Velocity and Intensity in Helioseismology

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A workshop

- --- Begin --
  - The intensity signal in helioseismology
    - Why?
    - What is it?
    - What information does it carry?
- --- End --

Helioseismology - Theory

60 *CHAPTER 4. EQUATIONS OF LINEAR STELLAR OSCILLATIONS*

4.2 The Oscillation Equations

4.2.1 Separation of variables

The displacement $\boldsymbol{\delta}\mathbf{r}$ is separated into radial and horizontal components as

$$\boldsymbol{\delta}\mathbf{r} = \xi_r \mathbf{a}_r + \boldsymbol{\xi}_\mathrm{h} \,. \tag{4.15}$$

The horizontal component of the equations of motion, (3.43), is

$$\rho_0 \frac{\partial^2 \boldsymbol{\xi}_\mathrm{h}}{\partial t^2} = -\nabla_\mathrm{h} p' - \rho_0 \nabla_\mathrm{h} \Phi' \,. \tag{4.16}$$

As the horizontal gradient of equilibrium quantities is zero, the horizontal divergence of equation (4.16) gives

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The equation of continuity, (3.41), can be written as

$$\rho' = -\frac{1}{r^2} \frac{\partial}{\partial r} (\rho_0 r^2 \xi_r) - \rho_0 \nabla_\mathrm{h} \cdot \boldsymbol{\xi}_\mathrm{h} \,. \tag{4.18}$$

This can be used to eliminate $\nabla_\mathrm{h} \cdot \boldsymbol{\xi}_\mathrm{h}$ from equation (4.17), which becomes

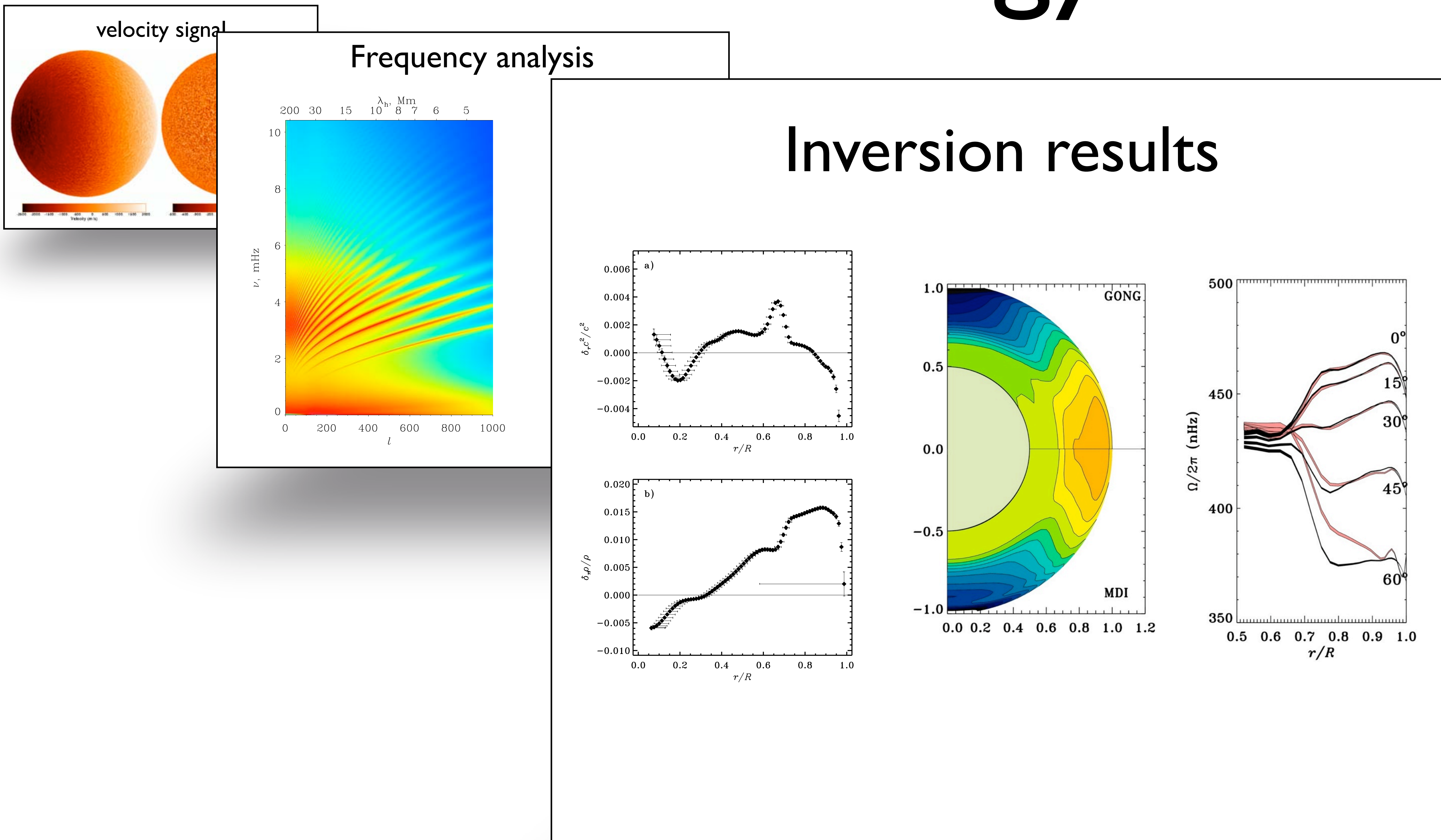
$$-\frac{\partial^2}{\partial t^2} \left[\rho' + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho_0 \xi_r) \right] = -\nabla_\mathrm{h}^2 p' - \rho_0 \nabla_\mathrm{h}^2 \Phi' \,. \tag{4.19}$$

The radial component of equation (3.43) is

- Equations are written in displacement, which is small.

  An 3 mHz oscillation with amplitude of 1 m/s has an displacement amplitude of about 50 m.

Helioseismology



Helioseismology
Why doing it differently?

❶ Helioseismology Why doing it differently?

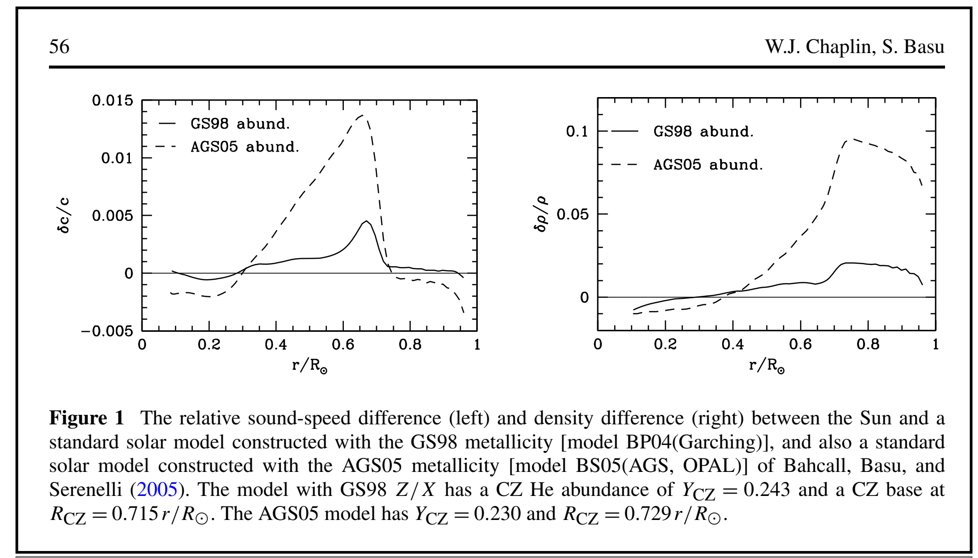
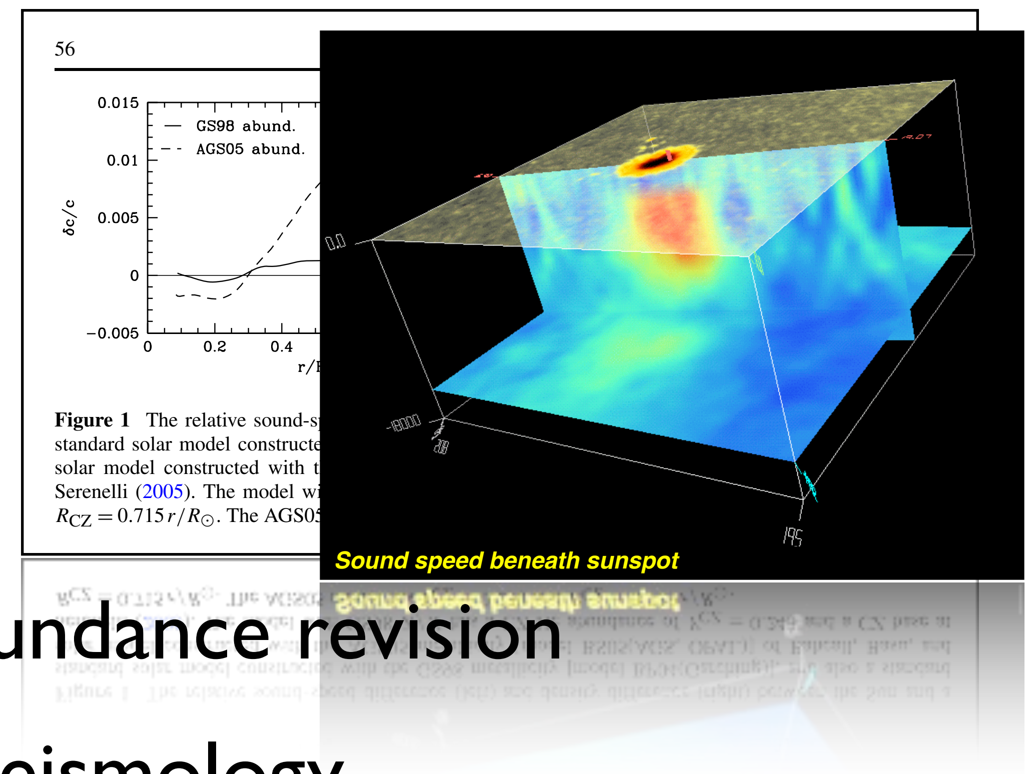

56 W.J. Chaplin, S. Basu

Figure 1 The relative sound-speed difference (left) and density difference (right) between the Sun and a standard solar model constructed with the GS98 metallicity [model BP04(Garching)], and also a standard solar model constructed with the AGS05 metallicity [model BS05(AGS, OPAL)] of Bahcall, Basu, and Serenelli (2005). The model with GS98 Z/X has a CZ He abundance of $Y_{CZ} = 0.243$ and a CZ base at $R_{CZ} = 0.715\,r/R_{\odot}$. The AGS05 model has $Y_{CZ} = 0.230$ and $R_{CZ} = 0.729\,r/R_{\odot}$.

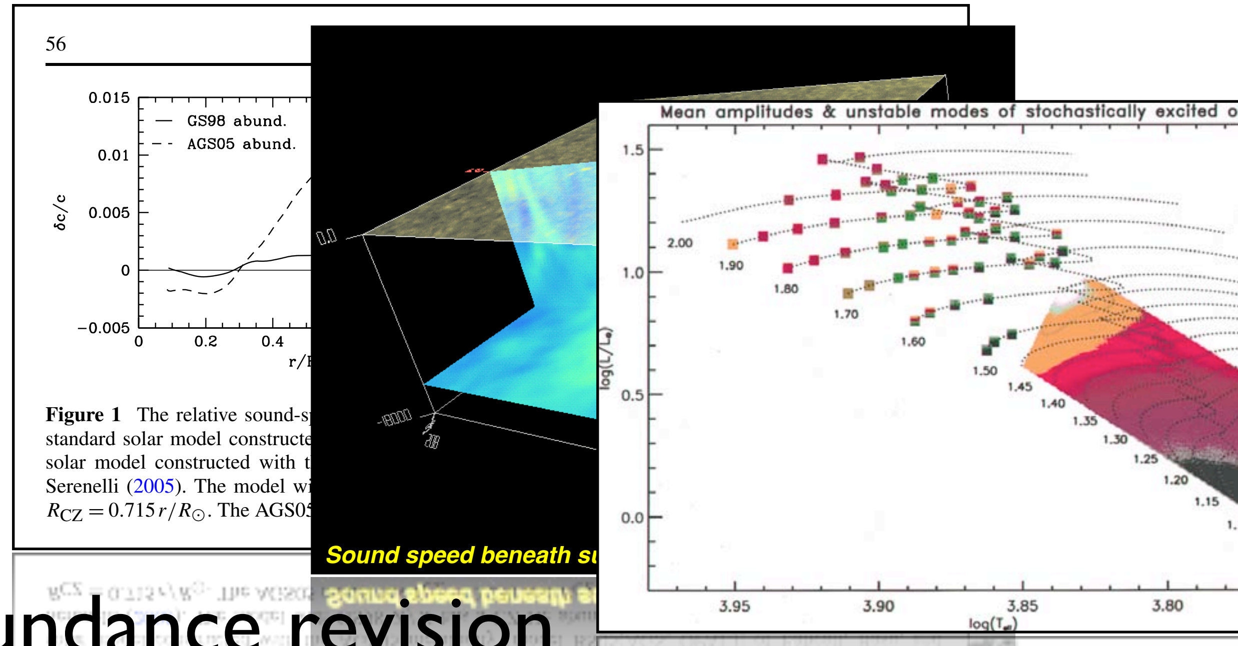
- Since we have done that for decades, what remains is...
  - Challenge of proposed abundance revision

❶ Helioseismology
Why doing it differently?



- Since we have done that for decades, what remains is...
  - Challenge of proposed abundance revision
    - Local helioseismology
      - ...

❶ Helioseismology
Why doing it differently?



- Since we have done that for decades, what remains is...
  - Challenge of proposed abundance revision
    - Local helioseismology
      - ...
      - Astroseismology

⚠ not only velocity, details of Solar case important!

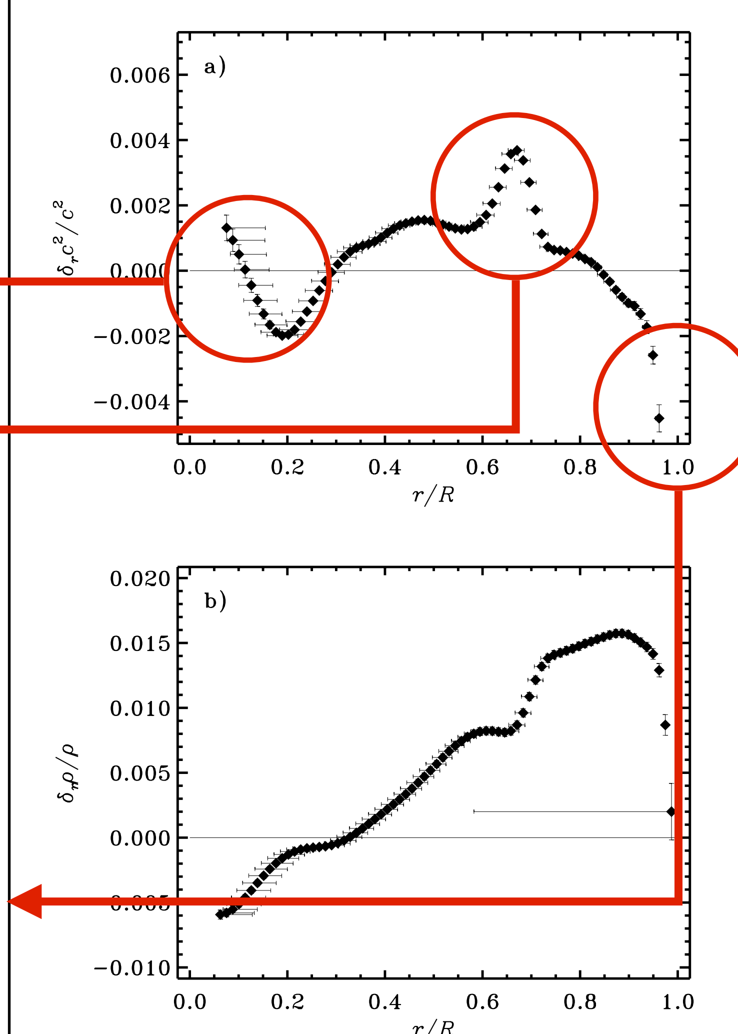
❷ Helioseismology
Why doing it differently?

- Because we know our limits since a decade...

◉ Solar nucleus

◉ Tachocline region

Inversion results



◉ Surface terms

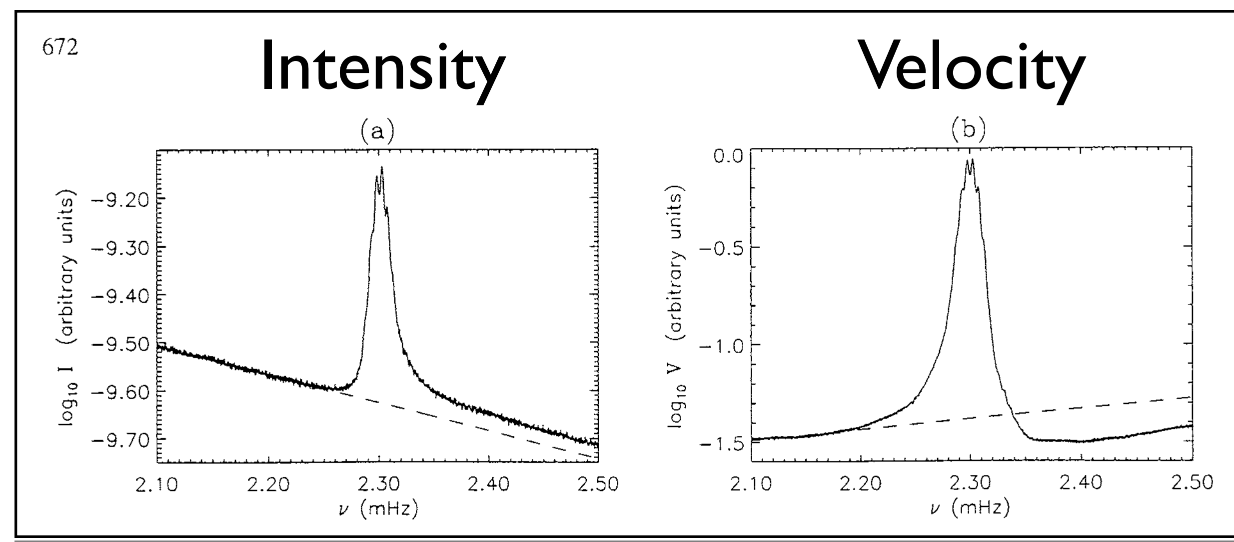
◉ Sources of oscillations

◉ Atmospheric Seismology

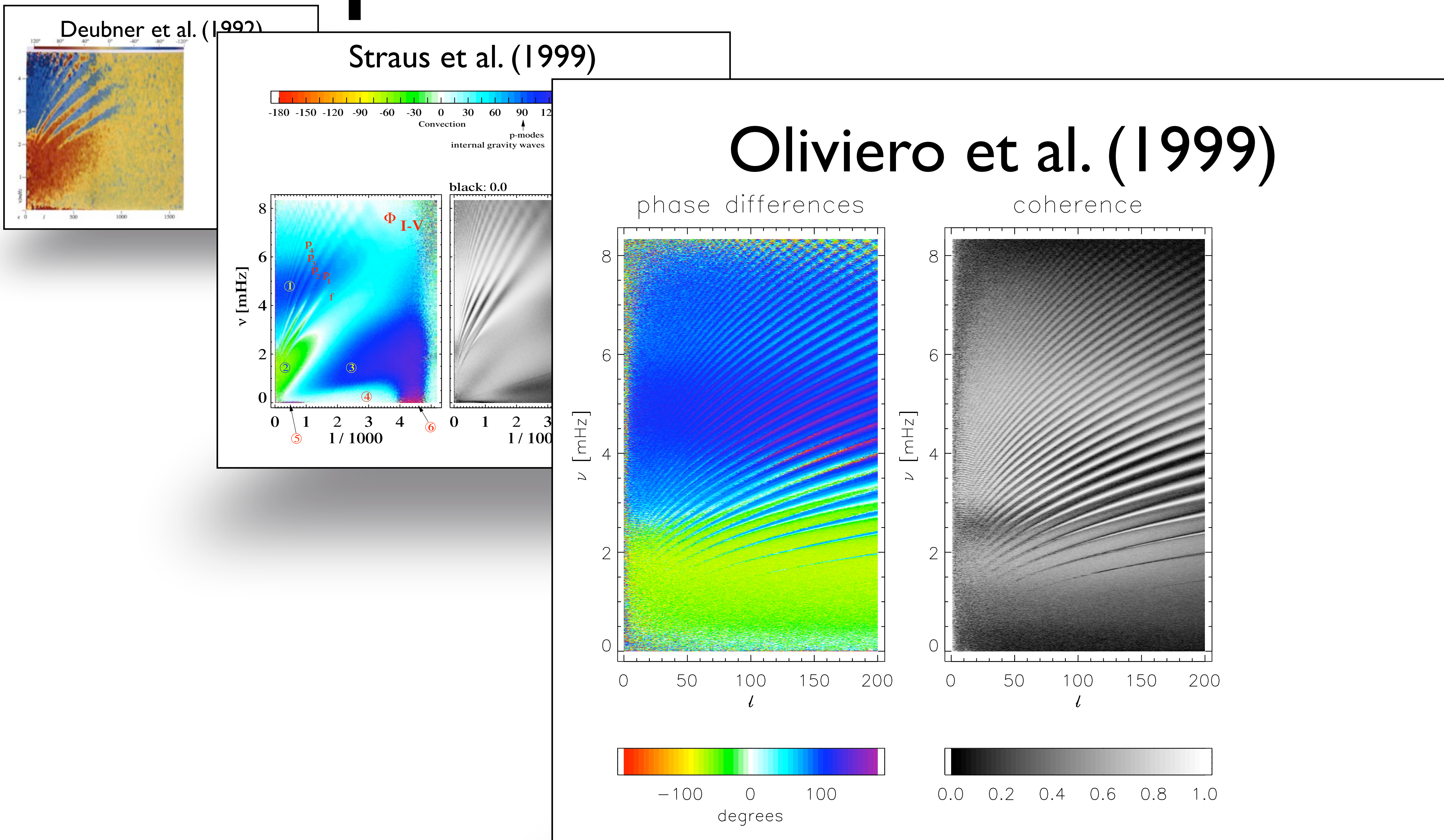
❸ Helioseismology
Why doing it differently?

- Because it is different...
  - line aymmetries (Duvall et al. 1993;...)

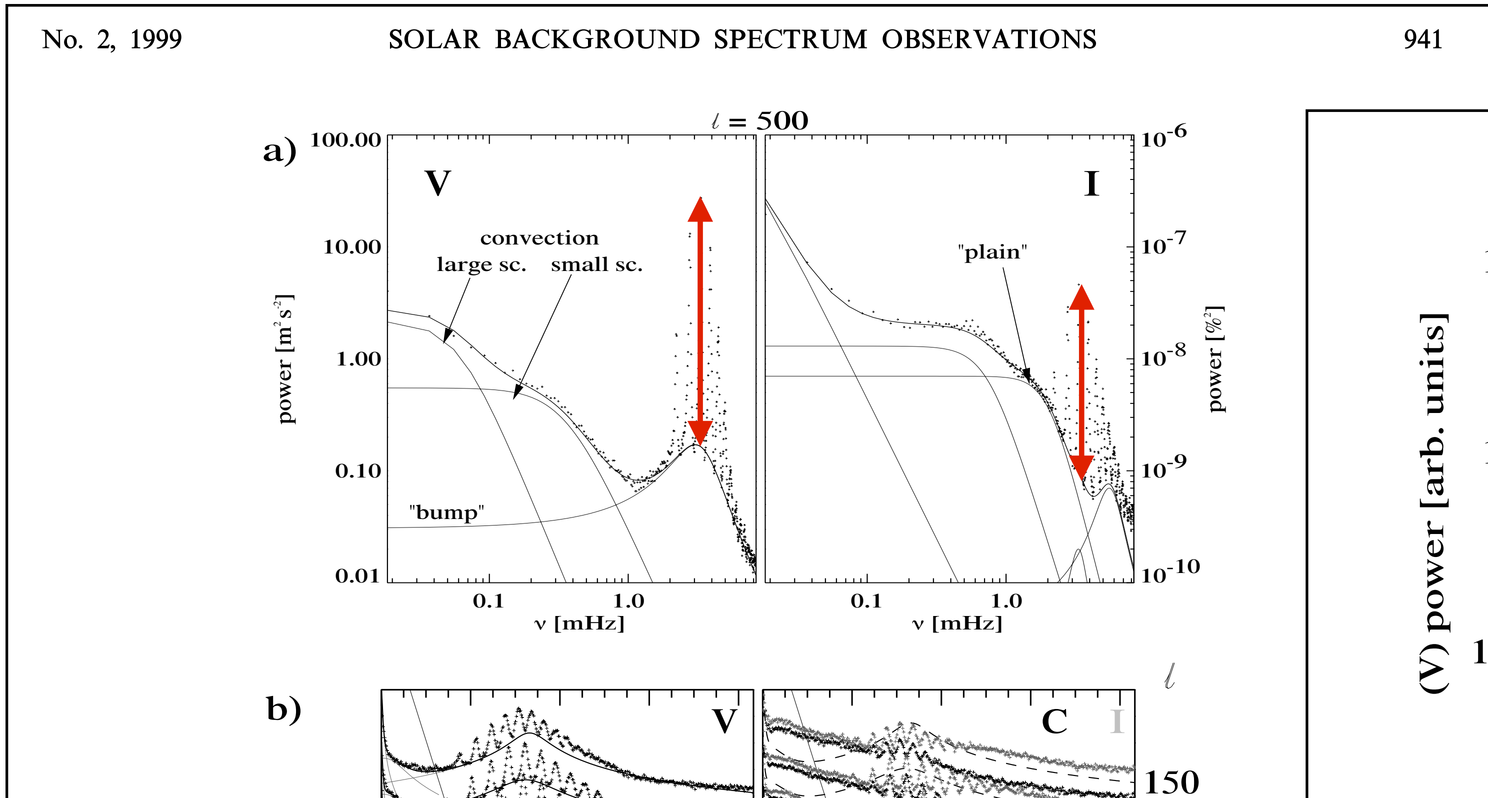
⇒new information to be unveiled...

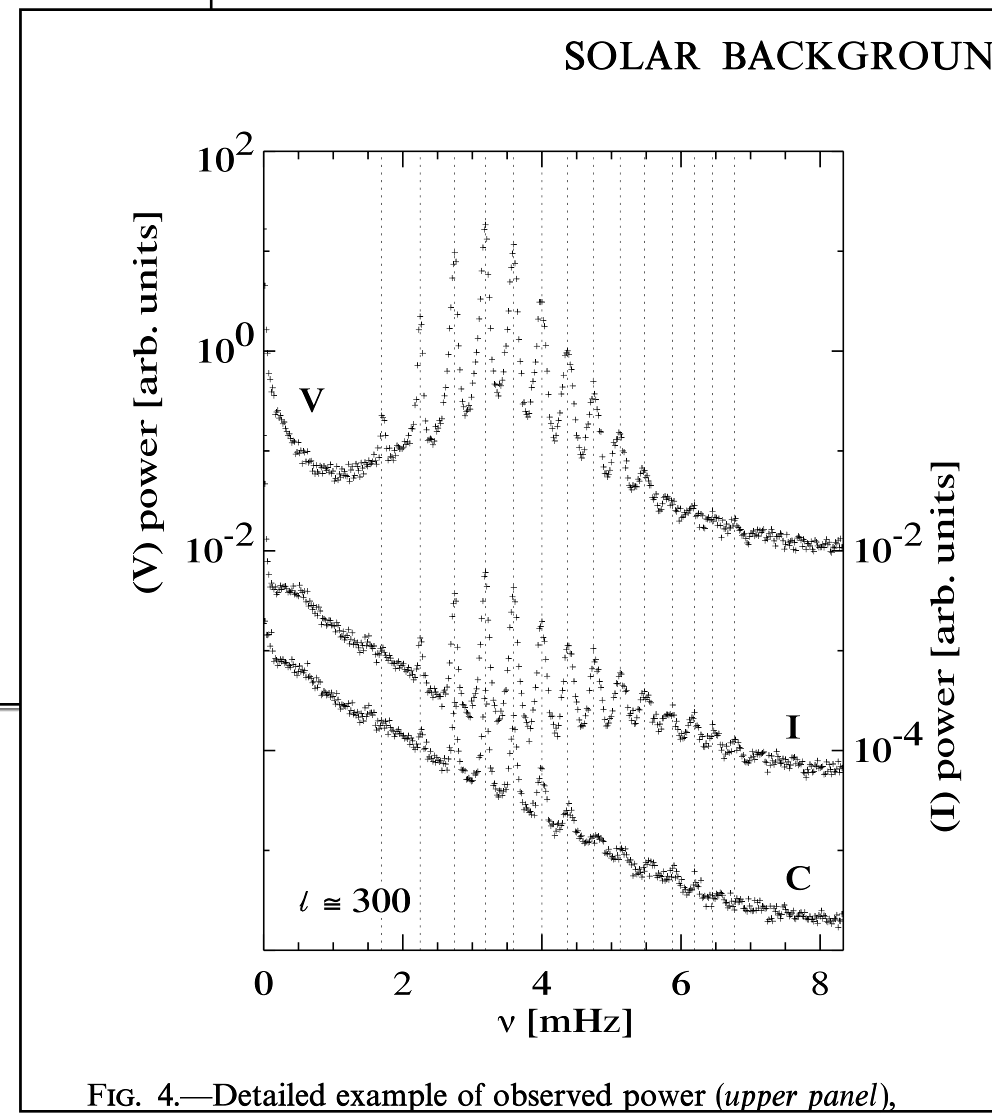


I-V phase difference



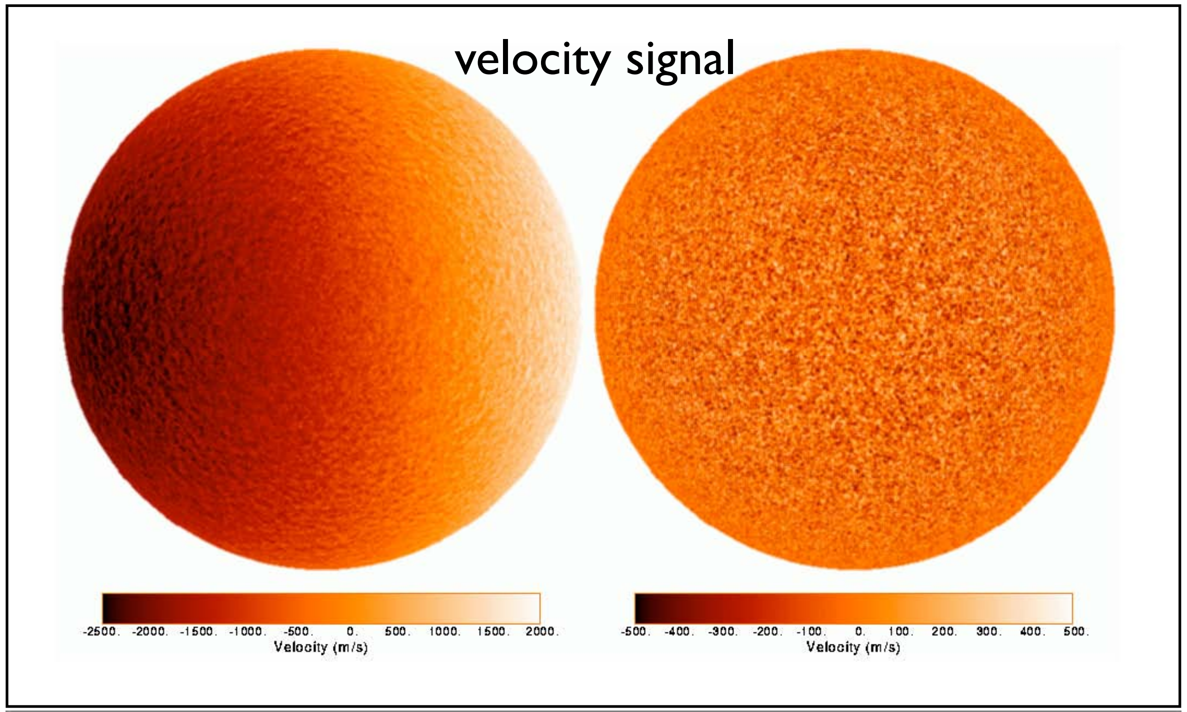
Velocity - Intensity: the Differences

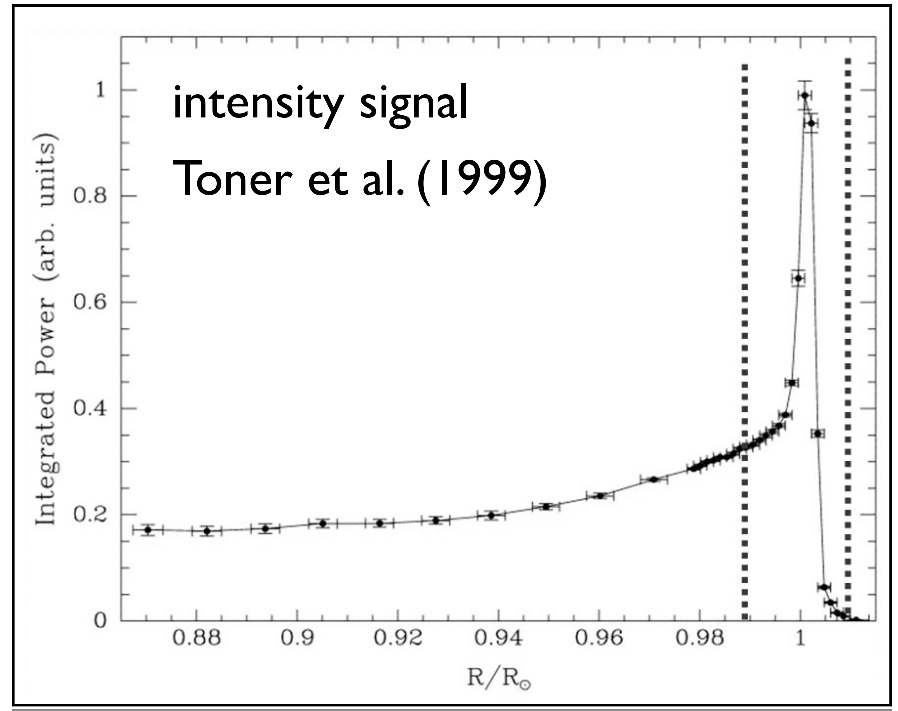




- Background
  frequency dependance, variation center-limb
- relative oscillation amplitude
  ⚠ is it only the relative amplitude?

Velocity - Intensity: the Differences II

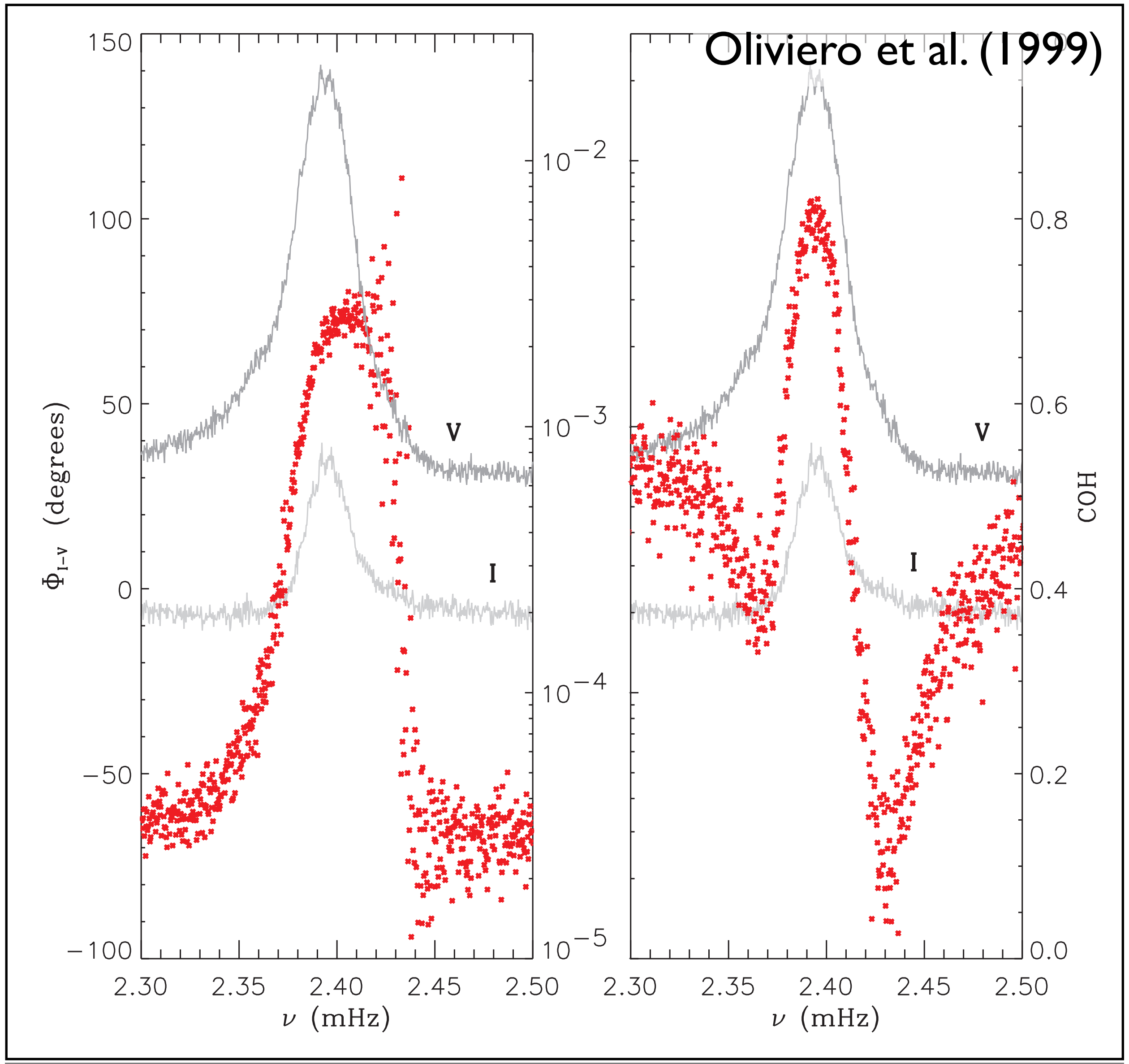




- Velocity signal
  - Background limb-dominated (supergranulation)
  - Oscillation signal center-dominated (geometrical effect)
- Intensity signal
  - Oscillation amplitude growing at limb

Velocity - Intensity: The Full Information

- Velocity has better S/N.
- Velocity and intensity have opposite asymmetry.
- I-V phase has shark-fin shape (~180°).



- Coherence has two asymmetric dips.

Mode profiles: Explanation

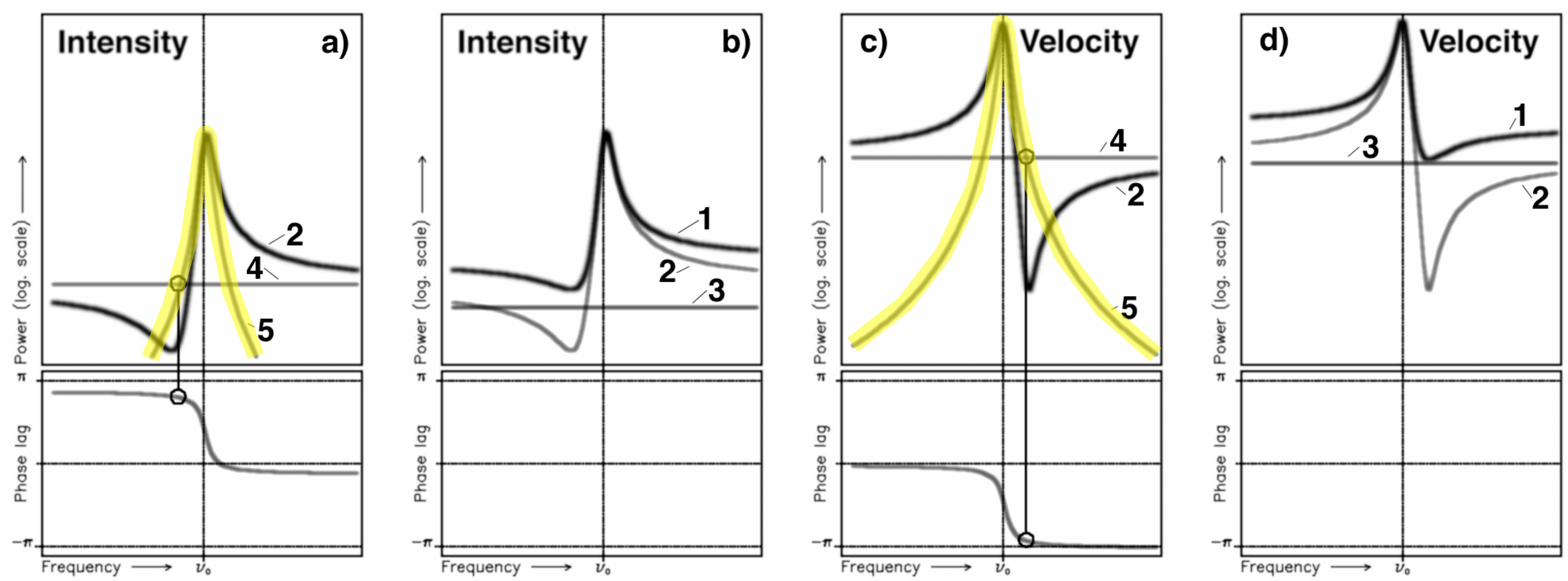


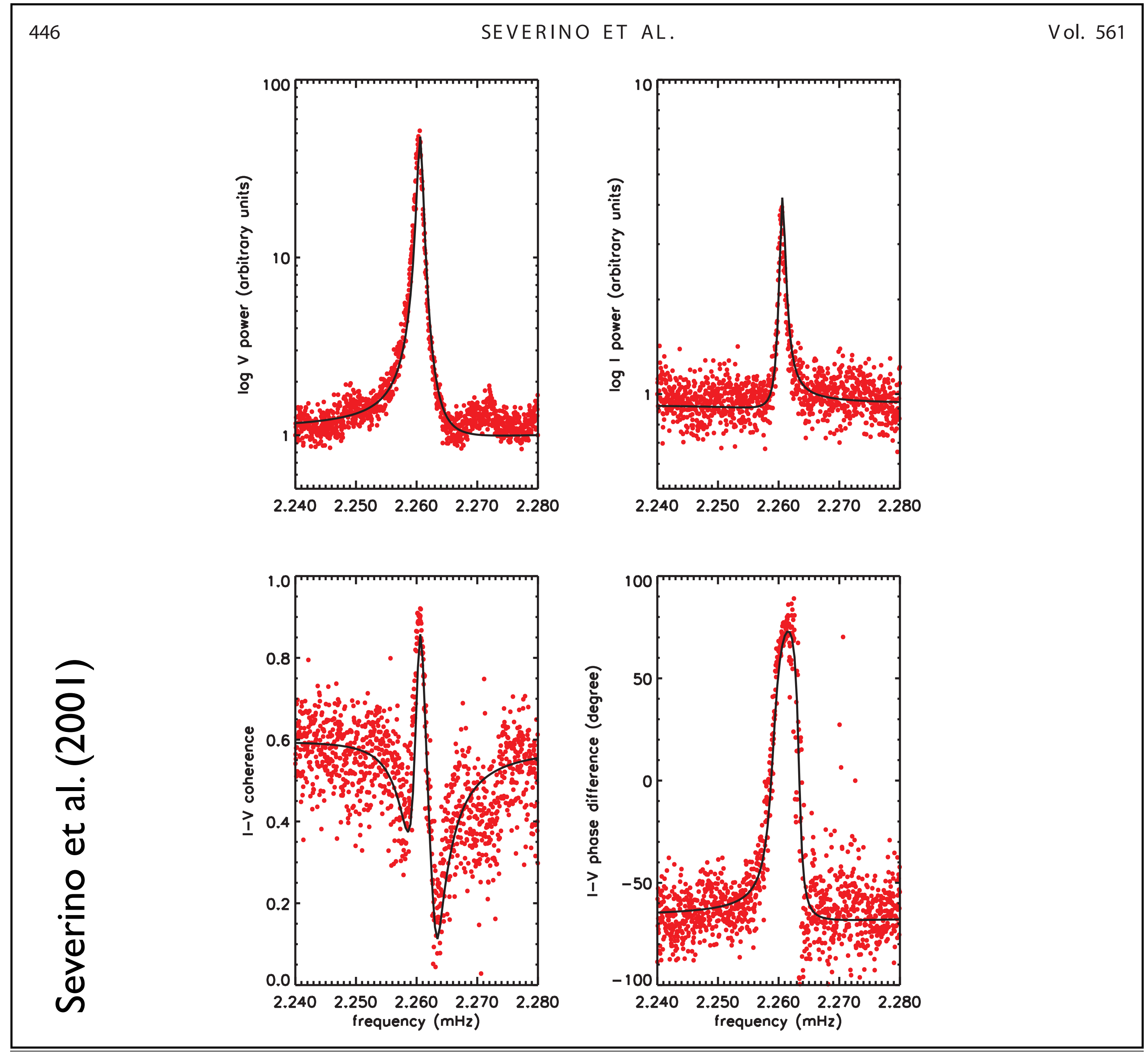
Figure 2. Components of the simple model (Magrì et al. 2000) to explain the asymmetry of p–mode profiles in power and coherence. Panel a: the p–mode signal and the correlated background in intensity (upper panel) with the phase lag between them (bottom panel); panel b: the correlated and uncoherent parts of the intensity signal; panel c: as a) for velocity; panel d: as b) for velocity. The single components are: 1) total; 2) total coherent; 3) uncoherent; 4) correlated background; 5) p–mode signals. Whereas the asymmetry in the power spectra is due to the asymmetry of the correlated *part of the signal, the two asymmetric dips in the coherence profile (see Fig. 3) are due to the variation of the fraction of the correlated part of the total signal with frequency (see text).*

- Mode
- Noise
- Correlated background
  e.g. excitation source, direct wave.
- Uncorrelated, coherent background

Mode profiles: Explanation

n=10, ℓ=17

Background amplitudes [fraction of mode]		
	velocity	intensity
Correlated	4%	19%
Uncorr.	11%	40%
Noise	9%	37%



Mode profiles: Limits

It is not (yet?) possible to distinguish different contributions to the asymmetry as they all behave identically:

- "Natural asymmetry" (e.g. excitation mechanism)
- Source depth (Kumar & Basu, 1999)
- Correlated background (e.g. direct wave)
- Opacity effects in intensity signal (Georgobiani et al., 2003)

What is the Intensity Signal?

60 *CHAPTER 4. EQUATIONS OF LINEAR STELLAR OSCILLATIONS*

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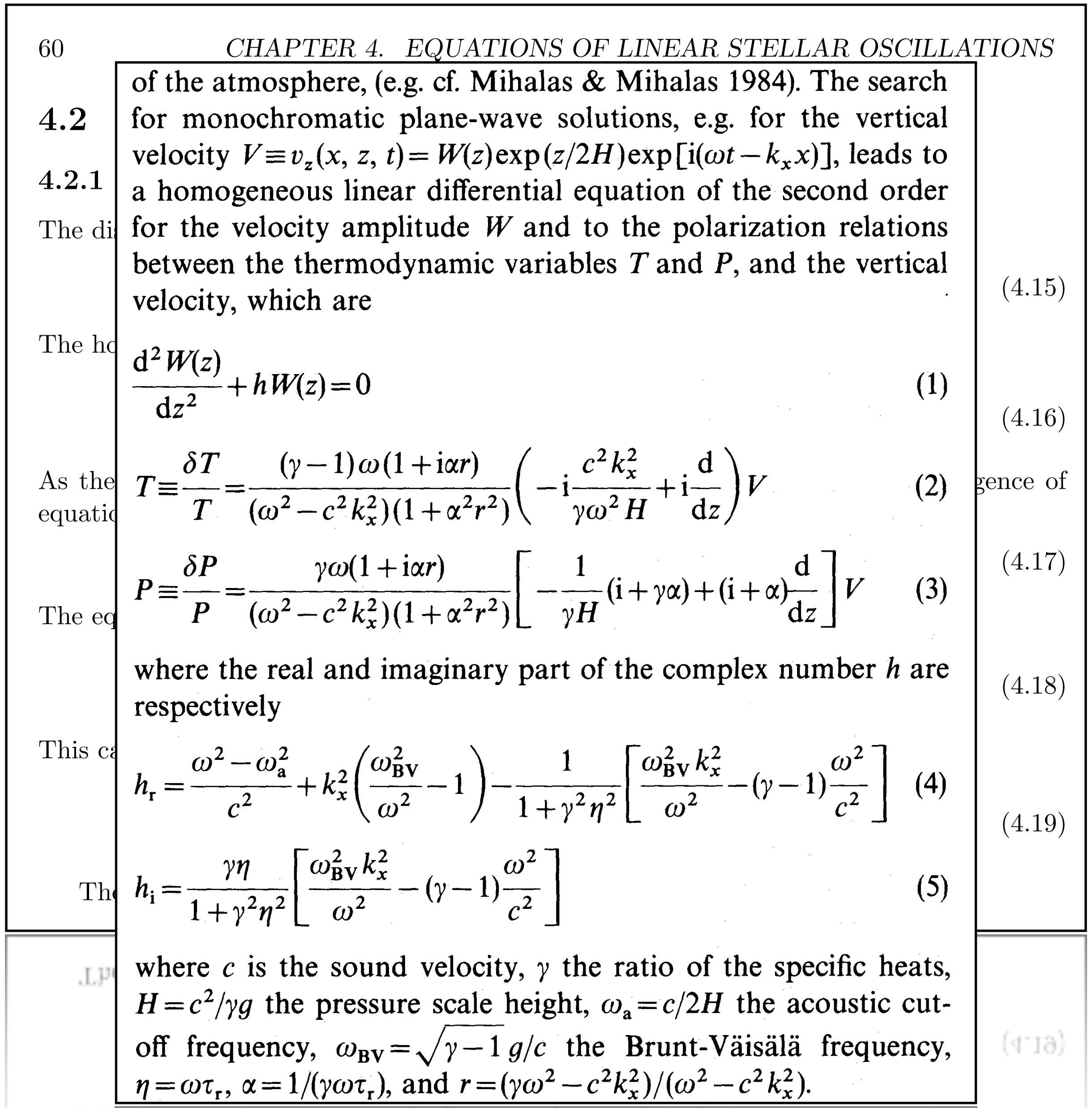
The radial component of equation (3.43) is

- "I ~ T⁴"
- Equations are written in Eulerian (or Lagrangian) space.

What is the Intensity Signal?



60 CHAPTER 4. EQUATIONS OF LINEAR STELLAR OSCILLATIONS

of the atmosphere, (e.g. cf. Mihalas & Mihalas 1984). The search for monochromatic plane-wave solutions, e.g. for the vertical velocity $V\equiv v_z(x,z,t)=W(z)\exp(z/2H)\exp[i(\omega t-k_x x)]$, leads to a homogeneous linear differential equation of the second order for the velocity amplitude W and to the polarization relations between the thermodynamic variables T and P, and the vertical velocity, which are

$$\frac{d^2W(z)}{dz^2}+hW(z)=0 \quad (1)$$

$$T\equiv\frac{\delta T}{T}=\frac{(\gamma-1)\omega(1+i\alpha r)}{(\omega^2-c^2k_x^2)(1+\alpha^2r^2)}\left(-i\frac{c^2k_x^2}{\gamma\omega^2H}+i\frac{d}{dz}\right)V \quad (2)$$

$$P\equiv\frac{\delta P}{P}=\frac{\gamma\omega(1+i\alpha r)}{(\omega^2-c^2k_x^2)(1+\alpha^2r^2)}\left[-\frac{1}{\gamma H}(i+\gamma\alpha)+(i+\alpha)\frac{d}{dz}\right]V \quad (3)$$

where the real and imaginary part of the complex number h are respectively

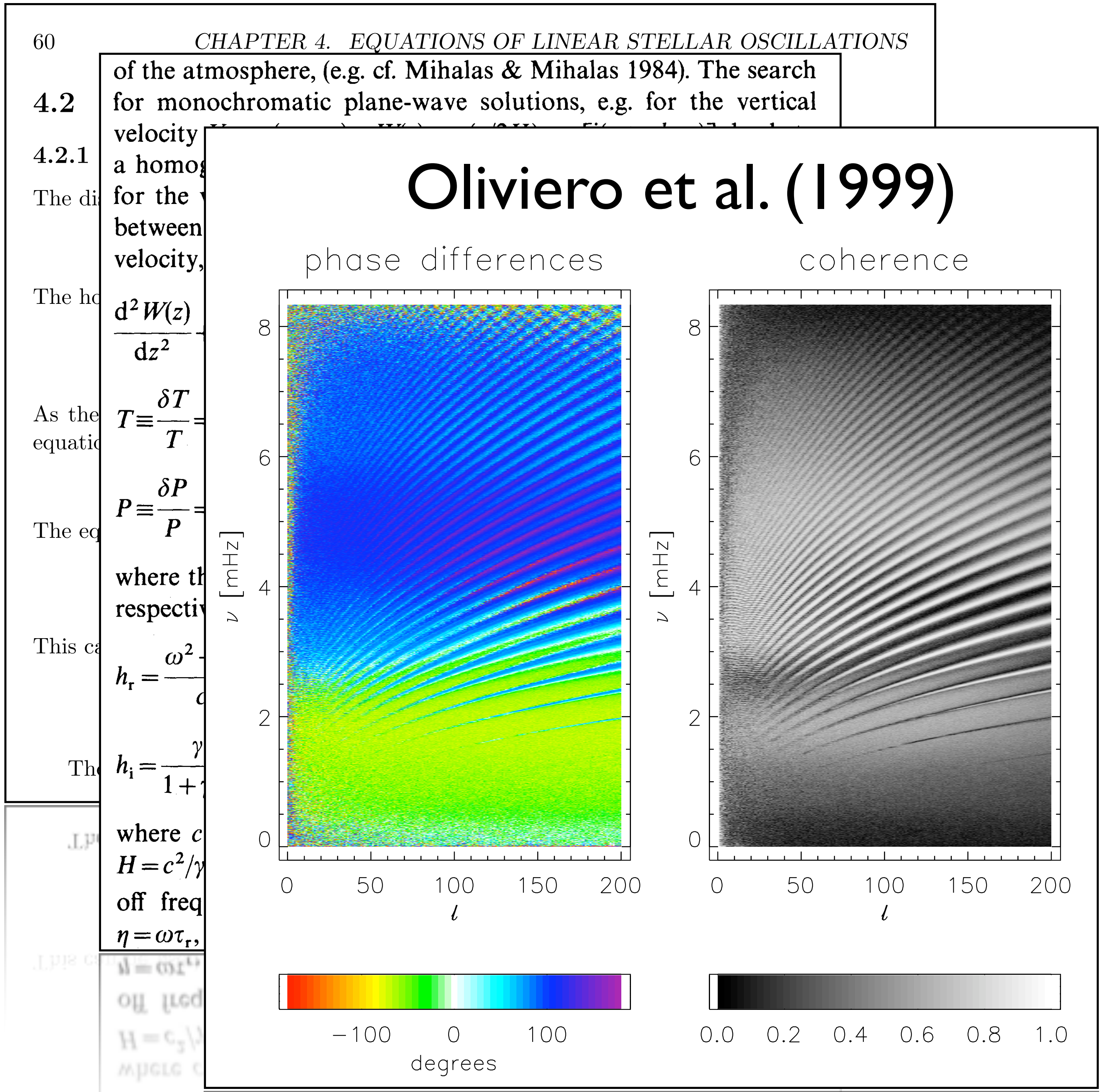
$$h_r=\frac{\omega^2-\omega_a^2}{c^2}+k_x^2\left(\frac{\omega_{BV}^2}{\omega^2}-1\right)-\frac{1}{1+\gamma^2\eta^2}\left[\frac{\omega_{BV}^2k_x^2}{\omega^2}-(\gamma-1)\frac{\omega^2}{c^2}\right] \quad (4)$$

$$h_i=\frac{\gamma\eta}{1+\gamma^2\eta^2}\left[\frac{\omega_{BV}^2k_x^2}{\omega^2}-(\gamma-1)\frac{\omega^2}{c^2}\right] \quad (5)$$

where c is the sound velocity, γ the ratio of the specific heats, $H=c^2/\gamma g$ the pressure scale height, $\omega_a=c/2H$ the acoustic cut-off frequency, $\omega_{BV}=\sqrt{\gamma-1}\,g/c$ the Brunt-Väisälä frequency, $\eta=\omega\tau_r$, $\alpha=1/(\gamma\omega\tau_r)$, and $r=(\gamma\omega^2-c^2k_x^2)/(\omega^2-c^2k_x^2)$.

- "I ~ T⁴"
- Equations are written in Eulerian (or Lagrangian) space.
- Rather simple polarization relations in an isothermal atmosphere.

What is the Intensity Signal?



- "I ~ T^4"
- Equations are written in Eulerian (or Lagrangian) space.
- Rather simple polarization relations in an isothermal atmosphere.
- Complex structure of phase spectra.

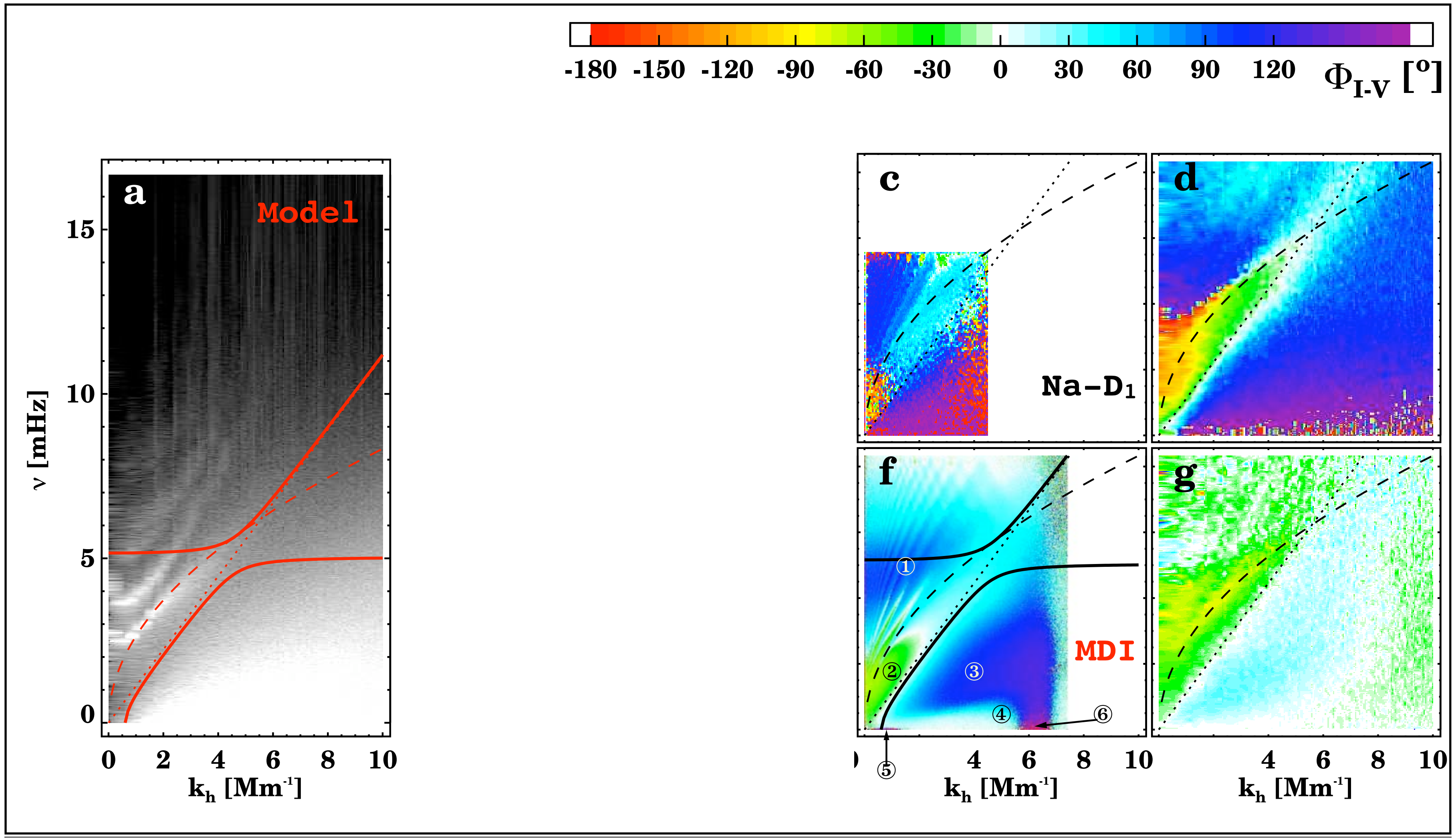
Intensity in Realistic Atmosphere

$$\tilde{v} = 1\,\mathrm{m\,s^{-1}},\ \nu = 3\,\mathrm{mHz}$$
$$\tilde{\xi}_\mathrm{r} = 50\,\mathrm{m}$$

$$T'\Big|_{z=\mathrm{const.}} \propto \underbrace{T'_\mathrm{wave}}_{\sim 10^{-3}\mathrm{K}} + \underbrace{\frac{\partial T}{\partial z}\xi_\mathrm{r}}_{\sim 1\,\mathrm{K}}$$

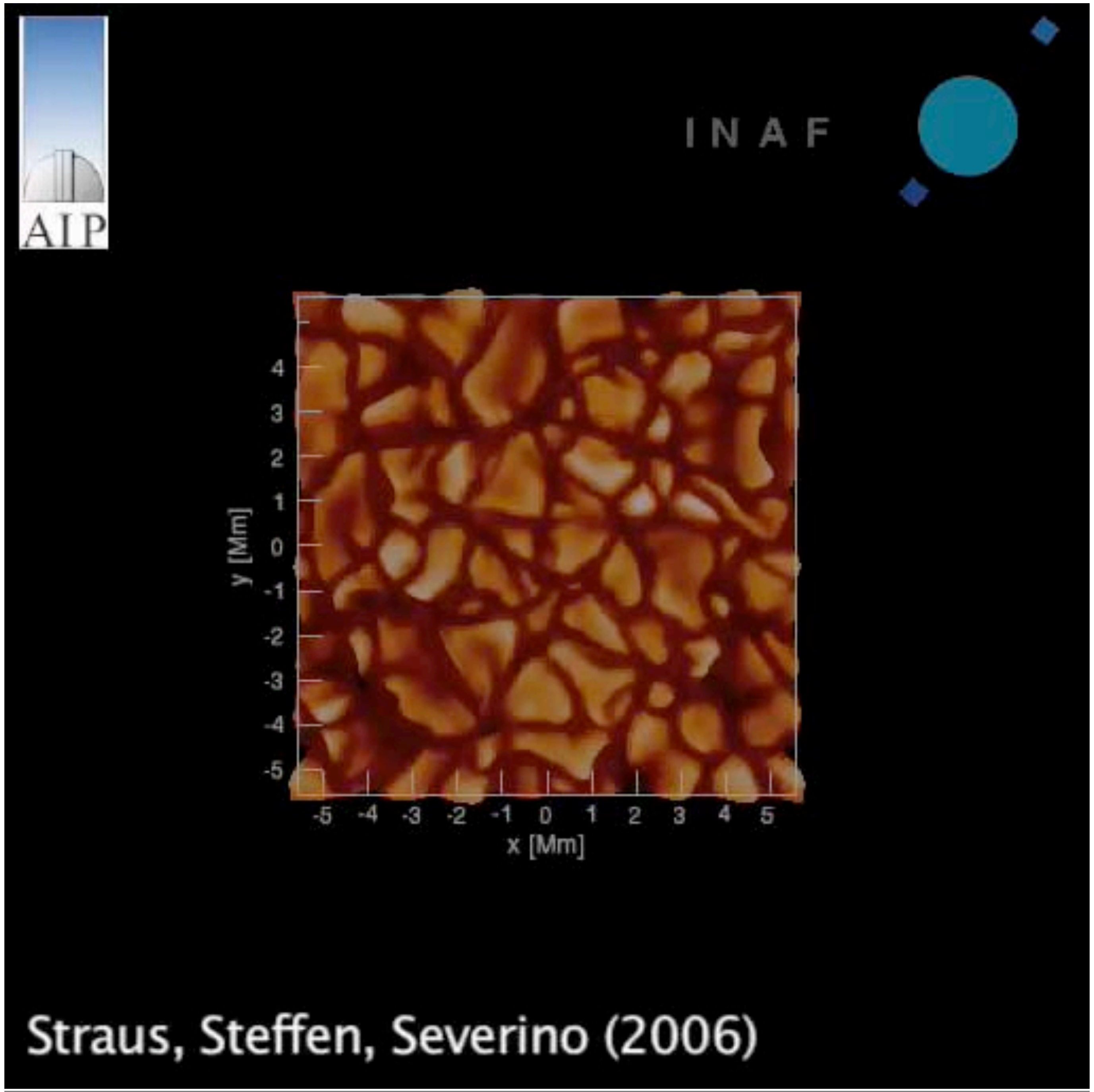
- Gradient-effect...
- ...dominates in the z-frame (=Eulerian frame) due to huge temperature gradient.
- In the z-frame we “observe” the displacement, not the wave fluctuation.

The z-frame



-180 -150 -120 -90 -60 -30 0 30 60 90 120
Φ_{I-V} [°]
a
Model
ν [mHz]
15
10
5
0
0 2 4 6 8 10
k_h [Mm^{-1}]
c
Na–D1
d
f
MDI
g
4 6 8 10
k_h [Mm^{-1}]
4 6 8 10
k_h [Mm^{-1}]

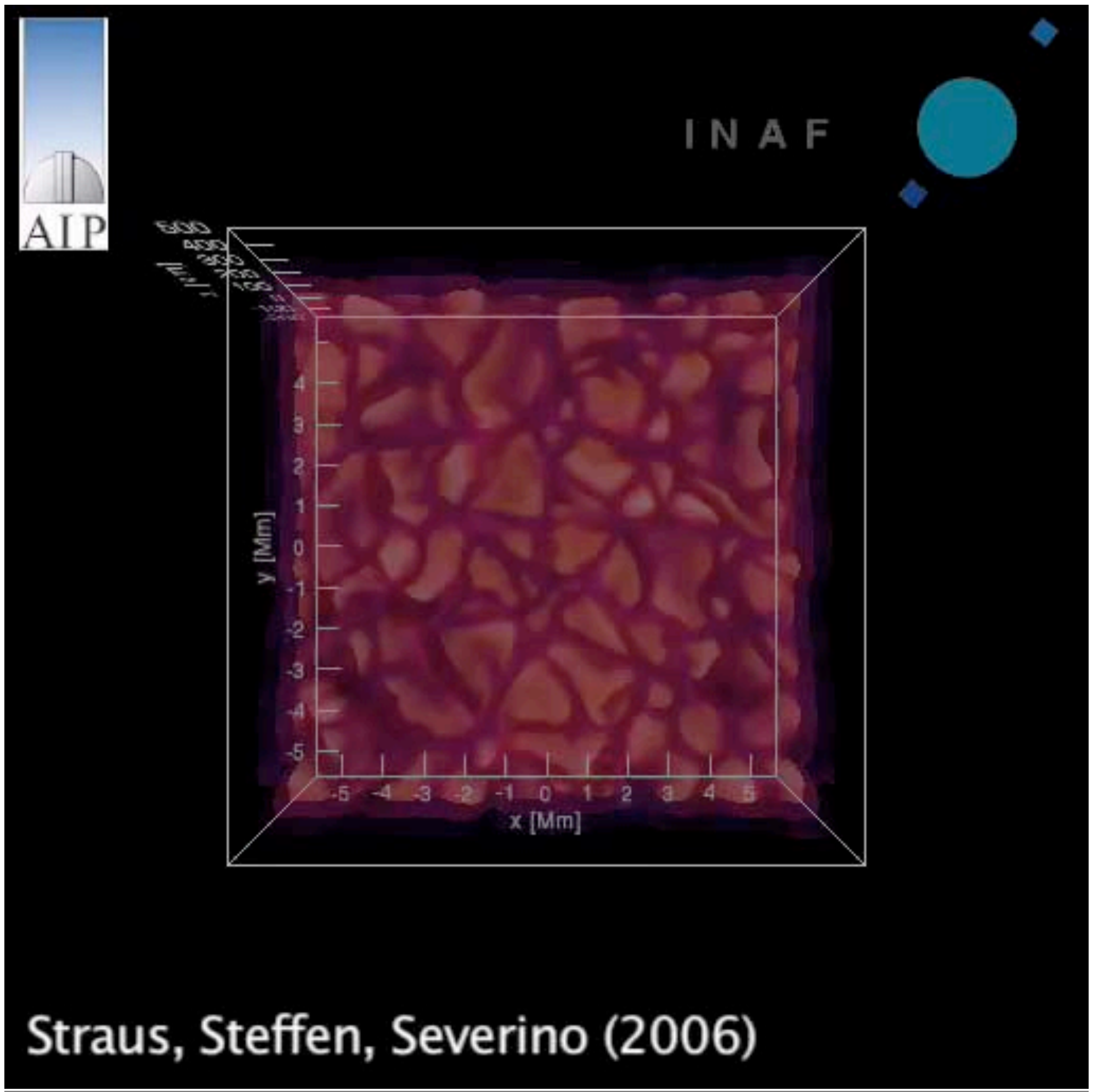
The intensity signal



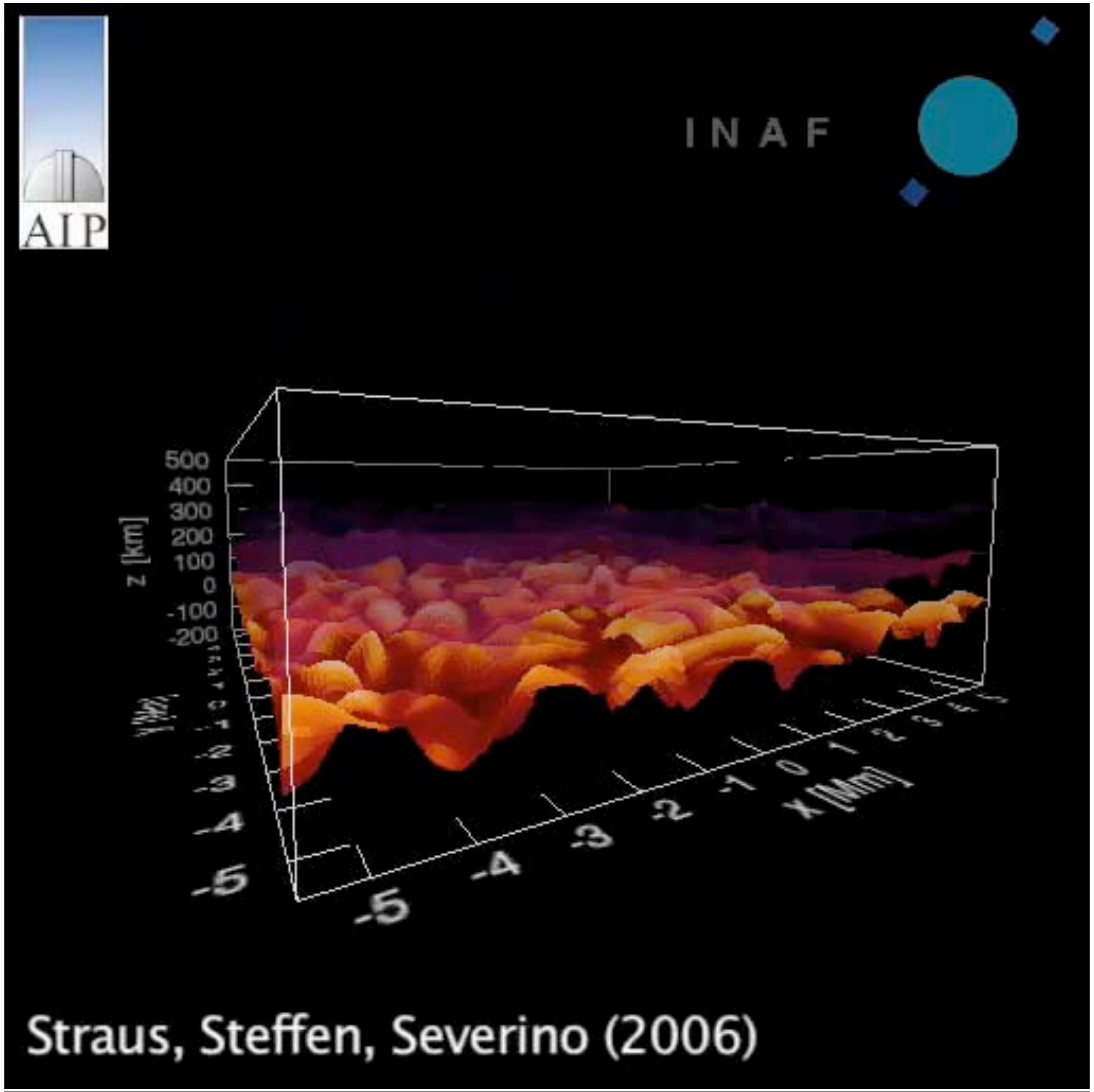
$$\Delta I \propto T'\Big|_{\tau=1}$$

Eddington-Barbier approximation

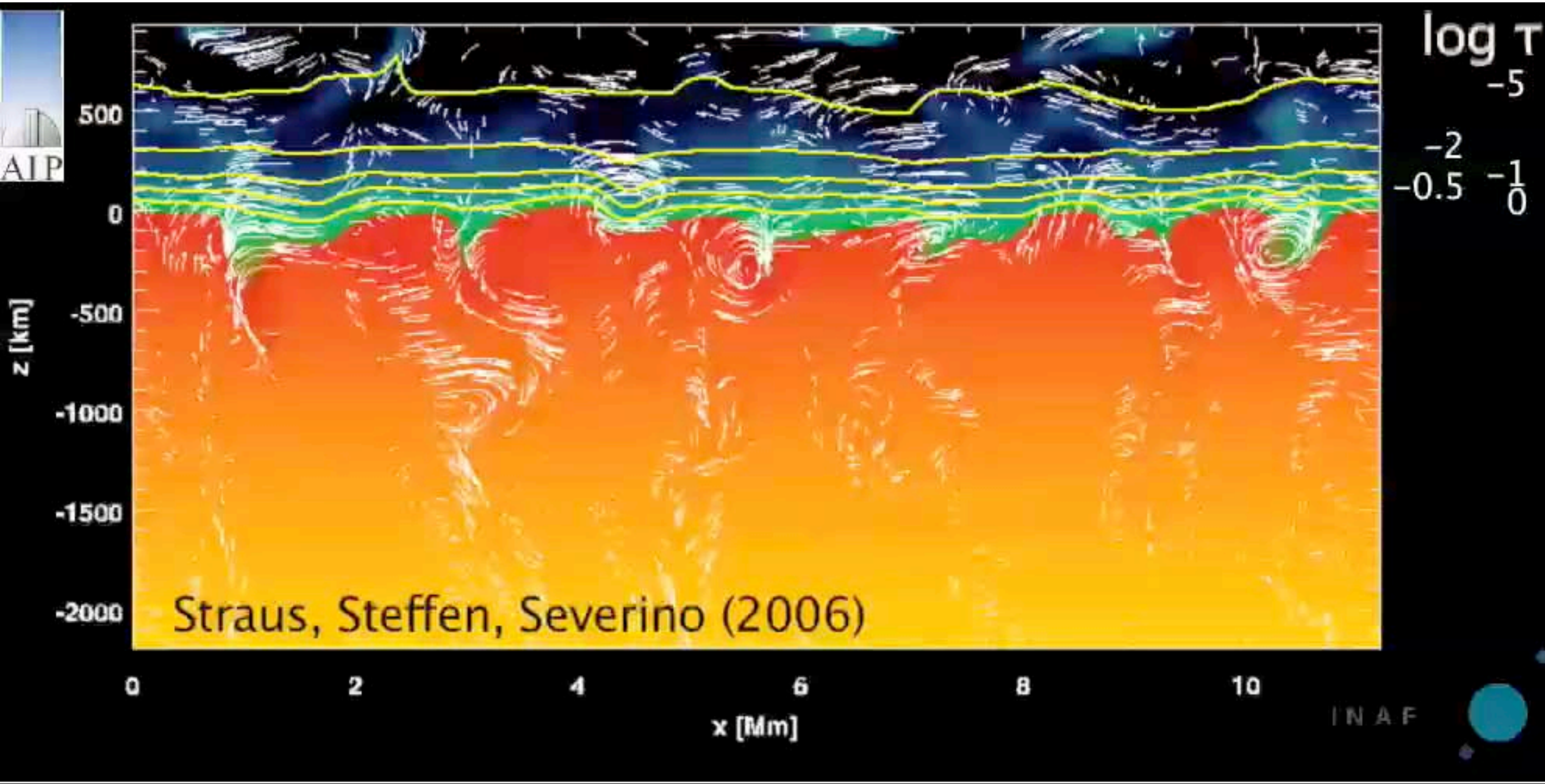
The τ-frame



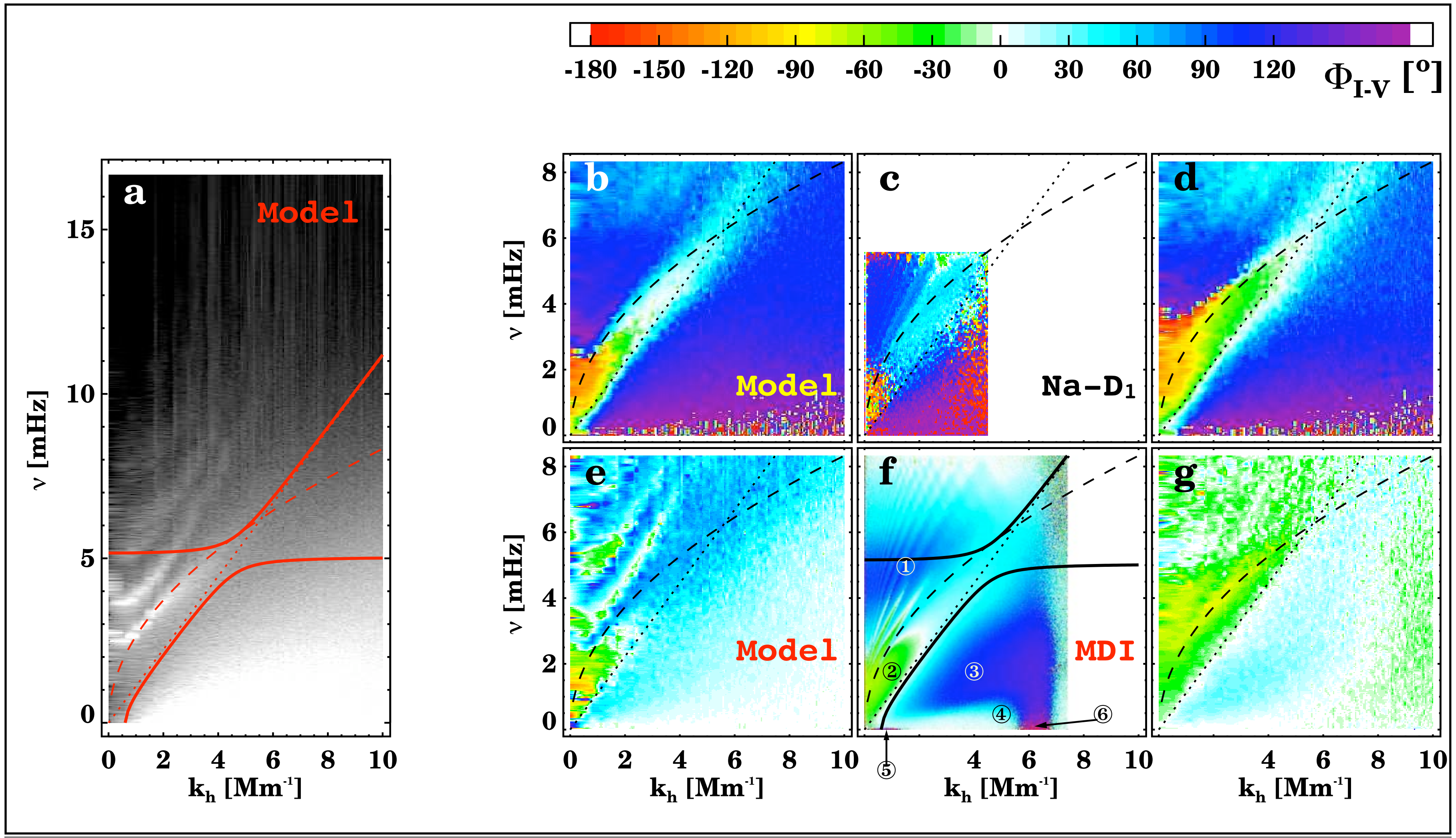
The τ-frame



The τ-frame



The τ-frame



-180 -150 -120 -90 -60 -30 0 30 60 90 120 $\Phi_{I\text{-}V}$ [°]
a
Model
b
Model
c
Na-D1
d
e
Model
f
MDI
g
ν [mHz]
k_h [Mm^{-1}]

The τ-frame

In the simple case that $\frac{\Delta T}{T}$ and $\frac{\Delta P}{P}$ are constant with h, we can derive an explicit, approximated expression for the observed height fluctuation:

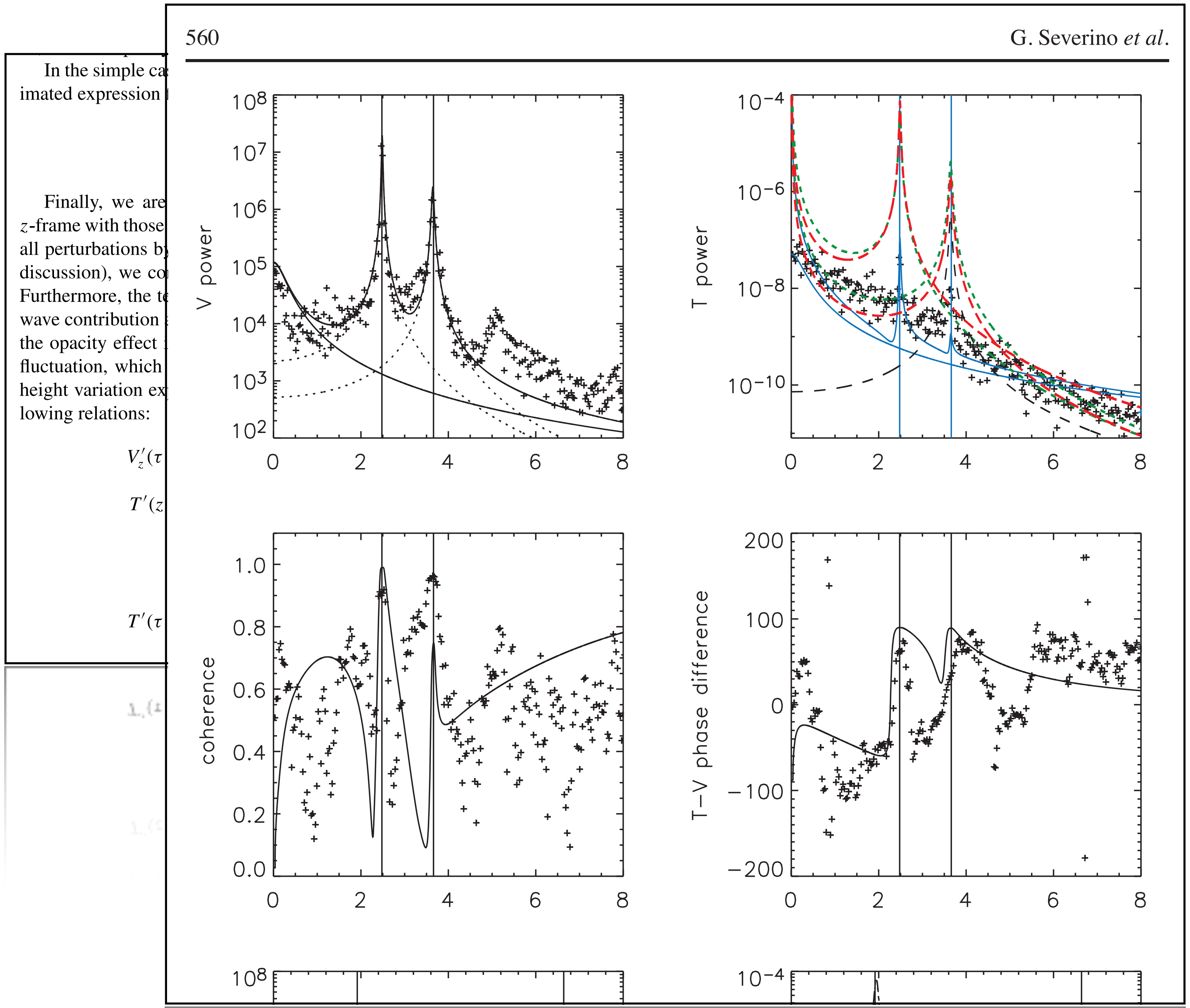
$$\Delta h \approx \left(5.76\frac{\Delta T}{T} + 1.74\frac{\Delta P}{P}\right)\frac{1}{k_1(h_1)}. \tag{25}$$

Finally, we are in a position to write the expressions linking the fluctuations in the z-frame with those in the τ-frame. Since we neglected the spatial phase change introduced in all perturbations by the height fluctuation occurring in the τ-frame (see Equation (16) and discussion), we conclude that the vertical velocity fluctuation is the same in both frames. Furthermore, the temperature fluctuation in the z-frame is due to the sum of an isothermal wave contribution and the T-gradient effect (Equations (8) and (9)), whereas in the τ-frame the opacity effect is also at work, producing an additional contribution to the temperature fluctuation, which is given to first order by the atmospheric temperature gradient times a height variation expressed, for example, by Equation (25). Therefore we can write the following relations:

$$\begin{aligned}
V_z'(\tau = \text{constant}) &= V_z'(z = \text{constant}),\\
T'(z = \text{constant}) &= T_{\text{iso}}'(z = \text{constant}) - \delta z\frac{\mathrm{d}T_0}{\mathrm{d}z}\\
&= T_{\text{iso}}'(z = \text{constant}) - \frac{V_z'(z = \text{constant})}{\iota\omega}\frac{\mathrm{d}T_0}{\mathrm{d}z},\\
T'(\tau = \text{constant}) &= T_{\text{iso}}'(z = \text{constant}) - (\delta z - \Delta h)\frac{\mathrm{d}T_0}{\mathrm{d}z}.
\end{aligned} \tag{26}$$

- The opacity effect reduces the gradient effect.
- The opacity effect can reverse the asymmetry.
- The simultaneous study of z- and τ-frame in the simulation can help to distinguish various asymmetry contributions.

The τ-frame



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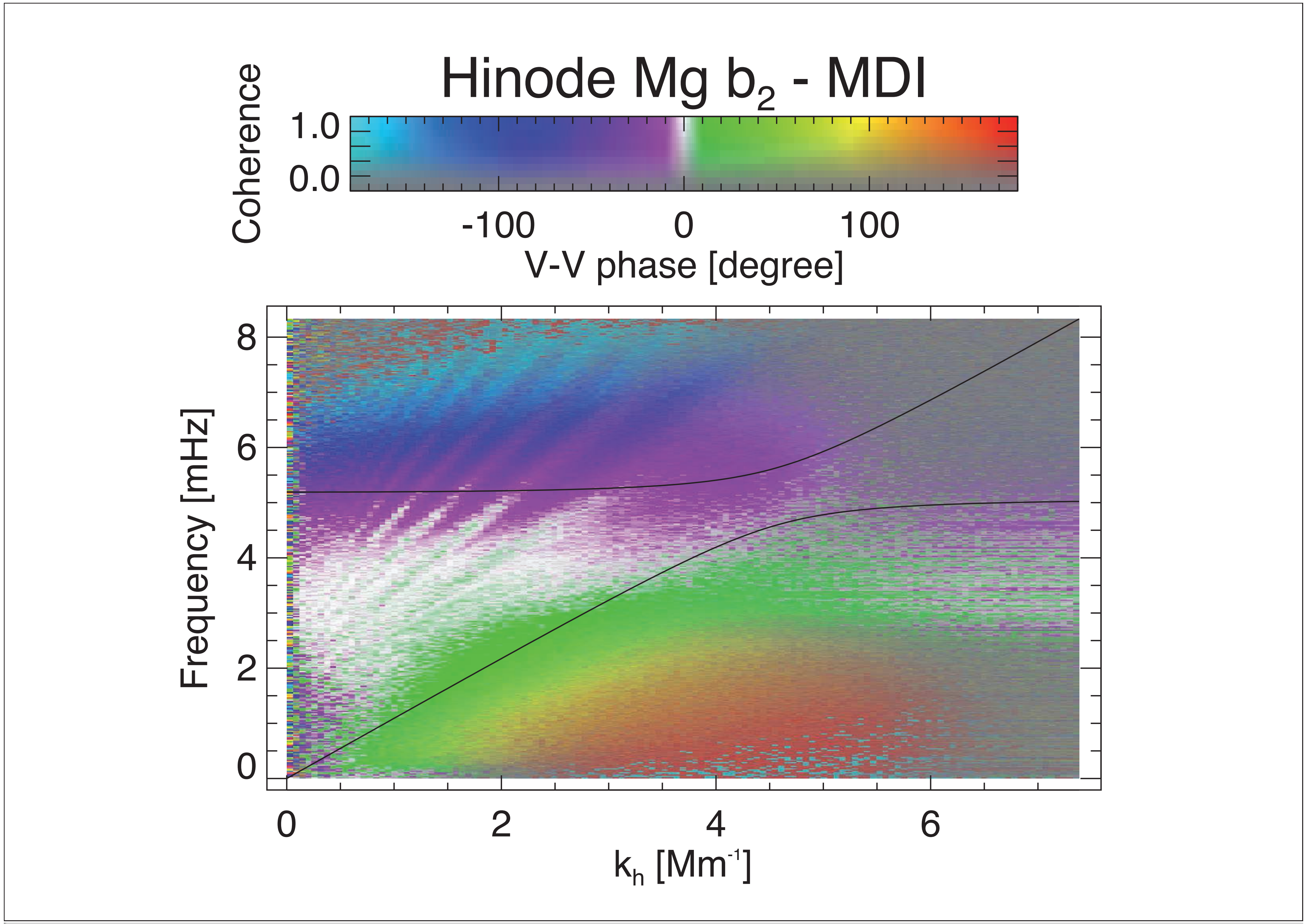
Gradient Effect and Opacity Effect at the Limb

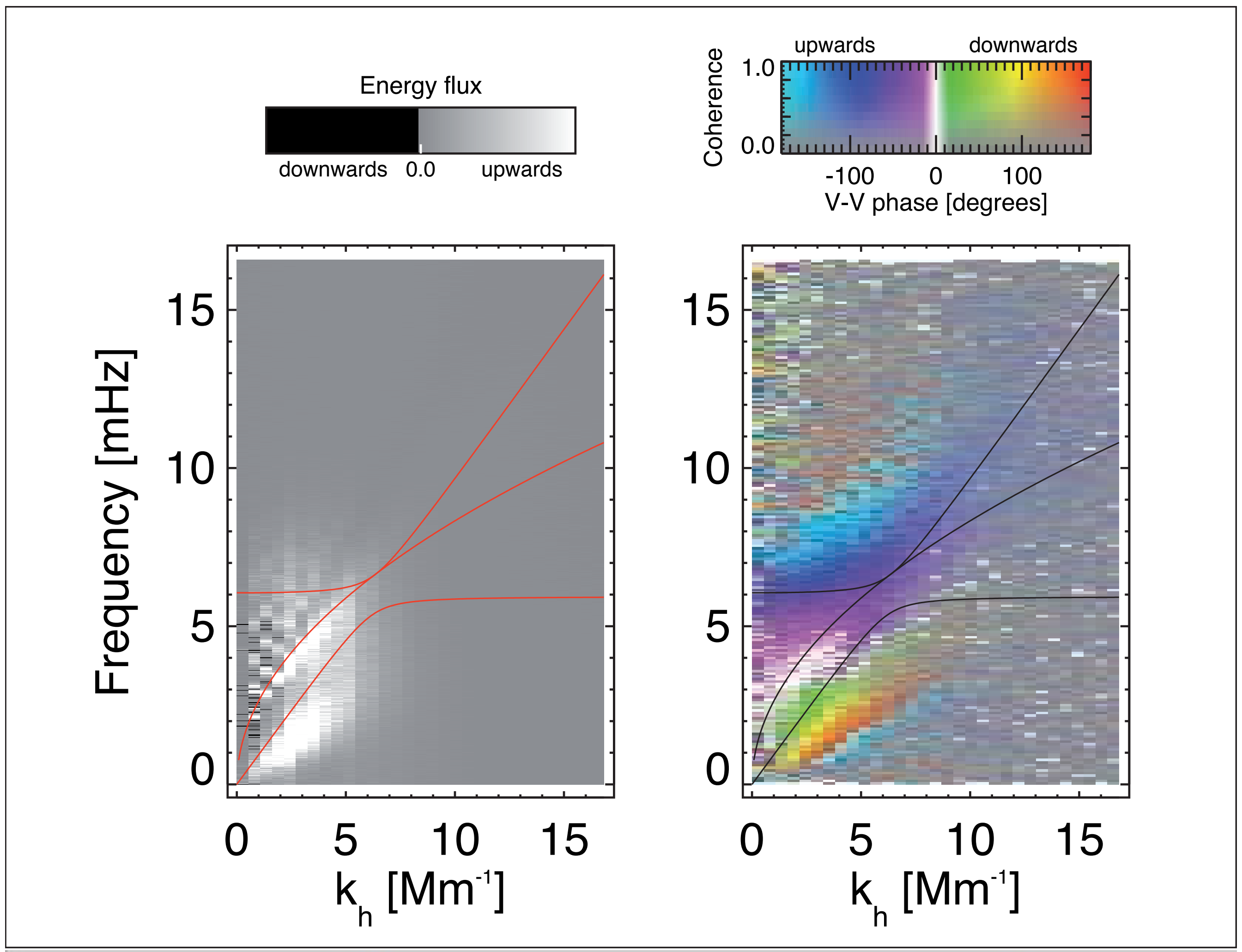
- The opacity effect is largely reduced at the limb as we observe the vertical oscillation from the side.
- The gradient effect is again dominating and is caused by an unresolvable displacement perpendicular to the limb.

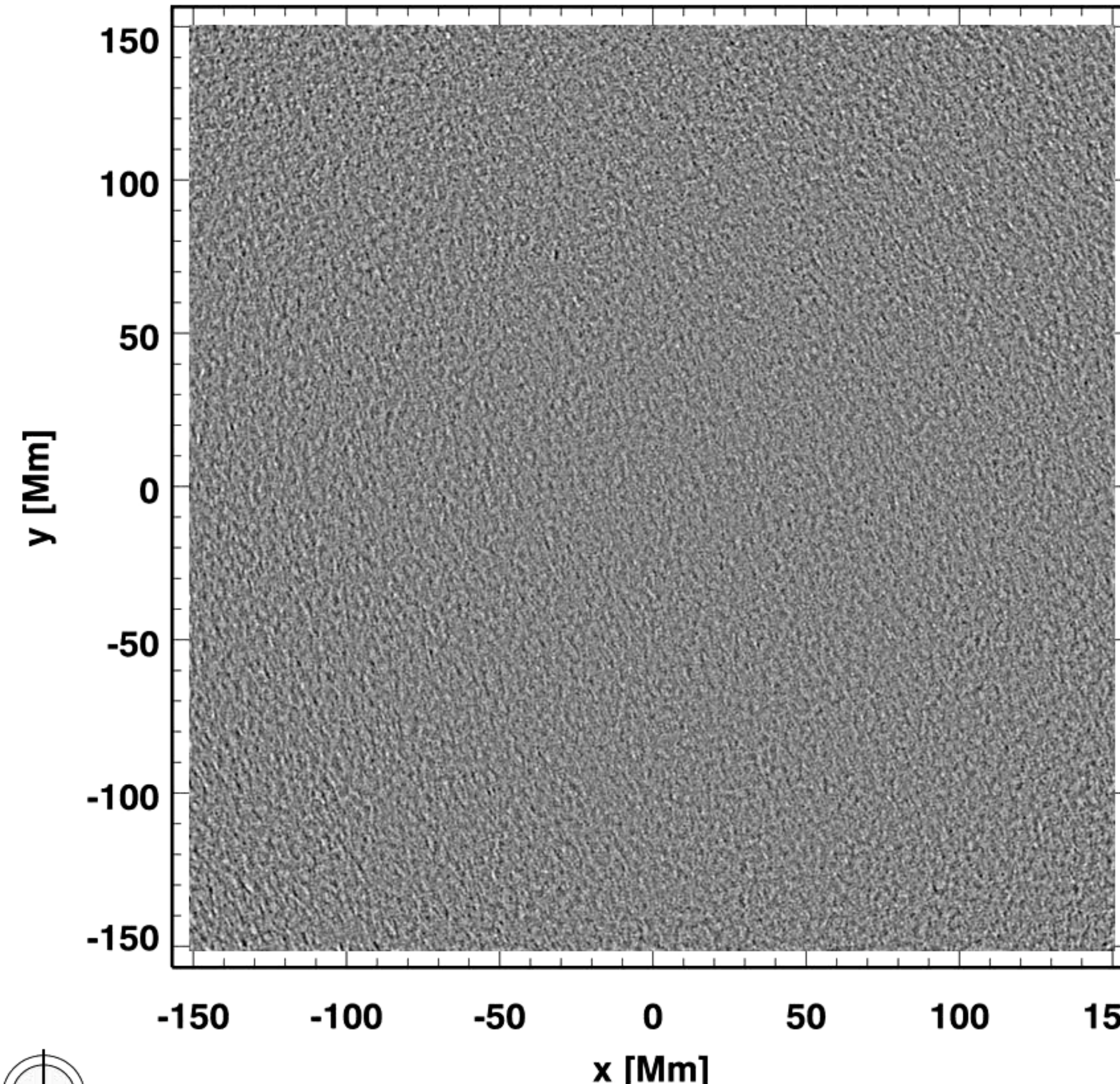
$\Rightarrow$ Oscillation power grows towards the limb.

The V-V phase difference

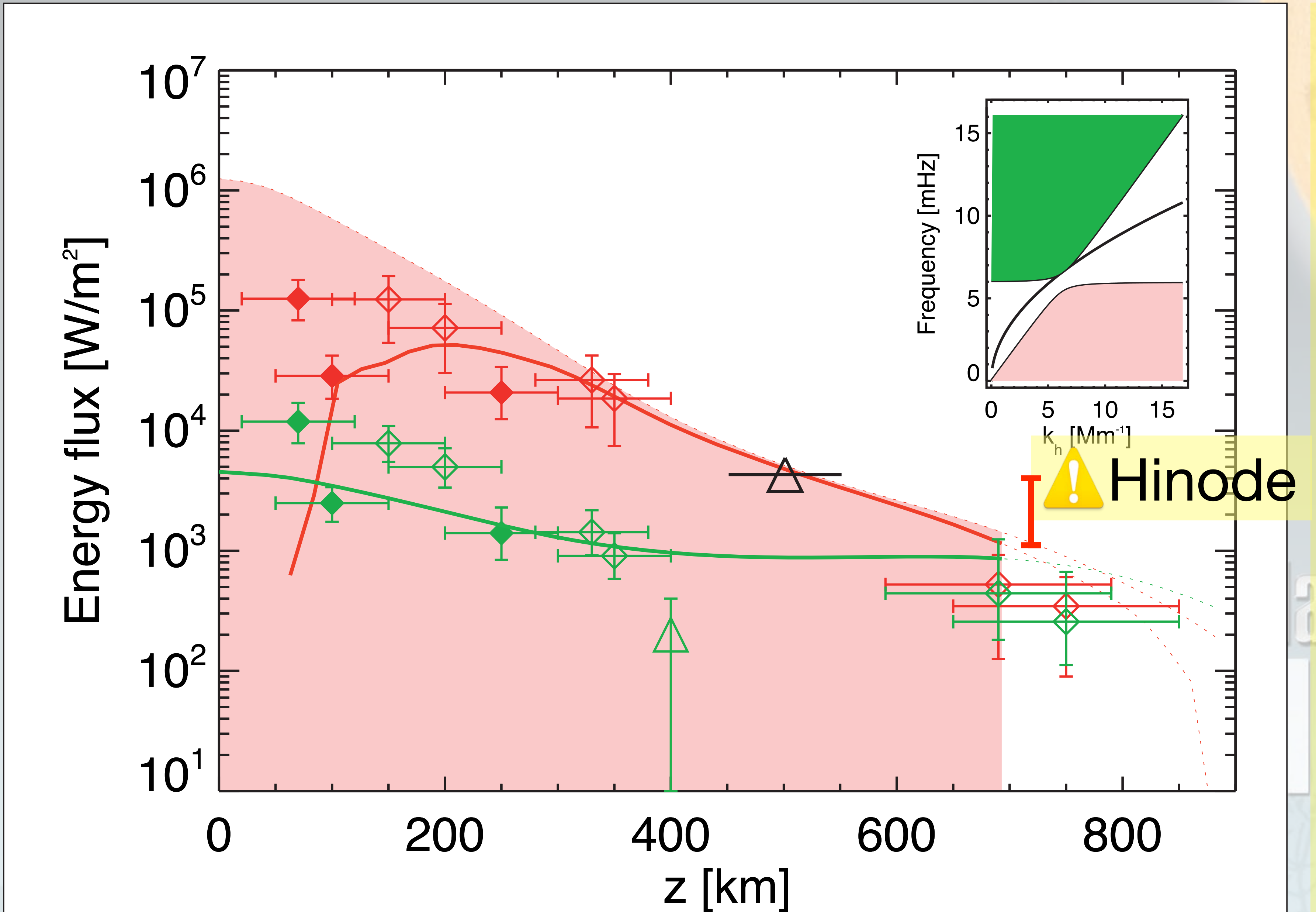


Atmospheric Gravity Waves





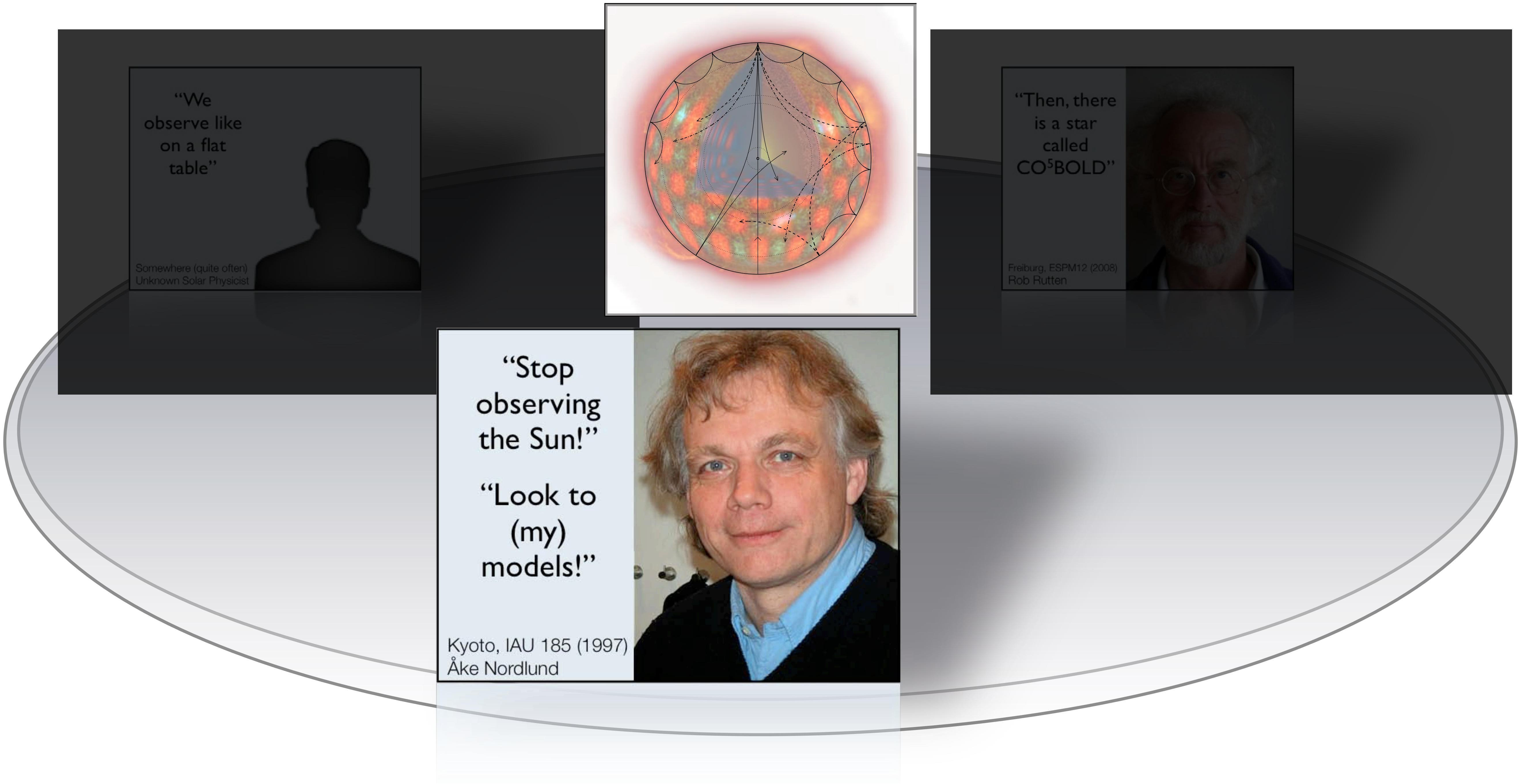
The energy balance



Straus et al. (2008) ApJ 681, L125

- Acoustic flux is *insufficient*.
- Internal gravity waves are the *only* important mechanism to the energy balance of the quiet, middle photosphere.
- At 300 km and above they carry completely the low-frequency energy-flux, i.e. no convection anymore.
- They carry 20 and more times the acoustic flux.

Future Plans

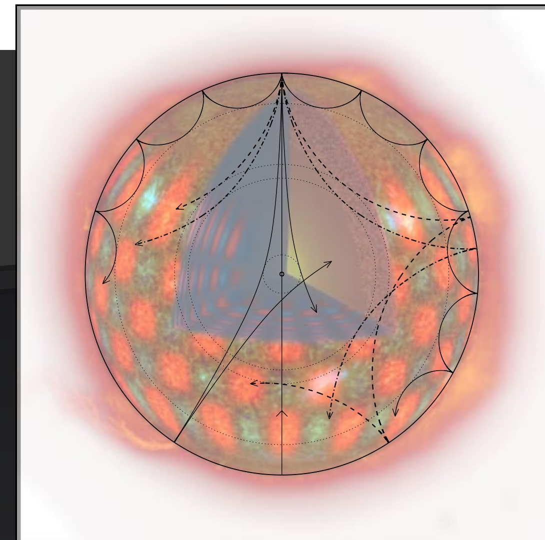


Future Plans

"Stop observing the Sun!"

"Look to (my) models!"

Kyoto, IAU 185 (1997)
Åke Nordlund



"We observe like on a flat table"

Somewhere (quite often)
Unknown Solar Physicist

"Then, there is a star called CO^{5}BOLD"

Freiburg, ESPM12 (2008)
Rob Rutten

Future Plans