FRAGMENT EJECTION VELOCITIES AND THE COLLISIONAL EVOLUTION OF ASTEROIDS

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Abstract. We have applied the algorithm developed by Petit and Farinella (1993) to model the outcomes of impacts between asteroids of different sizes, to show that a crucial feature of these models is the assumed relationship between velocity and mass of fragments ejected after a shattering impact. Not only how the mean velocity depends upon mass is important to determine the extent of fragment reaccumulation, but also the distribution of velocities about the mean values. The available experimental evidence on this issue is still sparse, and does not constrain the collisional models well enough to allow us to make reliable predictions on the outcomes of impacts between bodies of size much larger than the laboratory targets. As a consequence, when the collisional outcome models are used as an input for simulations of the asteroid collisional history since the origin of the solar system, the results show a strong sensitivity to the assumed velocity vs. mass relationship. This sensitivity is stronger in the diameter range (a few tens to a few hundreds of km) where the self–gravitational reaccumulation of fragments is most effective, but may extend also to much smaller sizes.

1. Introduction

Developing realistic models of the outcomes of asteroidal impacts is a basic prerequisite to understanding the collisional history of asteroid sizes and rotations since the end of the accretion phase 4.5×10^9 yr ago (Davis et al. 1989). Such impacts occur at velocities of several km/s, and most current knowledge on their outcomes is based on extrapolations of small–scale laboratory experiments (Davis et al. 1986; Fujiwara et al. 1989). Recently, it has become possible to measure at the same time the size and the ejection velocity for large numbers of fragments generated in the impact or explosive break–up of a solid target (Davis and Ryan 1990; Nakamura and Fujiwara 1991; Nakamura et al. 1992; Giblin et al. 1994), and a correlation beween these two quantities has been detected. For large–scale cratering events, a similar correlation had been found by Vickery (1986, 1987), through an analysis of the distribution of secondary craters surrounding some large impact craters on Mercury, the Moon and Mars. This is potentially an important finding, because fragment ejection velocities determine the extent to which material ejected from the target can be reaccumulated by self–gravity, so as to form a regolith layer (for cratering events) or a so–called *pile of rubble* (in the case of break–up; see Davis et al. 1979, Farinella et al. 1982).

Petit and Farinella (1993) have presented a self–consistent numerical algorithm aimed at predicting the collisional outcomes, based on a set of model input parameters which can be estimated from the available experimental evidence, and including the possible gravitational reaccumulation of ejected fragments whose velocity is less than a suitably defined escape velocity. The possible correlation between fragment ejection velocity and mass has been taken into account in different ways, including a probabilistic one. In that paper the effectiveness of the gravitational reaccumulation process has been analyzed for different choices of the collisional parameters and different assumptions on the relationship between fragment speed and mass. Both the transition size beyond which a large fraction of the fragment mass is likely to be reaccumulated and the collision energy needed to disperse "to infinity" most of the fragments were found to be sensitive functions of the assumed fragment velocity vs. mass relationship.

The purpose of this paper is that of assessing how much this sensitivity may affect the final size distributions of asteroids obtained from collisional evolution codes such as those described in Greenberg et al. (1978), Davis et al. (1979, 1985, 1989, 1994), Farinella et al. (1992) and Campo Bagatin et al. (1994). These codes simulate the collisional history of asteroids by evolving in time the numbers of objects residing in a set of discrete size bins. At any given time step, the expected number of collisions between bodies belonging to any pair of bins is computed, and in each case the collisional outcome algorithm provides the size distribution of the objects resulting from the impacts, which are then redistributed in the bins. In this way, given an initial size distribution (presumably resulting from the primordial accretion phase) and a suitable set of impact response parameters, the size distribution resulting from 4.5×10^9 yr of evolution can be calculated, and then compared with the real one as determined from astronomical observations.

The remainder of this paper is organized as follows. In Sec. 2 we recall briefly the main assumptions embedded in our collisional outcome model and the adopted parameter choice. In Sec. 3 we discuss some results from this model. In Sec. 4 we present several numerical simulations of the collisional evolution of the entire asteroid population, and comment on the importance of the assumed fragment velocity vs. mass relationship. Some conclusions from this work and the remaining open problems are summarized in Sec. 5.

2. The collisional outcome model

The collisional outcome algorithm of Petit and Farinella works in the following way. For each colliding body the available impact energy per unit volume is compared with a fragmentation threshold S (called *impact strength*), which is assumed to vary with size according to the strain–rate scaling theory of Housen and Holsapple (1990) and to include a gravitional self–compressional term in agreement with Davis et al. (1985) and Housen et al. (1991); this term is multiplied by a free input parameter γ. When a body is shattered, the mass ratio between the largest fragment and the parent body f_l is assumed to be a decreasing function of the impact energy according to an empirical relationship determined by Fujiwara et al. (1977), and the fragment mass distribution is modelled by a truncated power law whose lower cutoff is an input parameter M_{dust} and whose exponent ($-b$ for the cumulative distribution) becomes larger and larger as f_l decreases. On the other hand, in the cratering case the fragment mass distribution is assumed to have a fixed exponent, and the total mass excavated from the crater is determined from an empirical formula relating it to the impact energy (for small craters, this is a simple linear relationship; the ratio α between crater mass and projectile energy is called *crater excavation coefficient*, and is another input parameter of the model).

In order to determine how many fragments escape from the gravitational well of the colliding bodies, an *effective escape velocity* V_{esc} is computed by a suitable energy balance equation, assuming that the largest fragment has always a negligible kinetic energy and taking into account the self–gravitational energy of the fragments. It is assumed that the total kinetic energy of the fragments is given by a fixed fraction of the projectile's kinetic energy (this fraction is the so–called *anelasticity parameter* f_{KE}), and then this constraint is used in computing their velocity distribution, as explained below. The fragments whose velocity exceeds V_{esc} are assumed to escape "to infinity", the other ones are reaccumulated onto the largest fragment.

Most models of asteroid collisional evolution (e.g., Davis et al. 1985, 1989) neglect any possible correlation between the ejection velocity and the mass of the fragments (both from shattering and for cratering events), and just assume that the fraction of the total fragment mass ejected with speeds exceeding some given value v is $(v/V_{min})^{-k}$, V_{min} being a fixed lower cutoff to the ejection velocity. Such a relationship was observed for crater ejecta (with $k \approx 9/4$) by Gault et al. (1963); see Housen (1992) for a recent re–analysis of these experiments. On the other hand, recent experimental evidence (Davis and Ryan, 1990; Nakamura and Fujiwara, 1991; Nakamura et al. 1992) indicates that some correlation between fragment velocity and mass does exist, with the largest fragments on average going somewhat slower; this is also predicted by the semiempirical models of Paolicchi et al. (1989, 1992). It is easy to show (see e.g. Nakamura and Fujiwara 1991) that in this case the relationship given above for the mass fraction ejected at speeds $> v$ still holds provided the fragment speeds are proportional to the power $-r$ of their mass m, with $r = (1-b)/k$. The experimental results appear to be consistent with $r \approx 1/6$, although the dispersion of the data is considerable and smaller values of r cannot be ruled out (Takagi et al. 1992).

Petit and Farinella (1993) have used such a power relationship in either a *deterministic* way (i.e., assuming that a fragment of given mass has exactly the velocity value predicted by the empirical formula) or a *probabilistic* way (see below). Either r or k can be chosen as the input parameter specifying the adopted velocity vs. mass relationship, with the fragment velocity always bound to be less than a higher cutoff V_{max}.

In the *probabilistic case*, for every fragment mass m the $V_0 \propto m^{-r}$ relationship was assumed to hold for the r.m.s. value $V_0 = \sqrt{\langle v^2 \rangle}$ of a Maxwellian probability distribution function for the fragment velocity v:

$$P(v; V_0)dv = \sqrt{\frac{2}{\pi}}\,\frac{3\sqrt{3}v^2}{V_0^3}\,\exp\left(-\frac{3v^2}{2V_0^2}\right)dv \ , \tag{1}$$

The choice of a Maxwellian velocity distribution had the following motivations: (i) it roughly matches the (limited) experimental data available, which show a large dispersion of the velocities measured for every mass, rather than a strong correlation; (ii) a Maxwellian distribution for the magnitude of the velocity vector arises whenever the three components of it follow Gaussian distributions with zero averages and equal sigmas — the simplest assumption for a nearly–isotropic velocity field; (iii) Eq. (1) is consistent with a fairly simple method to compute the individual fragment velocities, as the mean kinetic energy for bodies of mass m is simply given by $mV_0^2/2$, so the energy conservation equation to derive the normalization of the V_0 vs. m relationship is the same as in the deterministic case. Thus for each fragment an ejection velocity value is drawn at random using the distribution (1), and is compared to V_{esc} to determine whether the fragment escapes or is reaccumulated (actually, this is done only when mass bins containing no more than 100 fragments are considered; otherwise Eq. (1) is integrated to obtain the fraction of escaping fragments for the given mass). Thus, whereas in the deterministic case when $r > 0$ all the fragments smaller than some threshold mass (corresponding to $v = V_{esc}$) are always reaccumulated, the transition is more gradual in the probabilistic case.

In this work, we have adopted the following "standard" set of input parameter values: density $\rho = 2500$ kg/m^3; impact velocity $V_{rel} = 5.85$ km/s; lower cutoff of the fragment mass $M_{dust} = 10^{-5}$ kg; higher cutoff of the fragment ejection velocity $V_{max} = 5$ km/s; pre–scaling impact strength (at laboratory target sizes of 0.2 m) $S_0 = 10^6$ J/m^3 (S was then scaled up assuming a value $\gamma = 1$ for the coefficient of the self–compressional term); anelasticity parameter $f_{KE} = 0.1$; crater excavation coefficient $\alpha = 10^{-4}$ s^2/m^2. This parameter choice is derived from the available astronomical and experimental evidence, as discussed in Fujiwara et al. (1989) and Davis et al. (1989). In particular, the S_0 value is within the typical range — spanning about an order of magnitude — observed for "rocky" materials such as basalt, granite, etc. (see Fujiwara et al. 1989, Fig. 2; note that Petit and Farinella 1993 adopted a value 3 times larger), whereas the value of f_{KE} is some 5 times larger than inferred from the experiments, but consistent with the properties of asteroid families (see Davis et al. 1989, Fig. 1). In view of their large uncertainty, we shall discuss in Sec. 4 some results obtained by choosing different values for these two parameters. For our numerical computations we used 50 logarithmic mass bins spaced by a factor 2 (i.e., a factor 1.26 in size), with central values of the corresponding radii ranging from 1.211×10^{-2}

km to 10^3 km. As concerns the fragment velocity vs. mass relationship, we have chosen as an input parameter either $k = 9/4$ or $r = 1/6$, using $r = (1 - b)/k$ to determine the missing coefficient, as explained earlier — note that this would give at the same time the "experimental" values $r = 1/6$ and $k = 9/4$ for $b = 5/8$, which is reasonable since it can be shown that as $M_{dust} \Rightarrow 0$, $b \Rightarrow (1 - f_l)$, and $f_l < 1/2$ is the definition of shattering impact (see Petit and Farinella 1993, Sec. 3).

The output of the collisional outcome algorithm can be written as a three–dimensional matrix, specifying how many objects (escaping fragments, cratered targets, reaccumulated rubble piles) end up in the i–th mass bin following a collision between two bodies lying in the j-th and k–th mass bins. This matrix can then be read by a program aimed at simulating the time evolution of an initial asteroid population by computing in any time step for every pair of mass bins the expected number of collisions, as in the work of Greenberg et al. (1978), Davis et al. (1979, 1985, 1989, 1994), Campo Bagatin et al. (1994) and Farinella et al. (1992). This will be further detailed below.

3. Results from the collisional outcome model

Let us start by commenting some plots that illustrate the results of the collisional outcome algorithm summarized in the previous Section. Fig. 1 shows, for a target 100 km in radius and a 10^{-3} projectile–to–target mass ratio, the fraction of reaccumulated mass vs. the logarithm of the impact energy, normalized to the threshold energy for fragmentation. A gap of two to three orders of magnitude is apparent between the fragmentation threshold and the energy needed to disperse to infinity a significant fraction of the target mass. As a consequence, with this parameter choice most asteroids of radius ≈ 100 km would have been shattered but not completely disrupted (i.e., they would have partially reaccumulated) as a consequence of the most energetic collisions undergone over the history of the solar system. Fig. 1 also shows that the reaccumulation process is confined to a narrower energy range in the $k = 9/4$ case, i.e. when we require that for every velocity V (exceeding a lower cutoff) the total mass ejected with higher speeds is $\propto V^{-9/4}$. The reason for this is that since $r = (1 - b)/k$ (see Sec. 2) and for such supercatastrophic collisions b is close to 1, the exponent r in this case is small, implying that the velocity of fragments has a weak dependence on their mass: thus, there is an abrupt transition between the case when nearly all the fragments are slower than V_{esc} and the opposite one. The transition is less sharp in the probabilistic case, since in this case for every mass the fragment velocities have a significant dispersion about the mean value — and it is much smoother when r (instead of k) is fixed to the value 1/6. In this case, even after a supercatastrophic impact a significant fraction of the target mass is reaccumulated, because most of the mass resides in small fragments and is forced to move comparatively fast, so that conservation of energy causes the largest fragments to be slow and reaccumulate.

The same phenomenon can be seen in Fig. 2, where the fraction of reaccumulated mass is plotted vs. target size. Again, for the $k = 9/4$ cases (but less for the probabilistic one), there is a comparatively sharp transition from negligible to substantial reaccumulation, which occurs when the velocity of most fragments becomes lower than V_{esc}. As remarked by Petit and Farinella (1993), this behavior would imply comparatively short collisional lifetimes for small asteroids, which would be unlikely to become rubble piles, but would mostly look like single, competent fragments. The opposite conclusion would follow from the $r = 1/6$ models, for which Fig. 2 shows that even small bodies are always reaccumulated; recall, however, that when V_{esc} is small only a small number of massive fragments may become reaggregated.

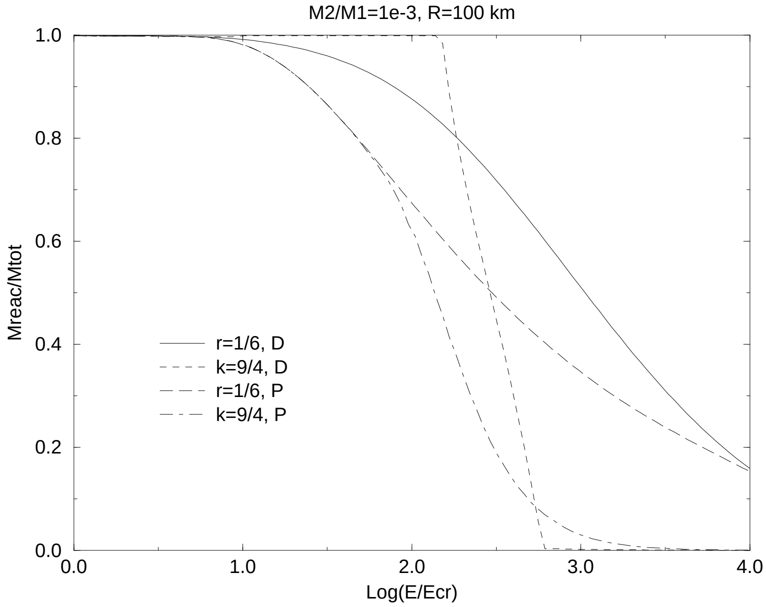

Figure 1. The fraction of reaccumulated mass for an impact with 10^{-3} projectile–to–target mass ratio against a target of radius 100 km, vs. the logarithm of the impact energy. The latter parameter is normalized to the minimum energy required for fragmentation. Different fragment velocity vs. mass relationships are used (see text), with D standing for "deterministic model" and P for "probabilistic model".

Fig. 3 shows another type of diagram, which should be compared with Fig. 5 of Davis *et al.* (1985) and Fig. 5 of Housen and Holsapple (1990). Here the lower (dashed) curve corresponds to the critical projectile–to–target mass ratio for fragmentation, which is a function of size because of the strength–scaling relationship. The other curves give the threshold mass ratio for collisional disruption, with one half of the target mass dispersed to infinity. While for small asteroids, whose gravity is not efficient in reaccumulating fragments, the fragmentation and disruption thresholds coincide, in agreement with Fig. 1 for asteroids ≈ 100 km across there is a large gap between the two thresholds, due to the gravitational reaccumulation process. In this case, the curves show that for the $r = 1/6$ models a sudden discontinuity appears when a critical size is exceeded, while for the $k = 9/4$ case there is a more gradual transition. This behaviour can be explained as follows: in the $k = 9/4$ case, when all the fragment velocities are close to each other, a modest specific energy (i.e., projectile–to–target mass ratio) increase may be enough to exceed V_{esc}; on the other hand, in the $r = 1/6$ case, after self–gravity has become strong enough to reaccumulate the largest fragments, a moderate increase in the specific energy causes a mass shift to fragments of smaller sizes, which receive a larger fraction of the available energy, thus preventing the largest fragments from starting faster (and being able to escape).

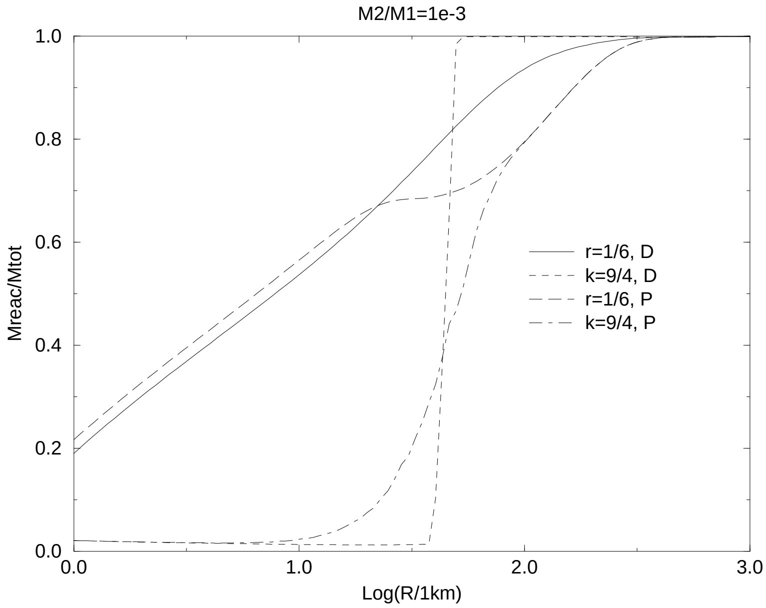

Figure 2. The fraction of reaccumulated mass for a shattering impact with 10^{-3} projectile–to–target mass ratio vs. the logarithm of the target radius. The same four models have been used as in Fig. 1.

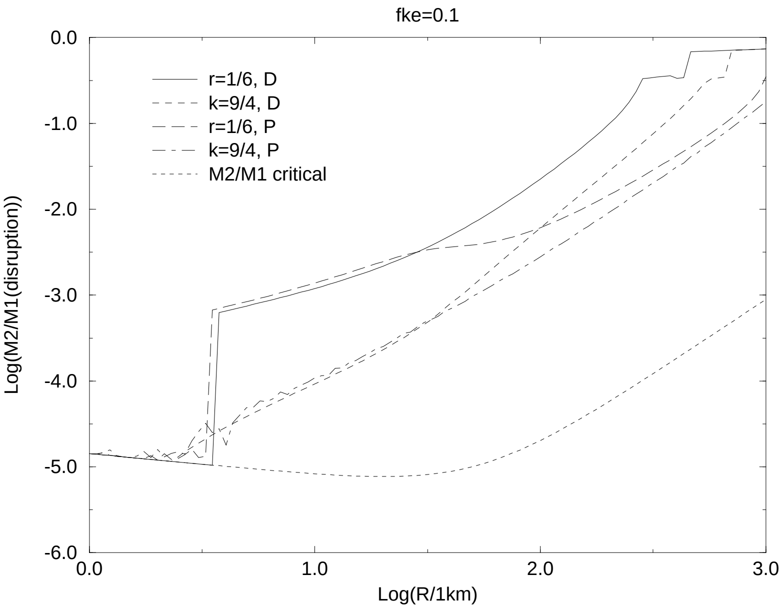

Figure 3. The critical mass ratio for *disruption* vs. the logarithm of the target size, for the four models used in Figs. 1 and 2. The bottom (dashed) line shows the projectile–to–target threshold mass ratio for *fragmentation*, derived from scaling laws for the impact strength S as described in the text.

4. Simulations of asteroid collisional evolution

We now come to the numerical simulations of the collisional history of the asteroid belt. In summary, we have developed a code which integrates numerically (by a simple variable–stepsize Eulerian technique) the following set of first–order differential equations:

$$\frac{dN_k}{dt} = \sum_{i,j=1}^{n_{bin}} f_{ijk} p_{ij} \frac{(N_i - \delta_{ij})N_j}{(1+\delta_{ij})} - B\frac{N_k}{D_k}, \tag{2}$$

where N_k are the populations of the n_{bin} ($= 50$) mass bins; f_{ijk} is the number of fragments injected into the k–th bin by an impact between two asteroids belonging to the i–th and the j–th mass bins, as determined by the collisional outcome algorithm described earlier; p_{ij} is the rate of impacts for a pair of asteroids belonging to the same two bins (derived from their impact cross section and the average *intrinsic collision probability* as estimated by Farinella and Davis 1992); δ_{ij} ($= 1$ if $i = j$ and 0 otherwise) accounts for collisions between asteroids residing in the same bin; and the last term in the right–hand member accounts for the removal of small particles due to the Poynting–Robertson drag, whose efficiency is assumed to be inversely proportional to the (mean) particle diameter D_k of the k–th bin (for the constant coefficient B the value $1/(4.5 \times 10^{10})$ m/yr has been adopted).

Note that using the same matrix f_{ijk} over the whole evolution time span requires that we assume that the impact response parameters defined in Sec. 2 do not change with time. Actually, one may expect that when an asteroid is converted by an energetic collision into a rubble pile (a weakly bonded aggregate of fragments), its subsequent behaviour becomes very different from the previous one. There is some experimental evidence that this is not the case, however, at least as far as the impact strength S_0 is concerned (Ryan et al. 1991). As for f_{KE} and k, the available data are too few to justify the development of a specific model for rubble piles.

As for the initial conditions, the starting population was built by joining two power law distributions at a transition diameter of 100 km, with 400 bodies assumed to exceed this size and being distributed according to the cumulative law $N(> D) \propto D^{-3.5}$ (with the only exception of the four largest size bins, containing Ceres, Pallas and Vesta, which were assumed to start already with 1, 0, 0 and 2 bodies). For $D < 100$ km, we used the cumulative distribution $N(> D) \propto D^{-3}$. This choice for the initial population is the same we have made in Davis et al. (1994) and Campo Bagatin et al. (1994), and similar to those made in Davis et al. (1989) and Farinella et al. (1992), and we refer to those papers for a discussion on its motivations. We stress that we did not aim in this work at obtaining a match to the real asteroidal size distribution or at exploring the full space of the model parameters, but only at assessing the importance of making different assumptions about the fragment velocity distribution.

Figs. 4 to 6 show the results of these simulations and their sensitivity to the assumed relationship between fragment velocity and mass. In Fig. 4 the two cases using a "deterministic" relationship (see Sec. 2) are compared to the "standard" case used in all previous work on asteroid collisional evolution, in which no dependence of velocity on mass was assumed (in this case a mass fraction $[1 - (V_{esc}/V_{min})^{-9/4}]$ was simply assumed to reaccumulate for $V_{esc} > V_{min}$, with the lower cutoff velocity V_{min} derived from f_{KE} through conservation of energy). Of course, the largest difference appears between the standard case and the $r = 1/6$ case, which corresponds to the strongest velocity vs. mass dependence. Whereas this difference is always considerable for asteroids ≈ 100 km in size, in the $r = 1/6$ case it is still significant at diameters down to ≈ 100 m. There are two concurring reasons for this behaviour: (i) in the $r = 1/6$ case some gravitational reaccumulation occurs even on very small targets (see Sec. 3, Fig. 2), because the largest fragments are always very slow, hence the response to impacts of small asteroids is affected by the behaviour of fragment velocities; (ii) since most small asteroids are first– or multi–generation fragments escaped after the break–up (and partial reaccumulation) of larger parent asteroids, their final abundance is sensitive to the efficiency of gravitational reaccumulation onto larger bodies. On the other hand, the $k = 9/4$ case differs from the standard one mainly in the size range (between 40 and 200 km in diameter) where reaccumulation is most important.

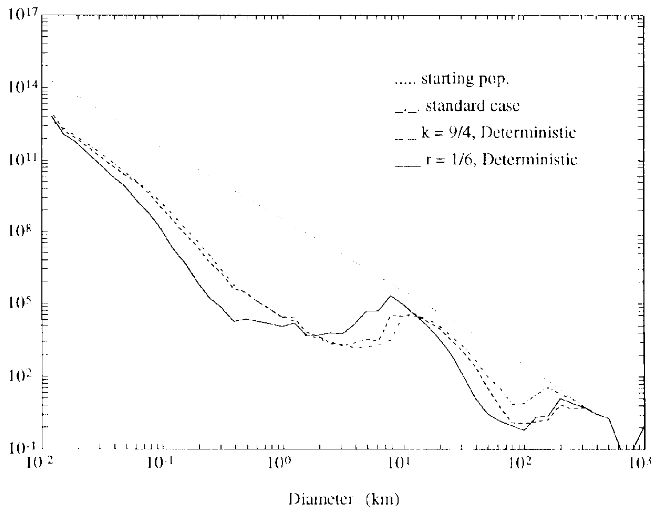

Figure 4. Results of numerical simulations of the asteroid collisional history. The dotted line corresponds to the assumed initial population, while the other curves are the final size distribution derived with different assumptions about the fragment velocity vs. mass relationship. Note that in this type of diagram, showing incremental populations of equal logarithmic bins vs. logarithm of their central diameter, a power–law size distribution appears as a straight line of slope coinciding with the index of the corresponding *cumulative* distribution.

Figs. 5 and 6 show the differences between the deterministic and the probabilistic cases for both the $r = 1/6$ and the $k = 9/4$ case. Although some difference propagates to smaller sizes, the largest discrepancy lies in the reaccumulation–effective diameter range around 100 km. These results show that the final size distribution of the observable asteroids depends sensitively not only upon the assumed relationship between *mean* fragment velocity and mass, but also on the dispersion of that velocity about the mean for any given mass. This highlights the crucial importance of obtaining new experimental data on fragment velocity distributions.

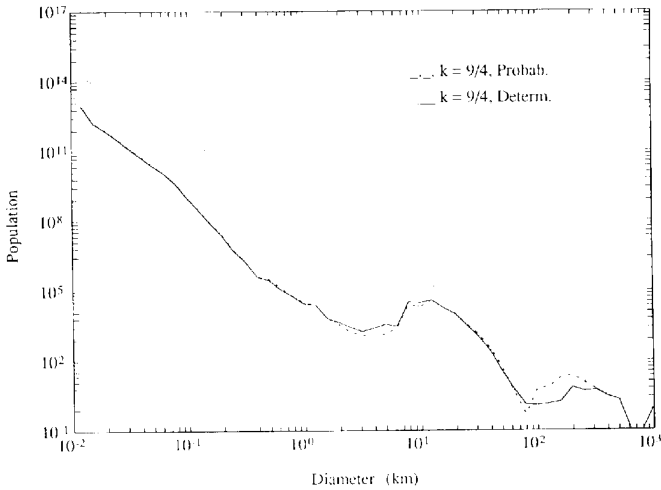

Figure 5. The same as Fig. 4, but comparing the results obtained with the deterministic and probabilistic models in the case $k = 9/4$ (see text).

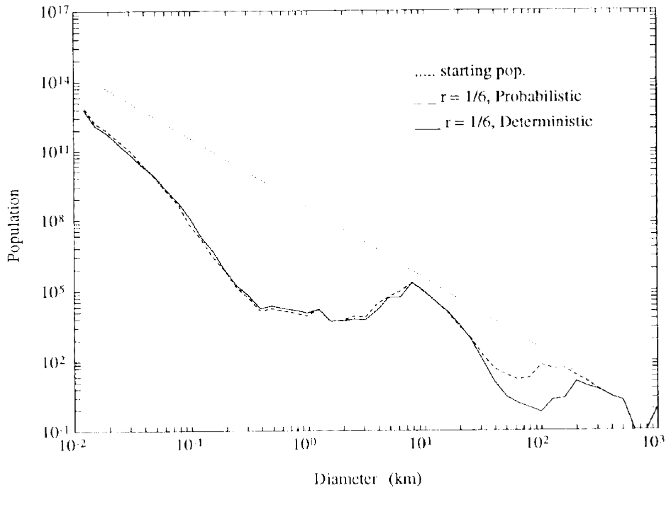

Figure 6. The same as Fig. 5, but for the case $r = 1/6$ (see text).

How does the sensitivity to the velocity vs. mass relationship discussed in this paper compare with the sensitivity to plausible changes of other input parameters which are needed to run the collisional outcome model? Figs. 7 and 8 show how the results of the "standard" case already displayed in Fig. 4 are affected when S_0 is changed by plus or minus an order of magnitude (Fig. 7), and f_{KE} is changed by plus or minus a factor of two (Fig. 8). Changing S_0 has very strong effects, as the final asteroid populations at some sizes (e.g., a few km) are changed by orders of magnitude. Note, however, that the resulting effects are not obvious: for instance, in the diameter range between about 10 and 50 km, the final populations are *more* depleted when the asteroids are assumed to be stronger. This is caused by the fact that these bodies are more frequently disrupted by km–sized projectiles, which on the contrary are much more abundant. This type of effect, together with the presence of a small–size cutoff in the evolving size distribution, is responsible for the "wavy" appearance of the final distribution, with the typical wavelength inversely correlated to S_0 (Campo Bagatin et al. 1994). Even if the results of the simulations are very sensitive to S_0, however, we recall that for any given material this parameter can be measured in the laboratory much more easily than the fragment velocities, as such measurements do not need complex fast–framing imaging devices, but only an *a posteriori* assessment of the impact outcome.

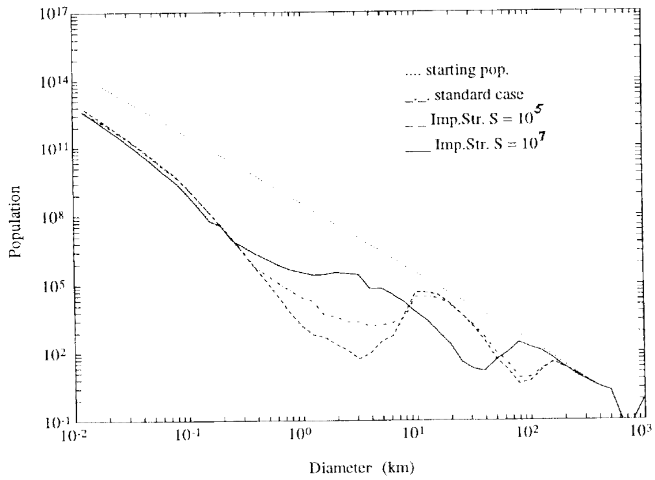

Figure 7. The same as Fig. 4, but comparing the standard case (with $S_0 = 10^6$ J/m^3) to the results obtained by changing S_0 by plus or minus an order of magnitude.

The results illustrated in Fig. 8 are even more interesting in the present context. The changes resulting when f_{KE} varies by a factor of four (i.e., typical fragment velocities vary by a factor of two, as f_{KE} is related to the total kinetic energy) are not negligible, and they can be interpreted easily. The final populations are almost insensitive to the value of f_{KE} at small sizes, where self–gravitational effects are negligible, whereas the location of the "bump" caused by the onset of reaccumulation (at diameters $\approx$ 10 km) is somewhat shifted. Also, for the largest asteroids (diameters > 100 km) higher fragment velocities result into a less effective reaccumulation, hence in a stronger depletion. However, it is worth remarking that the changes shown in Fig. 8 are smaller than those displayed in

Fig. 4 and 6, and similar to those displayed in Fig. 5. Thus, knowing how the fragment velocities are related to their mass and also how they are distributed for any given mass about the mean value, is at least as important as determining the average magnitude of these velocities (i.e., measuring f_{KE}).

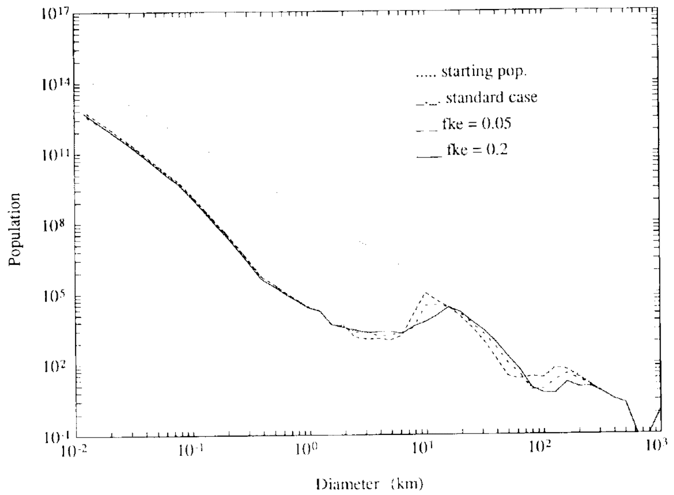

Figure 8. The same as Fig. 4, but comparing the standard case (with $f_{KE} = 0.1$) to the results obtained by changing f_{KE} by plus or minus a factor of two.

5. Conclusions

Numerical simulations of the asteroid collisional evolution are motivated by the desire to understand how the current asteroid population has been processed starting from an original population of newly accreted proto–asteroids, which would have been the building blocks for an asteroidal planet if the accumulation process had not been interrupted. However, to derive information about this primordial population, we need to use realistic physical models of how collisions have affected asteroids of different sizes, in particular how frequently they are disrupted and how many new smaller (fragment) asteroids are generated in this case.

In this paper we have shown that a crucial feature of these models is the relationship between fragment ejection velocity and mass. Not only how the mean velocity depends upon mass is important, but also the distribution of velocities about the mean values. The available experimental evidence on this issue is still sparse, and does not constrain the collisional models well enough to allow us to make reliable predictions on the outcomes of impacts between bodies tens to hundreds of km in size. This adds to the current uncertainty on the values of collisional parameters such as S_0 and f_{KE} and the appropriate scaling laws for them, and as a consequence we are still unable to simulate the evolution of the asteroid size distribution over billions of years having full confidence that the results will tell us the history of the real belt. Hopefully, further experimental and modelling work will improve this situation.

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