, 5/2, and 7/3 commensurabilities, for example, the motion of the secular angles $\omega-\omega_J$ and $\omega-\omega_S$ (ω is the longitude of perihelion and the subscripts J and S refer to Jupiter and Saturn, respectively) forces the eccentricity of a resonant asteroid to increase up to Mars-crossing or Earth-crossing values (Wisdom 1982, 1985; Yoshikawa 1989, 1990, 1991; Moons & Morbidelli 1995). Instability is therefore produced by close encounters with the terrestrial planets. In some cases, the eccentricity can increase even up to 1, forcing the resonant asteroid to collide with the Sun (Farinella *et al.* 1994; Morbidelli & Moons 1995; Froeschlé *et al.* 1995).

The most notable case among the Kirkwood gaps to be still partially unexplained is the one corresponding to the 2/1 resonance with Jupiter. As a matter of fact, the existence of a chaotic zone within the 2/1 resonance at small eccentricity has been first pointed out by Giffen (1973) and successively explored by Froeschlé & Scholl (1981), Murray (1986), and Wisdom (1987). Such a chaotic zone is associated to secondary resonances between the period of libration of the critical angle of the 2/1 resonance and the period of circulation of the secular angle $\omega-\omega_J$ (Henrard & Lemaitre, 1986; Lemaitre & Henrard 1990; Moons & Morbidelli 1993). In the planar problem, the chaotic zone is bounded by regular invariant curves, so that the eccentricity cannot increase to large values (Froeschlé & Scholl 1981); however, Wisdom (1987) showed numerically that some trajectories can go from this chaotic region to large eccentricities through a channel passing at large inclination. This channel has been recently understood by Henrard *et al.* (1995), as a bridge between secondary resonances and secular resonances (see also Morbidelli & Moons 1993). Nevertheless, at the light of Henrard *et al.* (1995), a regular region still seems to exist, characterized by eccentricity in between 0.25 and 0.45, small inclination and moderate amplitude of libration of the critical angle of the 2/1 commensurability. Therefore, the absence of a significant number of asteroids in the 2/1 mean motion resonance with Jupiter is still a puzzling mystery.

If the analytical explorations are not able to solve the problem completely, new possibilities are open now for numerical studies. The improvements in computer hardware and, particularly, the new symplectic algorithms for the numerical integration of N-body problems (Wisdom & Holman 1991; Levison and Duncan 1993) allow nowadays to integrate the motion of real and fictitious particles in the 2/1 resonance on time scales comparable to the age of the Solar System. Such numerical experiments can be very useful to point out what is the real structure of the dynamics within the 2/1 resonance and to guide the future development of semianalytic theories.

The most recent numerical works on the 2/1 resonance are, to my knowledge, those by Franklin (1994) and Ferraz-Mello (1994), who found that all their test orbits have a positive Lyapunov exponent, larger than 10^{-6} yr^{-1}. This fact should be interpreted as the indication that regular orbits on invariant tori either do not exist or fill a very small volume of the phase space (so that it is very improbable to choose initial conditions on them). However, the Lyapunov exponent does not have a uniform value all over the 2/1 resonance, being smaller than 10^{-5} yr^{-1} (Franklin 1994) for orbits with moderate libration amplitude and eccentricity in the interval (0.2–0.5), i.e., in the region where the semianalytic theory by Henrard *et al.* (1995) finds "stability." Unfortunately, the numerical integrations in Franklin (1994) and Ferraz-Mello (1994) were too short (a few millions of years) to observe escapes from the 2/1 resonance and no attempt was done to point out the existence and the time scales of diffusive phenomena.

The present paper summarizes the results of new numeri-

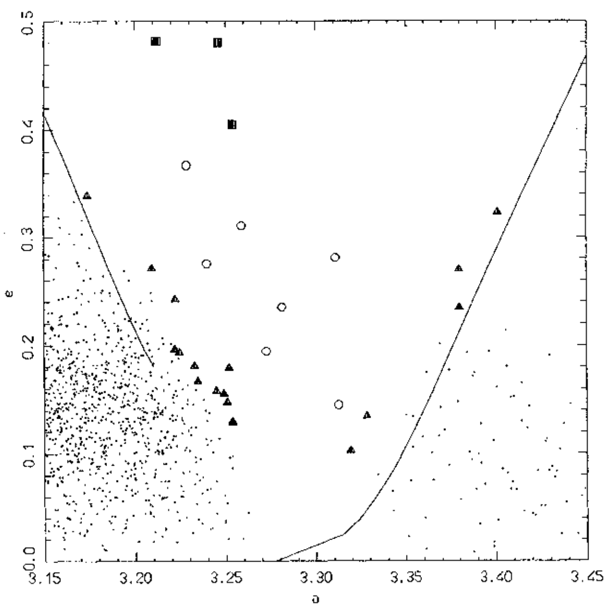


FIG. 1. The borders of the 2/1 resonance and the position of numbered and multiopposition asteroids, according to their semimean a and e. Triangles denote asteroids which are actually outside the 2/1 resonance, although they appear within its borders because of a coordinate projection effect. Squares denote asteroids which are presently in the 2/1 resonance, but which are ejected before 38 Myr. Circles denote asteroids which librate in the 2/1 resonance during the full 38 Myr integration. See Table 2 and text for comments.

cal investigations, performed using Levison & Duncan's "Swift" code (1993). In Sec. 2, the real asteroids which seem to be close/in the 2/1 resonance are integrated over 38 Myr; it is known, indeed, that the 2/1 resonance is not completely empty (for example, the asteroid 3789 Zhongguo is in the 2/1 resonance at small eccentricity and inclination), although it is still not clear how many asteroids are in the resonance and what is their long-term dynamical behavior. In Sec. 3, 24 fictitious objects, with initial conditions chosen on a regular grid in the phase space, are integrated over 1 Gyr and the evolution of their "proper elements" with respect to time is computed numerically, in order to discuss the existence and the relevance of diffusive phenomena. All these data will lead in Sec. 4 to some considerations on the origin of the few asteroids which are presently inside the 2/1 resonance. The conclusions will follow.

2. IS THE 2/1 RESONANCE A REAL GAP?

It is not a straightforward task to select from the asteroid' catalogues all the bodies which are inside the 2/1 resonance. A first rough selection is done in Fig. 1, which shows the borders of the resonance, computed with the usual tools of analytic Celestial Mechanics (see Morbidelli & Moons 1993), and the position of all numbered and multiopposition asteroids according to their semimean elements a and e (semimajor axis and eccentricity). Semimean elements are defined as obtained by averaging over the short periodic angles $n\lambda - m\lambda'$, with n, m integers and $n/m \neq 1/2$ (λ and λ' being the mean longitudes of the asteroid and of Jupiter, respectively), and are computed using a modified version of Milani & Knežević's (1990) program. The selection is rough, since in Fig. 1 some bodies can erroneously appear to be inside the 2/1 resonance just because of a projection effect. Indeed, the semimean a is characterized by large oscillations, forced by the motion of the critical angle of the 2/1 resonance $\sigma \equiv -\lambda + 2\lambda' - \omega$, which circulates (if the body is outside of the resonance) or librates (if the body is inside the resonance). The semimean elements of the asteroids, plotted in Fig. 1, correspond to their present values of σ. Conversely the borders of the resonance, defined as the limit between σ libration and σ circulation, are computed on the semimean a,e diagram setting always $\sigma=0$.

Semianalytical computations can be done to determine "resonant proper elements" (Morbidelli *et al.* 1995) for all asteroids, in order to plot their position in a compatible way with respect to the borders of the resonance, taking into account also secular perturbations, so as to avoid the projection effect. These computations show that the large majority of the objects which in Fig. 1 seem to be in the 2/1 resonance (i.e., those which are close to its borders) are, in reality, outside the resonance. As a check, I have integrated numerically, for a quite long time span (38 Myr), the equations of motion of the asteroids which, according to Fig. 1 seem to be significantly inside the 2/1 resonance, and analyzed their dynamical behavior. The numerical integrations have been done using symplectic Levison & Duncan's code (1994), with a step size of 30 days, and taking into account the perturbations of the four giant planets. The mass of the inner planets has been added to that of the Sun. The initial conditions assumed for the asteroids and the planets correspond to their osculating elements at JD=2449700.5. The osculating elements of the asteroids have been taken from Bowell *et al.* (1993).

The results are summarized in Fig. 1 and in Table 1. In Fig. 1 the dark triangles denote the integrated asteroids which turn out to be nonresonant. The critical argument σ circulates and these bodies seem to be inside the resonance in Fig. 1 just for the projection effect mentioned above. Similarly, all the asteroids which appear to be even closer to the resonance border should be, in reality, outside of the resonance. The dark squares denote asteroids which are presently inside the 2/1 resonance (σ librates) but on very unstable orbits, which escape from the 2/1 resonance (σ starts to circulate) within 38 Myr [see Fig. 2(a)] so that encounters with Jupiter occur (remark that the numerical integrator used here is not reliable in case of planetary close encounters). Table 1 reports the time of escape for these three bodies. The open circles denote seven asteroids which are in the 2/1 resonance during the full 38 Myr time span of the integration. Figures 2(b) and 2(c) report the evolution of their eccentricity, inclination, semimajor axis and σ.

Three of these asteroids [Fig. 2(b)] have an evidently irregular evolution (note that the amplitude of oscillation of a for $81EX_{11}$ is not conserved), so that their long-term association with the 2/1 resonance is questionable. Moreover, they are at large inclination (i being about 20°). Conversely, the remaining four asteroids [Fig. 2(c)] seem to have fairly regu-

TABLE 1. Semimean elements and dynamical character of the asteroids closest to the 2/1 resonance: NR means nonresonant; E*x* means escape after *x* Myr; *R* means resonant. In Fig. 1 NR corresponds to triangles, E to squares, and R to circles.

Number	Name	a	e	i	Dynamical character
225	Henrietta	3.380	0.27	21.0	NR
319	Leona	3.380	0.24	10.4	NR
1006	Lagrangea	3.174	0.34	12.5	NR
1362	Griqua	3.228	0.37	22.7	R
1373	Cincinnati	2.401	0.32	40.6	NR
1921	Pala	3.254	0.40	20.2	E21
1922	Zulu	3.246	0.48	36.2	E22
2393	Suzuki	3.225	0.19	11.0	NR
2667	1967 UO	3.233	0.18	1.6	NR
2921	Sophocles	3.245	0.16	1.1	NR
3688	Navajo	3.212	0.48	3.0	E30
3727	1981 PQ	3.328	0.14	4.5	NR
3789	Zhongguo	3.273	0.19	1.4	R
3821	1985 RC3	3.252	0.18	1.0	NR
4177	1987 SS1	3.311	0.28	17.6	R
6057	5182 T-3	3.319	0.10	17.2	NR
	75SX	3.313	0.15	2.3	R
	$78RG_1$	3.222	0.24	1.1	NR
	$80PB_3$	3.254	0.13	20.1	NR
	$84SY_5$	3.251	0.15	0.9	NR
	$85UQ_4$	3.254	0.13	1.3	NR
	$90TH_7$	3.281	0.24	2.0	R
	$92AK_1$	3.235	0.17	1.4	NR
	4559PL	3.249	0.16	1.5	NR
	$2672T_3$	3.209	0.27	13.2	NR
	$81EX_{11}$	3.259	0.31	19.5	R
	$81EG_{40}$	3.222	0.20	1.2	NR
	$93SK_3$	3.240	0.28	3.8	R

lar orbits (with the possible exception of asteroid $90TH_7$) and are at moderate eccentricity and inclination. Therefore one can conclude that there are at least four "good" inhabitants of the 2/1 resonance among the presently known asteroids with well determined orbits. The 2/1 resonance, therefore, is only a "partial" Kirkwood gap! Then some questions arise: what is the long-term dynamical evolution of these bodies? What is their origin? Why can we observe only such a small number of asteroids in the resonance at the present epoch? An attempt to answer requires a global investigation of the dynamics in the 2/1 resonance, through the long-term integration of many fictitious bodies.

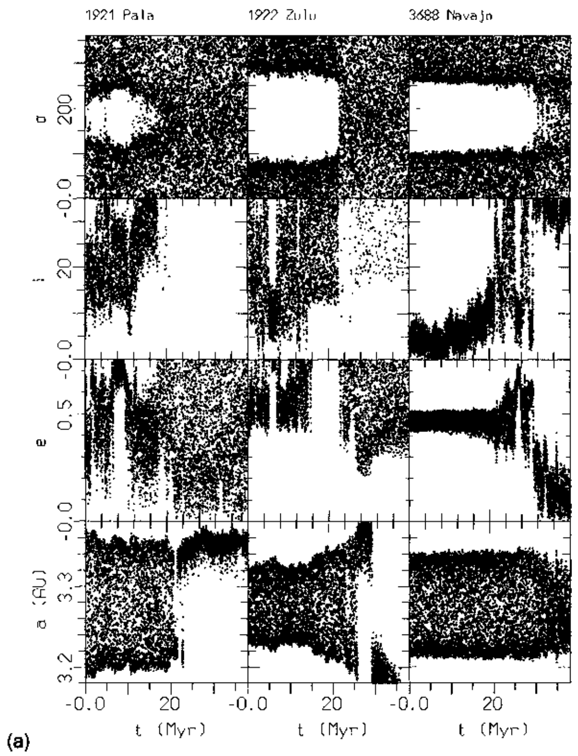
3. LONG-TERM CHAOTIC DIFFUSION IN THE 2/1 RESONANCE

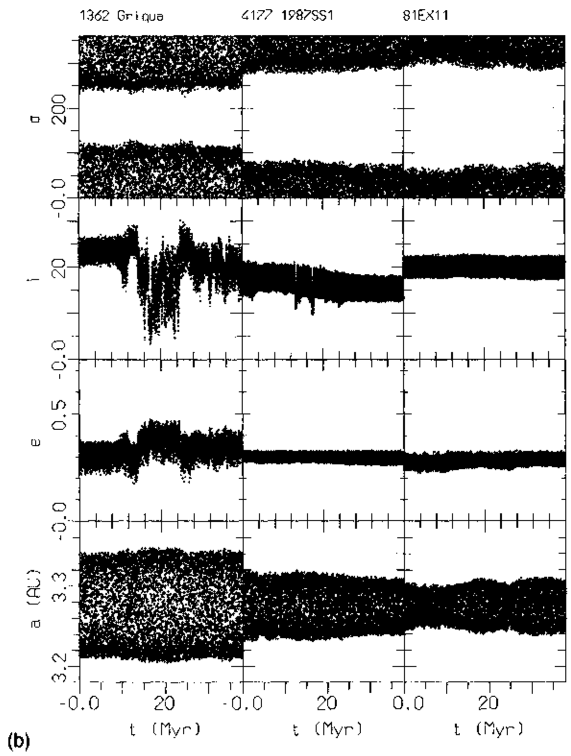
In order to explore the long-term dynamical behavior in the 2/1 resonance I have integrated, over 10^9 yr, 24 fictitious particles chosen on a regular grid as shown in Table 2. The bodies differ only for what concerns eccentricity and mean anomaly M, and, consequently, they have different values of the critical angle σ. The values of a,e,i,ω, and Ω have been chosen for simplicity equal to those of the asteroid 3789 Zhongguo, on date JD=2449700.5. Moreover, the value of a is close to the one of the exact resonance (note however that the value of the exact resonance can be defined only in the space of semimean elements and not in the space of osculating ones). As a consequence, the initial value of σ is close to the amplitude of the σ libration.

As in the previous section, I used for the integrations Levison and Duncan's code, with a step size of 30 days, and taking into account only the planets from Jupiter to Neptune (the masses of the inner planets being added to that of the Sun). The lifetime of each body is measured as the time necessary to become a Mars crosser. This has been done, although Mars is not very efficient in depleting the resonance from Mars-crossing bodies, since this limit (i.e., approximately $e\sim0.5$) is also the threshold for strong instability due to the overlapping of secular resonances (Morbidelli & Moons 1993).

Figure 3 shows the lifetime of each body with respect to its initial eccentricity and amplitude of libration. The size of the polygons is proportional to the lifetime of the corresponding body. The numbers allow to relate each dot to the complete initial conditions in Table 2, where the lifetime is also reported. Only four bodies survive for 1 Gyr in the 2/1 resonance (bodies Nos. 12, 16, 20, and 21), but the lifetime of quite a few bodies is very close to 1 Gyr: particle ten survives 950 Myr, particle 23 survives 970 Myr and particle 24 survives 810 Myr. Conversely, all bodies with initial small eccentricity ($e<0.15$) and/or large amplitude of σ libration have a very short lifetime, in agreement with the theoretical results on secondary resonances (Lemaitre & Henrard 1990) and on their overlapping with secular resonances (Henrard *et al.* 1995). It is interesting to remark that in the region defined by $e\geqslant0.2$, $\sigma\leqslant100°$, which corresponds to the part of the 2/1 resonance that Henrard *et al.* (1995) find to be stable and where Lyapunov exponents are small (Franklin 1994), escapes do not occur sooner than 361 Myr (particle No. 17). It is not surprising, then, that analytic theories do not find any mechanism of instability in this quasi-stable region. Indeed instability on such long time scales can be generated only by secondary and secular resonances of very high order, which are not usually accounted for by such theories.

The fact that some orbits survive for 1 Gyr inside the 2/1 resonance does not imply in principle that the dynamics of such orbits is regular. In order to point out the existence, the relevance and the time scales of diffusive phenomena within the 2/1 resonance, it is useful to define some numerically computable "proper elements" and to monitor their changes with respect to time. In this case I choose as proper elements the minimum value of a, the maximum value of e, and the maximum libration amplitude of σ, computed over an interval of 10 Myr. Among all possible numerically computable proper elements, these are the closest ones to those defined semianalytically by Morbidelli *et al.* (1995). Figure 4 shows the initial proper elements (σ,e) and (a,e) of the 24 bodies, i.e., computed over the first 10 Myr of integration. The numbers allow to relate each object in Fig. 4 with a corresponding object in Fig. 3. Again, the size of each polygon is linearly proportional to the lifetime of the body. The relative position of long living bodies is approximately conserved, passing from Fig. 3 to Fig. 4(a). conversely, the particles with short lifetimes are strongly chaotic already on a 10 Myr time scale, so that their proper elements are not significant; this is why their relative position is not conserved. Moreover, comparing Fig. 3 to Fig. 4(a) one sees that bodies with initial





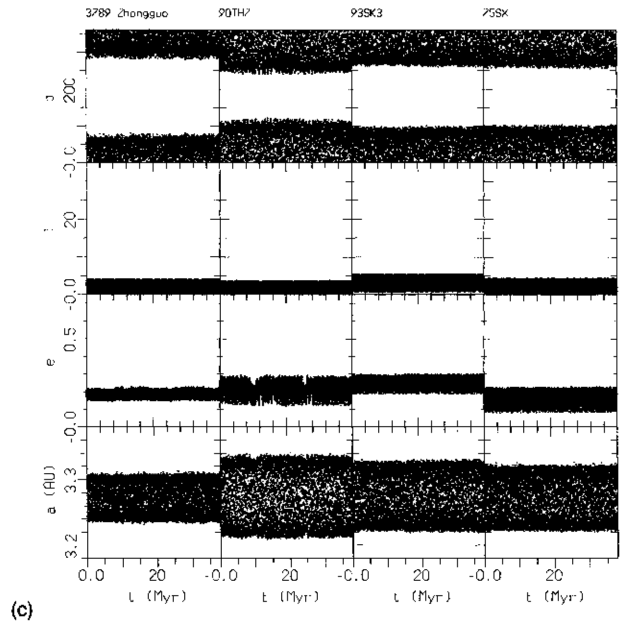


FIG. 2. (a) The evolution with time of the critical angle σ of the 2/1 resonance, of the inclination i, of the eccentricity e, and of the semimajor axis a for the asteroids 1921 Pala, 1922 Zulu, and 3688 Navajo. All three bodies escape from the 2/1 resonance (σ starts to circulate), and, due to their large eccentricity, they encounter Jupiter. Note that the results of symplectic numerical integrator used here are not reliable when planetary close encounters occur. (b) The same as (a) for the asteroids 1362 Griqua, 4177 1987SS$_1$, and 81EX$_{11}$. Remark the large inclination of the three bodies and their irregular evolution. (c) The same as (a) but for the asteroids 3789 Zhongguo, 90TH$_7$, 93SK$_3$, and 75SX. Remark the small inclinations the moderate eccentricities, and the quite regular evolution of the orbital elements.

σ close to zero have "proper" σ larger than 50°; such an increase of the amplitude of the σ libration is probably due to the effect of secular perturbations.

In order to compute the change of proper elements with time, the 10 Myr time span of computation is subsequently shifted, i.e., a "running box" with a window of 10 Myr is applied to the corresponding time series. For example, proper $a(t)$ is computed as the minimum of a over the interval $[t,t+10$ Myr$]$. In this way, I compute for each body one set of proper elements every 10^5 yr. Figure 5 shows the time evolution of the proper (a,e) for the four bodies which do not become Mars crossers during the full 1 Gyr time span of integration. The line denotes the border of the 2/1 resonance. As one clearly sees, all four orbits "diffuse," although they never leave the 2/1 resonance. The diffusion of the orbit of body No. 12 is quite moderate, while body No. 16 is almost Mars crosser at the end of the integration time span, the proper eccentricity being larger than 0.46. By com-

TABLE 2. Initial conditions at JD=2449700.5 of the test particles in the 2/1 resonance and their dynamical lifetime, i.e., the time spent in the 2/1 resonance before evolving to Mars-crossing orbit.

No.	e	M	σ	Lifetime (Myr)
1	0.054	275.72	120.14	38
2	0.055	315.72	80.14	19
3	0.055	355.72	40.14	21
4	0.105	255.72	140.14	13
5	0.105	295.72	100.14	27
6	0.105	335.72	60.14	62
7	0.105	30.72	5.14	59
8	0.155	275.72	120.14	12
9	0.155	315.72	80.14	97
10	0.155	355.72	40.14	949
11	0.205	255.72	140.14	71
12	0.205	295.72	100.14	>1000
13	0.205	335.72	60.14	407
14	0.205	30.72	5.14	469
15	0.255	275.72	120.14	52
16	0.255	315.72	80.14	>1000
17	0.255	355.72	40.14	361
18	0.305	255.72	140.14	21
19	0.305	295.72	100.14	763
20	0.305	335.72	60.14	>1000
21	0.305	30.72	5.14	>1000
22	0.355	275.72	120.14	12
23	0.355	315.72	80.14	969
24	0.355	355.72	40.14	810

parison, at the bottom of Fig. 5 the changes of the proper elements of the four giant planets are reported. For the planets, proper elements have been computed in the same way as for the bodies, but the semimajor axis has been scaled to enter Fig. 5. The orbits of the giant planets are very regular (Laskar 1990), so that their proper elements should not change with time. As a matter of fact, the evolution of the

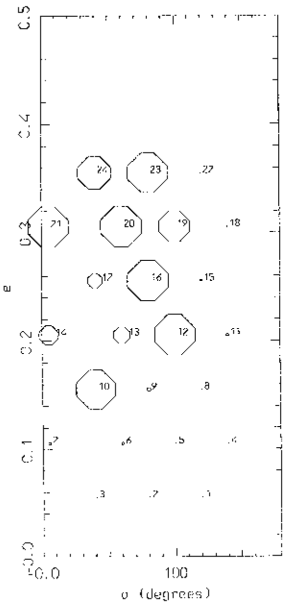


FIG. 3. Initial conditions σ, a of the 24 test particles (see Table 2). The size of the polygons is linearly proportional to their lifetime. Maximum size corresponds to 1 Gyr.

proper elements of Jupiter over 1 Gyr just results in one point: the conservation of its proper elements is remarkable. For Saturn, Neptune, and, particularly, Uranus, a time change of proper elements is visible. This is due to the existence of terms with period of the order of 10 Myr in the secular evolution of their orbits. Nevertheless, their apparent "diffusion" is much smaller than the one observed for the 2/1 resonant objects. This allows one to conclude that the diffusion observed for the resonant bodies is neither an artifact of the numerical integrator nor an artifact of the numerical computation of proper elements. We are confronted by a genuine dynamical phenomenon. One can therefore conclude that also the four long living orbits in the 2/1 resonance are chaotic and diffuse in a significant way.

Figure 6 shows the evolution with time of the proper elements (σ,e) and (a,e) for all 24 integrated bodies. This can be considered as a global chart of diffusion in the 2/1 resonance. Indeed, since proper elements are computed every 10^5 years, the regions where points accumulate correspond to the regions where the diffusion speed is smaller, so that the proper elements change slowly with time. Conversely, the regions where the points are more dispersed are the regions where the diffusion speed is larger and proper elements change fast. Since the evolutions of the 24 orbits would still leave some large parts of the 2/1 resonance unexplored. I have added in Fig. 6 the evolution of four more fictitious bodies, integrated during 500 Myr. Their suitably chosen initial conditions are listed in Table 3. It is certainly true that 28 is not a large number of particles. Nevertheless, the evolution of their proper elements covers quite completely the 2/1 resonant region in Fig. 6 (although some "holes" are unfortunately still visible). On the other hand, the numerical integration over 1 Gyr is very time consuming, so that it is important to keep the number of particles as small as possible.

In Fig. 6 bold dots represent orbits whose proper inclination is below 20°; smaller dots refer to proper inclinations larger than 20°. This has been done in order to give a hint of what happens in the third dimension (see also Fig. 7, referring to diffusion in the proper (e,i) space). Moreover, a dashed curve delimits what seems to be "by eye" the region characterized by the slowest diffusion speed, according to the density of bold points. The position in the proper element space of the four "good" asteroid 3789 Zhongguo, 90TH$_7$, 93SK$_3$, and 75SX is also reported; the stars in Fig. 6(b) denote the "proper elements" of the members of the Themis family which are close to the 2/1 resonance. The Themis family extends from a=2.8 AU to a=3.2 AU approximately and, as remarked by Morbidelli *et al.* (1995), is truncated at the border of the 2/1 resonance [the continuous line in Fig. 6(b)].

It is very instructive to look at the computer screen while Fig. 6 is forming, since one can see which are the preferential paths for escaping orbits. Orbits escape either diffusion towards large librations (small proper semimajor axis) or towards small eccentricities. The regions at small eccentricity (proper $e<0.2$) and the regions at large amplitude of libration are the most unstable parts of the 2/1 resonance, where diffusion accelerates a lot; the eccentricity can increase very

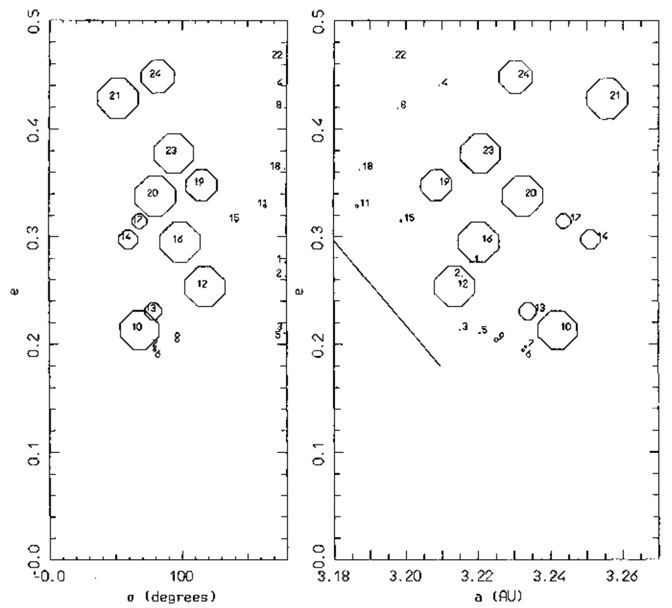


FIG. 4. The proper elements of the 24 test particles, computed during the first 10 Myr of integration. Picture a (on the left) refers to the amplitude of σ libration and e; picture b (on the right) refers to a and e.

rapidly, often together with increments of the inclination, and the orbits become Mars crossing (see also Henrard *et al.* 1995). Conversely, orbits which happen to diffuse towards the central part of the 2/1 resonance experience a sort of sticking phenomenon. For example, orbit No. 10, starting from proper $a=3.242$ AU, $e=0.213$ [see Fig. 4(b)] initially migrates towards larger eccentricity; thus, the orbit is sticked at proper $a\sim3.24$ proper $e\sim0.24$ for about 800 Myr; then it manages to diffuse again to small eccentricities where the amplitude of libration and, subsequently, the eccentricity and the inclination increase, and it becomes Mars crossing in less than 100 Myr. Particle No. 20, starting from proper $a=3.233$ AU, $e=0.338$ [Fig. 4(b)] diffuses towards the right part of the proper (a,e) diagram and manages to survive 1 Gyr in the 2/1 resonance (see Fig. 5).

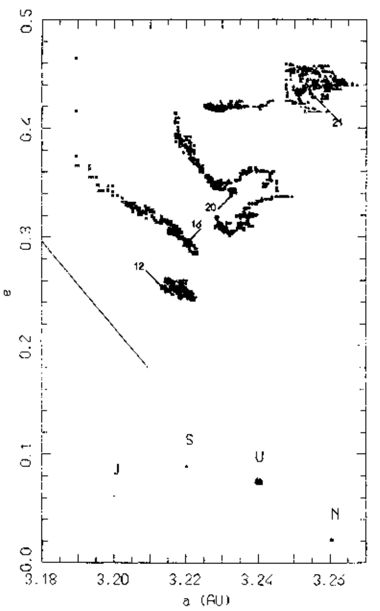


FIG. 5. Diffusion of the proper elements of test particles Nos. 12, 16, 20, and 21 over 1 Gyr. The numbers point to their initial proper elements. The line denotes the border of the 2/1 resonance. At the bottom, the evolution of the proper elements of Jupiter, Saturn, Uranus, and Neptune; their semimajor axis has been scaled to enter the picture. The evolution of planetary proper elements is a check which shows that the diffusion of the test particles is a genuine dynamical phenomenon and not an artifact of numerical computations.

Looking at Fig. 7 one sees that there are three main channels for large increase of the inclination: at $e=0.2$, at $e=0.27$–0.35, and at $e=0.45$. Inspired by the results in Henrard *et al.* (1995), one can conjecture that the first one (which concerns mainly orbits at large amplitude of libration) is related to the secondary resonances between the period of circulation of the argument of perihelion and the period of libration of σ; the second one (which concerns orbits with moderate amplitude of libration) could be related to the ν_{16} secular resonance; the last one (which concerns again orbits with large amplitude of libration) could be related with the Kozai resonance. Unfortunately, in the present numerical integrations, the angles have not been monitored with enough time resolution, so that the data are not sufficient to prove this conjecture. As in Fig. 6, a dashed curve delimits what seems to be the region characterized by the slowest diffusion speed, according to the density of points.

4. ON THE NATURE AND THE ORIGIN OF THE 2/1 RESONANT ASTEROIDS

In the light of the results illustrated in the previous section, one can now speculate about the long-term dynamical

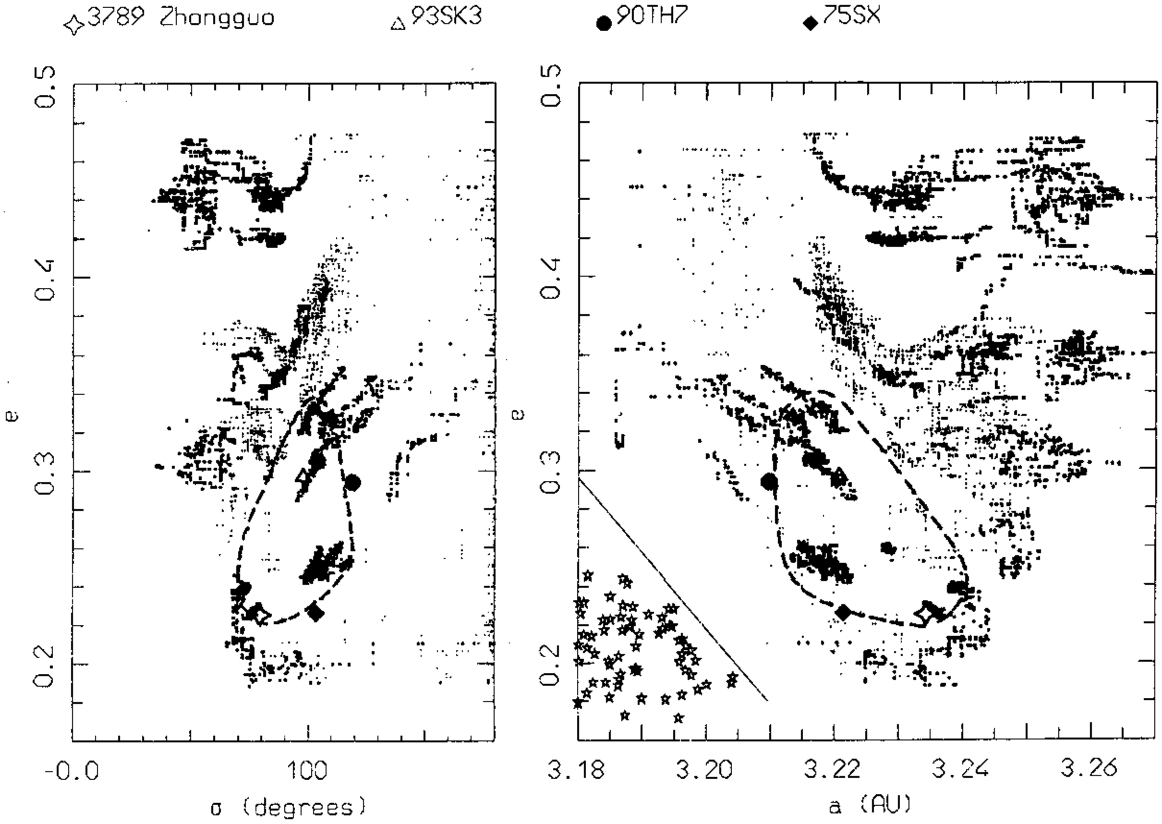


FIG. 6. Diffusion of the proper elements of the 24 particles of Table 2 and the 4 particles of Table 3. The regions where points accumulate are the regions where the diffusion speed is lower; conversely the regions where points are dispersed are those characterized by fast diffusion. Bold points refer to orbits with $i<20°$; light points to orbits with $i>20°$. The present position in the proper elements space of the asteroids 3789 Zhongguo, $93SK_3$, $90TH_7$, and 75SX is also marked. The dashed curve delimits what seems to be "by eye" the region characterized by the slowest diffusion speed. In picture b (right-hand side), the line denotes the border of the 2/1 resonance and the stars mark the position of the members of the Themis family (see Morbidelli *et al.* 1995).

behavior of the resonant asteroids 3789 Zhongguo, $90TH_7$, $93SK_3$, and 75SX, as well as on their origin. One could be tempted to integrate numerically the evolution of these objects over a very long time scale, but this work would be, at this stage, quite useless. Indeed, since in the 2/1 resonance all orbits seem to diffuse chaotically, numerical integrations are not deterministic and cannot reproduce the real evolution of the asteroids under study. It is true that numerical integrations show possible evolution paths, but in the present case these paths have been already explored by looking at the evolution of the 24+4 fictitious bodies discussed above.

Figures 6 and 7 show that the asteroids 3789 Zhongguo, $90TH_7$, $93K_3$, and 75SX are more or less connected to the "best" region of the 2/1 resonance (i.e., the region characterized by the slowest diffusion speed, delimited by the

TABLE 3. Initial conditions at JD=2449700.5 of the four additional test particles in the 2/1 resonance.

No.	e	M	σ
a	0.230	325.72	70.14
b	0.230	345.72	50.14
c	0.330	353.72	42.14
d	0.255	30.72	5.14

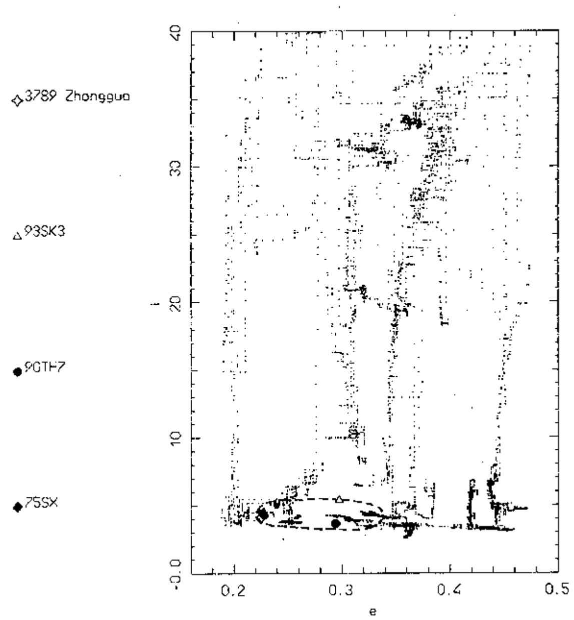


FIG. 7. The same as Fig. 5 but for proper eccentricity and inclination.

dashed curve). Therefore, they should stay inside the 2/1 resonance over timescales of hundreds of millions of years. Conversely, their belonging to the 2/1 resonance over timescales of the order of billions of years is questionable, since most of the test orbits escape in a shorter time. Therefore, if these asteroids have an age of the order of 1 Gyr or more, they must be the remnant of a much larger population. With these considerations in mind, one can outline three scenarios.

(i) The asteroids that we observe now can be the remnant of a much larger original population of resonant bodies of cosmogonic origin (i.e., as old as 4–5 Gyr). One should explain, however, why they are of very small size (a few kilometers in diameter). Indeed, there is no apparent reason why chaotic diffusion should "privilege" small bodies, depleting the resonance of all the big ones. A statistical argument could be invoked, based on the fact that, in the original population of asteroids, the number of bodies should increase rapidly with decreasing size. The small size of the presently observed asteroids can suggest also a possible origin as fragments injected into the 2/1 resonance by collisions between non-resonant asteroids, as considered by the two following scenarios.

(ii) The origin of the resonant asteroids could be connected to the origin of the Themis family. As shown in Fig. 6(b) (see Morbidelli *et al.* 1995 for a detailed discussion), the Themis family is truncated at the border of the 2/1 resonance. Therefore the family must have injected, at the time of its original breakup, several members inside the resonance. These resonant members should have covered in the proper elements space the region with proper e roughly ranging from 0.18 to 0.26 and proper a up to 3.27, the smallest fragments being the farthest (since typically fragment ejection speeds increase for smaller sizes: see Giblin *et al.* 1994). Then, as one sees from Fig. 6(b), most of the fragments of the Themis family might have been injected in the region where diffusion is very fast and, as a consequence, they would have been ejected. However, some of them should have reached the quasistable region bordered by the dashed line. This hypothesis is also supported by the fact that the inclinations of the orbits in the quasistable region are consistent with the inclinations of the Themis family members. The age of the Themis family is usually considered to be older than 1 Gyr (Marzari *et al.* 1995), so that most of its quasistable resonant members should be escaped, by now. It is possible, however, that the four asteroids that we observe at the present epoch (i.e., a few times the average dynamical lifetime in the quasistable region) are just the last "survivors." Chaotic diffusion would explain also why two of the 4 asteroids (namely $90TH_7$ and $93SK_3$) are no longer on the ideal prolongation of the Themis family inside the 2/1 resonance, but are at a somewhat larger proper eccentricity.

(iii) One could explain the existence of the four resonant asteroids in the quasistable region of the 2/1 resonance on the base of a statistical argument. Since the quasistable region is sticky for the dynamics, then, statistically, bodies which are temporarily associated to the 2/1 resonance should spend most of the time there. Therefore this is the region where one has the largest probability to observe bodies at any given time. This conjecture is also supported by the fact that three of the four asteroids are just at the borders of the quasistable region, as if they had been stuck there, coming from the "fast" diffusion regions (in a similar way to what happens to the fictitious particle No. 10, described in the previous section). According to this scenario, it is not necessary to invoke a very energetic event such as the formation of the Themis family to explain their origin. Indeed, the breakup of the Themis family is probably the only event that could inject bodies *directly* in the quasistable region; however, if the bodies that we observe now have reached the quasistable region diffusing through the strongly chaotic regions, they could have been injected into the 2/1 resonance in a more recent epoch and during much less energetic events. They could even be dormant comets, the eccentricity of which has been decreased by chaotic dynamics. It is well known, indeed, that several short-period comets are in temporary mean motion resonances with Jupiter.

It is very hard to privilege one of the three scenarios outlined above, and this is not the purpose of the present paper. There is a large source of uncertainty, since matters such as the size distribution of primordial bodies, the collision rates, and the age of the Themis family, are poorly determined. Last but not least, we do not have a complete picture of the actual population of the 2/1 resonant asteroids. Indeed, kilometric size objects are far beyond the completeness limit for discovery in this region of the asteroid belt (Zappalá & Cellino 1995). If new small asteroids will be discovered in the quasistable region, then the Themis family scenario will be more strongly supported.

5. CONCLUSION

Although the future evolution of chaotic orbits cannot be exactly determined, numerical integrations are surely able to discriminate between resonant and nonresonant bodies, regular and chaotic orbits, and show possible evolutionary paths. Therefore, in the light of the numerical investigations described in the previous sections (which take into account the perturbations produced by the four major planets) one can draw the following conclusions. The 2/1 resonance with Jupiter is a partial Kirkwood gap. There are presently ten asteroids in the resonance with $e<0.5$; of these bodies, six have a strongly irregular orbit (three of them are ejected in a few tens of million years in the present integrations) and/or a quite large inclination. The remaining four asteroids, named 3789 Zhongguo, $90TH_7$, $93SK_3$ and 75SX, have a quite regular orbit with moderate eccentricity and inclination.

From the analysis of the evolution of the orbits of several fictitious particles, it turns out that there are no regular orbits (i.e., invariant tori) within the 2/1 resonance. All test orbits diffuse in the space of suitable proper elements and their evolution fills almost completely the phase space. Diffusion is fast for orbits with small eccentricity and for orbits with large amplitude of libration; conversely there are regions at moderate eccentricity and moderate amplitude of libration where diffusion is very slow (although it is still there). Therefore, some test particles can survive even 1 Gyr in the 2/1 resonance without becoming Mars crossers (this happens to 4 over 24 particles in the integrations reported here). This slow diffusion region corresponds approximately to the zone which is found to be stable in low-order analytic theories (Henrard *et al.* 1995), and where Lyapunov exponents are small (Franklin 1994). This is not surprising, since such a slow diffusion can be forced only by secondary and secular resonances of very high order, which are not usually accounted for by semianalytic theories, and can force only weak hyperbolic divergence of nearby orbits.

What seems to be the most stable region in the 2/1 resonance has been surrounded by a dashed line in Figs. 6 and 7. The asteroids 3789 Zhongguo, $90TH_7$, $93SK_3$, and 75SX seem to be associated with this region. Test particles close to these bodies are "stuck" to this region during several hundred million years before they exit the quasistable region and become strongly unstable. Therefore, the age of these bodies, from the dynamical point of view, can be easily of the order of a few times 10^8 yr. Morbidelli *et al.* (1995) have pointed out that in the proper elements space, 3789 Zhongguo is on the ideal prolongation of the Themis family inside the 2/1 resonance, and therefore conjectured that Zhongguo is a member of the Themis family injected into a stable island of the 2/1 resonance. In the light of the present study, also 75SX is also on the prolongation of the Themis family, while $90TH_7$ and $93SK_3$ are at somewhat larger eccentricities.

The Themis family is usually considered to be older than 1 billion years; therefore, if Zhongguo and/or the other three bodies are members of Themis family, they must have been lucky enough to survive longer than the average lifetime of test particles in their vicinity. On the other hand, it is unlikely that the Themis family injected only a small number of km-sized bodies in the quasistable region of the 2/1 resonance; therefore, the asteroids 3789 Zhongguo, $90TH_7$, $93SK_3$, and 75SX could be just the last survivors of a larger population which has been, by now, depleted, because the average dynamical lifetime in the 2/1 resonance is shorter than 1 Gyr.

In conclusion, the present paper neither contradicts the conjecture by Morbidelli *et al.* (1995) on the origin of Zhongguo, nor proves it in a definite way. The fact that three additional asteroids have been discovered recently close to Zhongguo (in the phase space) shows that we are probably far from having a complete picture of the real population of bodies within the 2/1 resonance, which will be achieved only by decreasing the size limit of completeness for asteroid discovery. As well, a deeper knowledge on the structure and the age of the Themis family will allow to make more specific simulations on the depletion rate of its 2/1 resonant members.

Conversely, in the light of the numerical results of this paper, it is understood that the 2/1 resonance is depleted of at least most of primordial bodies of cosmogonic origin (i.e., as old as 4–5 Gyr), in particular taking into account that such bodies should have formed mainly in the small eccentricity region, where the diffusion speed is faster.

I am grateful to M. Moons and J. Henrard for their advice. I am in debt also with Ch. Froeschlé, G. B. Valsecchi, P. Farinella, and F. Franklin for their suggestions.

REFERENCES

Bowell, E., Muinonen, K., & Wasserman, L. H. 1993, in Asteroid Comets Meteors, edited by Milani, DiMartino, and Cellino, p. 477
Farinella, P., Froeschlé, Ch., Froeschlé, C. Gonczi, R., Hahn, G., Morbidelli, A., & Valsecchi, G. B. 1994, Nature, 371, 314
Ferraz-Mello, S. 1994, AJ, 108, 2330
Franklin, F. 1994, AJ, 107, 1890
Froeschlé, C., & Scholl, H. 1981, A&A, 93, 62
Froeschlé, Ch., Hahn, G., Gonczi, R., Morbidelli, A., & Farinella, P. 1995, Icarus, 117, 45
Giblin, I., Martelli, G., Smith, P. N., Cellino, A., DiMartino, M., Zappalà, V., Farinella, P., & Paolicchi, P. 1994, Icarus, 110, 203
Giffen, ? 1973, A&A, 23, 387
Henrard, J., & Lemaitre, A. 1986, Celest. Mech., 39, 213
Henrard, J., Watanabe, N., & Moons, M. 1995, Icarus, 115, 336
Kirkwood, D., Proceedings of the American Association for the Advancement of Science, 1866
Laskar, J. 1990, Icarus, 88, 266
Lemaitre, A., & Henrard, J. 1990, Icarus, 83, 391
Levison, H. F., & Duncan, M. J. 1993, ApJ, 406, L35
Marzari, F., Davis, D., & Vanzani, V. 1995, Icarus, 113, 168
Milani, A., & Knežević, Z. 1990, Celest. Mech., 49, 247
Moons, M., & Morbidelli, A. 1993, Celest. Mech., 57, 99
Moons, M., & Morbidelli, A. 1995, Icarus, 114, 33
Morbidelli, A., & Moons, M. 1993, Icarus, 102, 316
Morbidelli, A., & Moons, M. 1995, Icarus, 115, 60
Morbidelli, A., Zappalá, E., Moons, M. Cellino, A., & Gonczi, R. 1995, Icarus (in press)
Murray, C. 1986, Icarus, 65, 70
Wisdom, J. 1982, AJ, 85, 1122
Widsom, J. 1985, Icarus, 63, 272
Wisdom, J. 1987, Icarus, 72, 241
Wisdom, J., & Holman, M. J. 1991, AJ, 102, 1528
Yoshikawa, M. 1989, A&A, 213, 436
Yoshikawa, M. 1990, Icarus, 87, 78
Yoshikawa, M. 1991, Icarus, 92, 94
Zappalá, E., & Cellino, A. 1995, ASP Conf. Ser. (in press)