Direct numerical simulation of Alfvén-wave filamentation in Hall-MHD

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The description of a magnetized plasma at spatial and temporal scales approaching the ion inertial length and the inverse ion gyromagnetic frequency Ω_i requires that the effect of ion inertia be retained through the Hall term in a generalized Ohm's law. This leads to the Hall-MHD equations

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\frac{\beta}{\gamma} \nabla \rho^\gamma + (\nabla \times \mathbf{b}) \times \mathbf{b} \tag{2}$$

$$\partial_t \mathbf{b} - \nabla \times (\mathbf{u} \times \mathbf{b}) = -\frac{1}{R_i} \nabla \times \Big(\frac{1}{\rho} (\nabla \times \mathbf{b}) \times \mathbf{b} \Big) \tag{3}$$

$$\nabla \cdot \mathbf{b} = 0. \tag{4}$$

As usual, ρ is the density of the plasma, $\mathbf{u}$ its velocity and $\mathbf{b}$ the magnetic field. The equations are written in a non-dimensional form, taking as unity the Alfvén speed $c_A = \mathcal{B}_0/\sqrt{4\pi\rho_0}$, where $\mathcal{B}_0$ is the magnitude of the ambient magnetic field and ρ_0 the mean density of the plasma. Furthermore, $\beta = c_s^2/c_A^2$ where c_s is the sound velocity and γ is the polytropic gas constant. The parameter $R_i = \Omega_i L/c_A$ (where L denotes a reference scale) provides a non-dimensional measure of the ion-cyclotron frequency.

These equations admit exact solutions in the form of dispersive monochromatic circularly polarized Alfvén waves propagating along an ambient field. When subject to purely transverse perturbations, small-amplitude waves can experience self-focusing and collapse into intense magnetic filaments, a process asymptotically described by the two-dimensional nonlinear Schrödinger equation [1]. In fact, such a filamentation instability usually coexists with other longitudinal and oblique instabilities. In order to investigate the competition between these effects, the Hall-MHD equations were integrated numerically in the case of an initially weakly perturbed circularly polarized Alfvén wave propagating along an ambient field. A main observation concerns the strong sensitivity of the resulting dynamics to the spectral extension of initial noise.

While an analytic study of the oblique instabilities is cumbersome, the longitudinal and transverse instabilities are easily characterized in the case of a small-amplitude pump [2]. As displayed in Figs. 1, for right-hand and left-hand polarized pumps, the decay, modulational and long-wave (in the sense that the unstable daughter waves can have an arbitrarily small wavenumber and the side-band modes be arbitrary close to the pump) modulational instabilities in the longitudinal direction occur in regions of the (β, k/R_i)-plane separated by the group and phase velocities v_g and v_ϕ of the pump. The above figures also indicate the parameter domains of the transverse instability.

Figure 2 displays the results of a direct numerical simulation, using $\beta = 1.5$ and a left-hand polarized pump with wavenumber $k = 1$. For these parameters, no longitudinal instability is present but the oblique instabilities extend from large to small scales. The initial noise being confined at relatively large scale, we observe the formation of magnetic filaments slightly perturbed by the oblique unstable modes, while the density fluctuations and the induced longitudinal magnetic field display spiral structures. As exemplified in

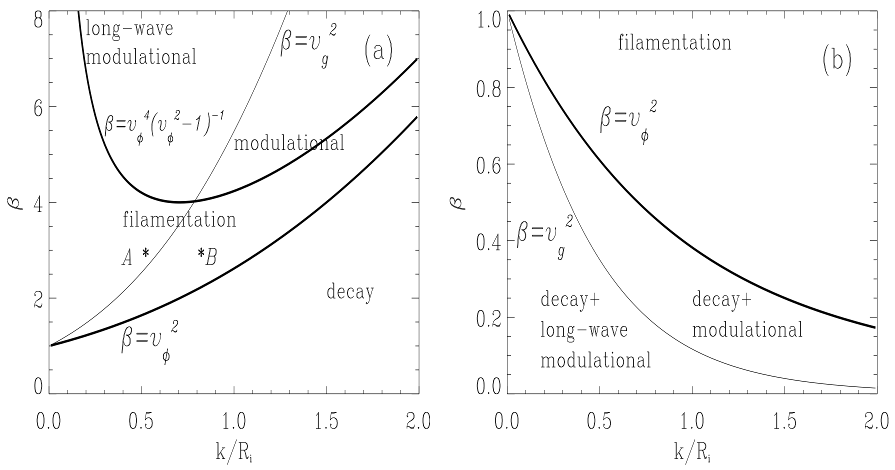


Figure 1: *Longitudinal (thin lines) and transverse (thick lines) instability regions in the plane* (β, k) *for a small-amplitude Alfvén wave with right-hand (a) or left-hand (b) polarization.*

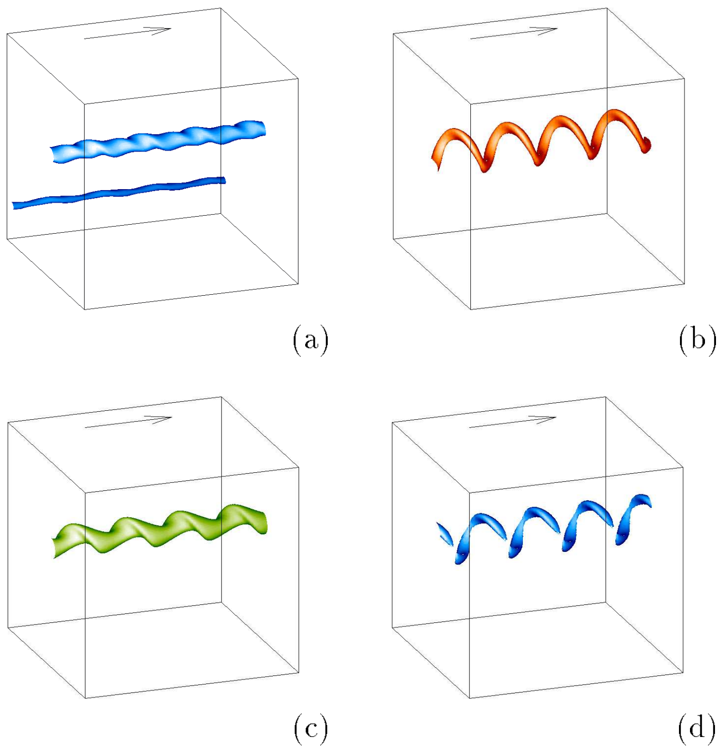
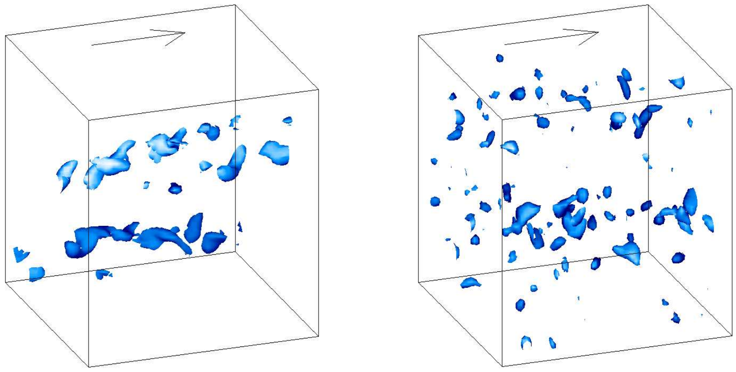


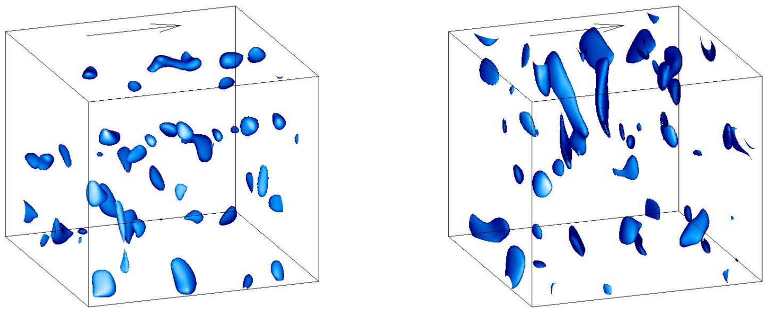
Figure 2: *Isosurfaces of* $b_y^2 + b_z^2$ *(a),* ρ *(b),* u_x *(c) and* b_x *(d) for an initially left-hand polarized Alfvén wave with amplitude 0.1 and wavenumber* $k = 1$*, perturbed at large scale when* $\beta = 1.5$.

Figures 3 and 4, these structures are progressively destroyed as the spectral range of the initial noise is extended or when the wave amplitude is increased. In the latter case, strong gradients still develop in the transverse directions but without the strong amplitude growth characteristic of wave filamentation [3]. Other simulations are presented in [4]. It is shown that filamentation of a small-amplitude wave can occur even in the presence of a linearly dominant large-scale longitudinal instability (conditions of point A of Fig. 1a),



(a) (b)

Figure 3: *Same as Fig. 2a, for initial noises including smaller and smaller scales.*



(a) (b)

Figure 4: *Same as Fig. 2a, for a pump wave of amplitude 0.35 (a) and 1 (b).*

while pancake-like magnetic structures mostly transverse to the direction of propagation are formed when unstable longitudinal daughter modes at scales comparable to that of the pump are initially excited (point B in Fig. 1a).

An other important issue concerns the dynamics of long-wave pumps that are often encountered in space plasmas. When the amplitude of the pump being kept small, its wavelength is increased, the problem is amenable to a reductive perturbative expansion that, for $\beta \neq 1$, isolates the Alfvén-wave dynamics by prescribing a slow evolution in a frame moving at the Alfvén velocity (taken here as unity). This condition that eliminates the perturbations propagating at other characteristic velocities leads in one space dimension to the so-called derivative nonlinear Schrödinger equation (DNLS) [5] for the transverse magnetic field b, defined as the complex field $b_y + ib_z$ rescaled by the wave magnitude $\epsilon^{1/2}$ and viewed as a function of the stretched variables $\xi = \epsilon(x-t)$ and $\tau = \epsilon^2 t$. In the case of a multidimensionally perturbed wave trains, mean fields resulting from the averaging of magneto-sonic waves along the direction of propagation are to be retained. One obtains [6], [3]

$$\partial_\tau b + \partial_\xi \left(\frac{1}{2}bP + (\bar{u}_x + \frac{1}{2}\bar{b}_x)b\right) - \frac{1}{2}\partial_\perp P + \frac{i}{2R_i}\partial_{\xi\xi} b = 0 \tag{5}$$

$$\partial_\tau \bar{u}_x = \frac{1}{2}\left(\partial_\perp^* \langle bP\rangle + \partial_\perp \langle b^* P\rangle\right) \tag{6}$$

$$\partial_\xi \widetilde{b}_x + \frac{1}{2}\left(\partial_\perp^* b + \partial_\perp b^*\right) = 0. \tag{7}$$

In the above equations, bars or brackets $\langle\cdot\rangle$ indicate averages over the ξ-variable and the notation $\partial_\perp = \partial_\eta + i\partial_\zeta = \epsilon^{3/2}(\partial_y + i\partial_z)$ has been introduced. Like the mean longitudinal velocity $\bar{u}_x$, the induced longitudinal magnetic field is rescaled by a factor ϵ. It is furthermore separated in mean and fluctuating parts in the form $\bar{b}_x + \tilde{b}_x$ with $\bar{b}_x = \frac{1}{1+\beta}(-\frac{\langle|b|^2\rangle}{2} + E_M)$. The constant E_M, defined as the average over the whole domain of the magnetic energy density $\frac{|b|^2}{2}$, is retained to ensure that $\bar{b}_x$ is zero in one space dimension. In addition, the fluctuations of magnetic pressure are given by $P = \frac{1}{2(1-\beta)}(2\tilde{b}_x + |b|^2 - \langle|b|^2\rangle)$. These equations select Alfvén waves propagating forward along the ambient field and are thus restricted to plasmas with $\beta > 1$ where decay instabilities are absent. Numerical simulations of these equations show that for right-hand polarization, no small-scale oblique instability develops and Alfvén wave filamentation is obtained even with a broad spectrum initial noise. This is consistent with the observation that as the pump wavelength is reduced, the oblique and transverse unstable modes obey different scalings. As a consequence, the oblique instabilities for a right-hand polarized wave are filtered out by the long-wave asymptotics. In the case of a left-hand polarized pump in contrast, oblique instabilities remain present, leading to a dynamics similar to that observed with a finite-wavenumber pump. Further developments concern in particular the inclusion of kinetic effects in the long-wave equations by extending the approach of [7] to wave-train dynamics, in order to study their influence on filamentation.

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