A new insight in Uranus rings: a wavelet analysis of the Voyager 2 data

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Abstract. A new signal processing analysis, based on the wavelet transform has been developed. It allows the detection and the reconstruction of fine structures in a very noisy signal. It removes the noise and gives a quantified level of detection of the structures against chance fluctuations. This powerful method has been applied on the PPS Voyager 2 data on the Uranus rings. A preliminary catalog of structures found in the σ Sagitarii occultation experiment, is proposed here.

1. Introduction

The data obtained in the PPS (recording of brightness of star δ Sco using a photopolarimeter) experiment of Voyager 2 have revealed a whole hierarchy of structures in planetary rings, at all scales and with a wide range of amplitudes. In order to determine physical parameters of the rings and embedded (or shepherding) satellites we have to be able to determine precisely the density profile of features with a signal to noise ratio of the order of 1. In this paper, we briefly describe a new method of signal processing based on the wavelet transform. The method has be fully described in Bendjoya *et al.* 1993. We will recall here the main philosophy of the analysis and we will propose, as an example of application, an inventory of the fine structures detected in the σ Sagitarii occultation experiment of Voyager 2.

2. Philosophy of the method

The signal processing tool presented here is particularly well suited for the detection and reconstruction of structures (bumps or depletions) embedded in a very noisy signal. Simulations have been performed and have shown that the method remains powerful for signals with a signal-to-noise ratio of order of one. The method gives a reconstruction of the underlying structures precise enough to derive physical parameters with a relative uncertainty of order of 10 %. Moreover the detection of these structures can be quantified against chance fluctuations by giving a level of significance.

The method relies on the following model: a signal is a sum of details at different scales i.e. different characteristic sizes. A natural assumption is that the significant information is not equivalently displayed at all scales. For example,

the small scales are generally related to the noise while a structure lives on a set of larger scales, determined by its shape, depth and width.

The idea is then to decompose the signal onto a peculiar band pass filter basis which allows one to separate the details at the different scales. This basis is called the wavelet basis and the decomposition of the signal onto this basis is called the wavelet transform. The wavelet transform acts like a local mathematical zoom which allows detection and reconstruction of a structure without noise by taking into account only the details corresponding to that structure.

A signal is defined with a cutoff frequency f: the sampling step. The procedure of the wavelet transform consists in applying a low pass filter with a cutoff frequency of $f/2$. We thus obtain a smoothed function of the signal. A "good" choice (Daubechie, Mallat) of the low pass filter allows the derivation of a band pass filter just by taking the difference point by point between the signal and the smoothed function. This band pass filter extracts the details contained in the frequency band $[f/2, f]$. By iterating the same filtering on the smoothed function, we get on one hand a new smoothed function with a cutoff of $f/4$ and on the other hand the details we lost when passing from a resolution to the next one. Fig. 1 illustrates the whole procedure.

Figure 1. Scheme of the wavelet transform algorithm.

The band pass filter bank provides a basis onto which the signal is decomposed. This basis is the wavelet basis and is chosen so that each element is a

zero mean function. Since the successive steps are obtained by difference we can reconstruct the signal by simply adding the last smoothed function with all the details. This is the scheme of a multi resolution analysis. Fig. 2 shows concretely how it works: starting from the signal one can see the successive smoothed functions and the details at different scales. Since the wavelet basis is composed by zero mean functions, the details (also known as the wavelet coefficients) have a zero mean.

Figure 2. Left: the original signal sampled at frequency f (top), the details in the frequency range $[f/2; f]$ (middle) and the details in the frequency range $[f/4; f/2]$ (bottom). Right: the successive smoothed signals at frequencies $f/2$ (top), $f/4$ (middle) and $f/8$ (bottom).

Since the noise is generally associated with the smallest scales the idea is to reconstruct the signal by adding the last smoothed function and a set of selected coefficients discarding those corresponding to the noise. In order to select these coefficients, we do a thresholding in the wavelet plane. The thresholding of the

wavelet coefficients is done by comparing for each scale the wavelet coefficients derived from the signal with those derived from a pseudo random signal. This later is built randomly from the signal following the same histogram of values. In such a way the noise is not derived from a model and no assumption is made on it. Due to the shape of the wavelet (zero mean function) a bump of the size of the studied scale will give high positive coefficients. Similarly, a depletion will give large amplitude negative coefficients. the threshold is computed from the distribution of the wavelet coefficients of the pseudo random signal. When looking for bumps, the threshold is the mean value of the considered scale plus a given number n times the r.m.s. When looking for depletions, the threshold is the mean value of the considered scale minus a given number n times the r.m.s. Among the wavelet coefficients computed from the real signal, only those greater (in absolute value) than the threshold will be kept for the reconstruction. By construction the pseudo random signal and the signal itself are similar at the small scales (scales of the noise). A sufficiently high threshold (n of order of 4) will discard the wavelet coefficients of these smallest scales and hence the noise. On the other hand at larger scales the pseudo random signal will not provide high (in absolute value) coefficients but the true signal will. If there is a structure, it will be conserved for the reconstruction. Fig. 3 displays a scheme of the thresholding procedure.

Figure 3. Scheme of the thresholding procedure.

In fact the reconstruction is made after a second thresholding more permissive than the first, whose role is to remove the noise. This second thresholding

(typically n between 1 and 2) allows to add locally details to the structures detected after the first thresholding. Finally the level of detection (or of confidence against chance fluctuations) is given by the second thresholding for the finest details and by the first thresholding for the structure as a whole.

This is the main difference with the more classical filtering methods such as Fourier Transform or binning which perform a global filtering and are unable to add details (small scales or high frequency) only where it is needed.

3. Results

3.1. A simulated structure

In order to show the efficiency of the method we have built an artificial structure with a three peak shape (fig. 4a) and plunged it in a gaussian noise in order to get a signal to noise ratio (S/N) of 1.2 (fig. 4b). We define the signal to noise ratio (S/N) as the ratio between the amplitude of the structure (here, 1) and the standard deviation of the gaussian noise we add. This definition gives a local meaning to S/N. In our case, we are not interested in large scale structures but rather in local ones. The signal shown in fig. 4b has a signal to noise ratio of 1.2, since the standard deviation is 0.833 and the amplitude of the noiseless signal is 1. Fig. 4c displays the reconstructed signal (solid line) superimposed to the original structure (dashed line).

The choice of such a shape for the simulated structure has been suggested by the models of gravitational interactions. The rings are known to be maintained by the gravitational interaction with satellites (Goldreich and Tremaine 1978, Shu 1984, Shu *et al.* 1985 a,b). The effect of an embedded satellite is to clear a gap (provided its mass is large enough) and retain a faint ringlet on the same orbit as itself. The profile of such a ringlet can have two or three peaks (Spahn *et al.* 1992).

The first thresholding of the wavelet coefficients is 4 times the r.m.s of coefficient distribution from the pseudo random signal. We recall that this first thresholding remove the noise and detect the structure. The second thresholding has been automatically chosen by the procedure equal to 1.5. This second pass add locally details to the detected structure. The value of this second thresholding corresponds to a level of significance against chance fluctuation of: 93.3%. One can see the great ability of the method to remove noise, to detect and to reconstruct precisely fine structures. The two bumps on each side of the reconstructed three peak depletion are due to the property of the wavelets to be zero mean functions. This phenomenon is all the more so marked since the underlying structure is thinner.

Figure 4. a) the simulated three peaks structure; b) the analysed signal with $S/N = 1.2$; c) the superposition of the reconstructed signal and the initial structure (dashed line).

The wavelet analysis method has been performed for different shapes of the three–peak structure, different signal to noise ratios and different realizations of the noise (see Bendjoya *et al.* 1993) leading to very good detections and reconstructions.

3.2. The Voyager 2 PPS data

The Voyager 2 PPS data analysed here were obtained from the occultation of the star σ *Sagitarii* by the Uranus' rings. During the occultations the spacecraft passed behind the ϵ and δ rings in a nearly tangential occultation as viewed from Voyager as it approached Uranus. Every 10 msec the PPS recorded the brightness of the occulted star, with successive data points separated by 10 m

in distance from Uranus' center for ϵ ring and 3.8 m for δ ring. These data have been provided to us by Mark Showalter from the PDS.

We have made the analysis on subsets of 32,000 data points with an overlapping of 16,000 points. This overlapping enables us to avoid the edge effects such as false detection or missing detection. The detection and reconstruction have been performed in a fully automated manner. Local analyses have been performed around the detected structures. The data are provided with the number of the record and the photon counts. The conversion to the radii (from the center of the planet) is made assuming an uninclined equatorial ring (French *et al.* 1988).

3.3. Inventory of the Uranus rings structures in the σ Sagitarii occultation experiment

Results are displayed in fig. 5 and in table 1. In this table the widths have been computed at the widest location of the reconstructed structure. We recall that all these reconstructed structures have been detected with a threshold of 4 σ. The thresholds given in column 4 of table 1 correspond to the second thresholding and give the level of confidence of the details added to the global structure.

We only present here the structures found both in ingress and egress. We will not discuss here the possible origin of the fine structures especially inside the ϵ and δ rings. A forthcoming paper will be devoted to a comparison between theoretical models and the detection of these fine details. Nevertheless one can obviously see how the noise is removed and how the quality of the reconstruction can bring clues to the modelling of the formation and the stability of the rings. Moreover the detection and reconstruction of these fine structures can be indicators of the presence of embedded moonlets.

Among the 1,400,000 data points, the analysis has provided the detection and reconstruction of the well known ϵ and δ rings plus three new ringlets. We have called these ringlets respectively I_{wv}, II_{wv} and III_{wv}. The "$_{wv}$" subscript stands for "wavelet" and its role is to avoid any confusion with possible previous classifications.

The ringlet I_{wv}, located at 50685 km (ingress) and 50680 km (egress) from the center of Uranus could be associated with Ring I detected by Lane *et al.* (1986) from the occultation of β Persei. However they found a width of 16 km where we get a width of the order of a few hundred meters. But they did not detect the ring in the σ Sagitarii occultation, due to their method of analysis based on binning. This would be consistent with a narrowing of the thick part of the ring. In such a case our method allows detection of features not accessible to older methods.

The newly detected ringlet II_{wv}, located at 50,023 km is quite wide since its wider part is almost 6 km at ingress and a little more than 2 km at egress. The very good agreement in position between ingress and egress is a factor in favor of the detection of a real feature, in addition to the rather large value of the second threshold which give a high confidence in the reality of the details of the profiles (94.5% at ingress and 98.6% at egress). As for the optical depth of this ringlet, it is comparable at egress to the optical depth of ring I_{wv} at ingress.

Although the last new detected ringlet, located at 47,977.4 km, is very narrow, it has been found both at ingress and egress and it correspond to a rather large number of data point since the precision in radial distance at that location is 1.4 m. And even if the level of confidence in all the fine details is not extremely high (90.3% and 91.9%), the structure itself has been detected at 4σ, i.e. with a confidence of 99.997%.

Table 1. List of detected features

Name	Midpoint range	Width (km)	Threshold
ϵ (I)	50,870	30.7	1.4
ϵ (E)	50,748	21.7	1.4
I_{wv} (I)	50,684.9	0.25	1.6
I_{wv} (E)	50,680.3	0.075	1.5
II_{wv} (I)	50,023.4	5.8	1.6
II_{wv} (E)	50,023.1	2.26	2.2
δ (I)	48,300.5	7.6	1.4
δ (E)	48,301.7	7.6	1.1
III_{wv} (I)	47,977.4	0.10	1.3
III_{wv} (E)	47,972.7	0.11	1.4

Figure 5. Reconstruction of structures detected in the σ Sagitarii occultation data both for ingress (left) and egress (right).

4. Conclusion

We presented here a powerful signal processing tool for detection of faint structures in 1 dimension data set. It allowed us to analyse the σ Sagitarii occultation observations performed by Voyager 2. In addition to the well known ϵ and δ rings, we also found 3 new probable rings. One of these (ring I_{wv}) is probably the same as Ring I detected by Lane *et al.* (1986) in the β Persei. In the ϵ ring, we are able to separate a plateau in the center of the ring and peaks on each side. By comparison, the standard methods exhibit a noisy reconstructed signal

which makes it difficult to separate real peaks. The detection of the new features may need to be confronted to the analysis of β Persei occultation observations. This analysis, which we are undertaking now, will allow us to explore the Uranus ring system closer to the planet. The present analysis was performed on a data set extending only down to 47900 km from the center of Uranus. The analysis of both data set will allow us to compare observation with the predictions of theoretical models.

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