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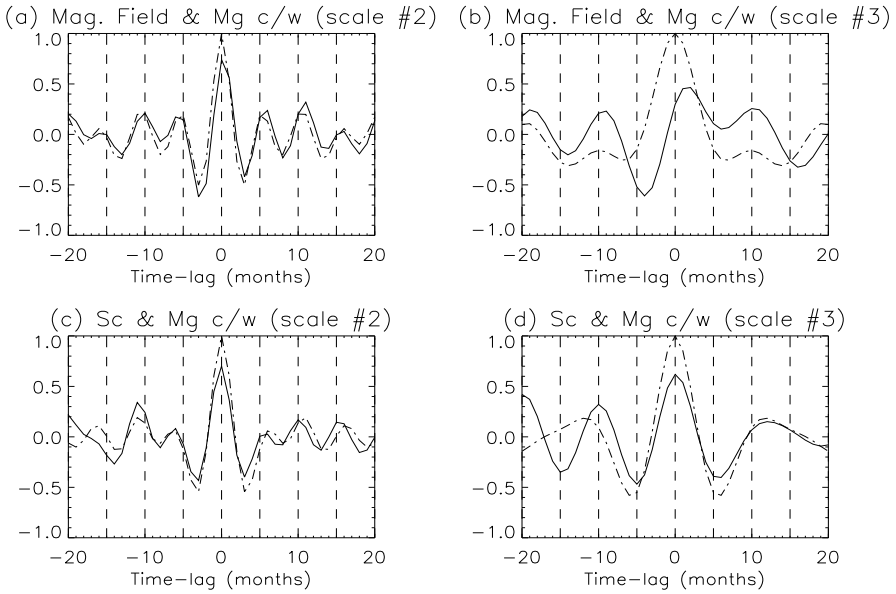

Figure 4. Cross-correlation of the second and third scales of the wavelet transforms (see text) of the detrended time series.

4. Conclusion

Variations observed in total solar irradiance on time scales of months to years have been compared to the changes in its surrogates, such as the full disk magnetic flux, PSI, and the Mg c/w ratio. It has been shown that there is a -3 months time delay between the monthly averages of the full disk magnetic flux, total irradiance corrected for sunspot darkening, and the Mg c/w, when irradiance is leading the magnetic flux variability. The physical origin of this anticorrelation at -3 months is not yet understood. It is a question whether it is related to the 5-6 months periodicity found in various data sets (e.g. Bai & Sturrock, 1987; Pap et al., 1990) or it represents a real time delay between the magnetic flux, S_c and Mg c/w. The cross-correlation between several scales of the wavelet transfrom of the detrended data indicate that on long time scales the correlation between the magnetic flux and S_c as well as the magnetic flux and the Mg c/w is quite different from the autocorrelation of the magnetic field. It is also interesting that there is a +10 months long phase shift between S_c and the Mg c/w. This indicates that the response of the chromospheric layers to the magnetic field variations is quite different than that of the photosphere, and therefore the use of chromospheric proxies for modelling the changes in total irradiance may not be adequate. Further studies of the time delays between the magnetic flux and solar radiation emitted from different layers of the solar atmosphere will lead to a better understanding of the dynamics taking place below, in and above the photosphere.

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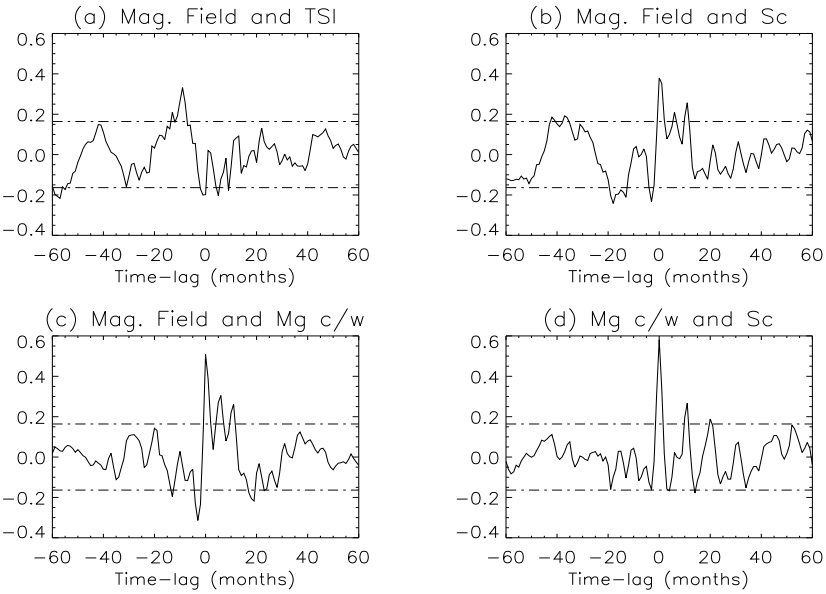

Figure 3. Cross-correlation between the data sets detrended with wavelet technique.

field strength. As can be seen, the cross-correlation between the magnetic field and Mg c/w is similar to the pattern of the autocorrelation of magnetic field, except that the value at 1 month time delay is almost as strong as the one at 0 time-lag. We note that similar results have been found for the first scale wavelet transforms (2 – 4 months periodicity) in the case of both the Mg c/w ratio and S_c.

The results of the cross-correlation between the third scale (corresponding to 8-16 months periodicities) of wavelet transforms of the magnetic field and Mg c/w are presented in Fig. 4b, where the dashed line shows again the autocorrelation for the magnetic field strength. In this case, the maximum of the cross-correlation is found for 1–2 months time delay. Furthermore, an anti-correlation for -4 months time-lag is seen in the cross-correlation but not in the autocorrelation of the magnetic field. These results indicate that the changes in the Mg c/w ratio (and S_c) and the magnetic flux are similar on time scales less than 8 months, but the differences become more obvious if we compare longer term variabilities.

The solid lines in Figs. 4c and 4d present the cross-correlation between the second and third scales of the wavelet transforms of the Mg c/w and S_c, respectively. The dashed lines show the autocorrelation of S_c. In the case of the second scale of the wavelet transform, the autocorrelation and cross-correlation curves fit each other quite well, indicating the similarity of the variations in the Mg c/w data and total irradiance corrected for sunspot darkening on time scales shorter than 8 months. As Fig. 4d shows, the cross-correlation function follows the autocorrelation of S_c to -5 months time-lag.

the significant level for the cross-correlation between the irradiance values and the full disk magnetic flux is 0.164, and it is represented by dashed lines in Fig. 3.

The results of the cross-correlation between the Nimbus-7/ERB total irradiance and the full disk magnetic flux are presented in Fig. 3a. As can be seen, there is a slight anti-correlation between these two indices with zero time-lag. We note that this anti-correlation is more pronounced in the case of the PSI function (r = -0.7, not shown here), indicating that sunspots reduce total solar irradiance and they can explain about half of its short-term variability (c.f. Chapman, 1987). Although the magnetic field is the strongest in sunspots, its value is also enhanced in the presence of bright magnetic features which increase total irradiance. Therefore, the anti-correlation between the magnetic flux and irradiance deficits is less visible. It is interesting to note that a significant positive correlation has been found between total irradiance and the magnetic field at -10 months time-lag. Similar results have been found in the case of the SMM/ACRIM I total irradiance (not shown here).

The cross-correlation between S_c and the magnetic flux is presented in Fig. 3b, whereas Fig. 3c shows the same for the Mg c/w and the magnetic flux. As can be seen, the cross-correlation diagrams for S_c (Fig. 3b) and the Mg c/w ratio (Fig. 3c) show a similar pattern. Positive correlations are found at 0, 1, 5, and 10 months time-lags, whereas anticorrelation is seen at time-lag of -3 months. The positive correlation with 0 time-lag indicates that the weaker magnetic fields of plages and the active network increase both S_c and the Mg c/w ratio. A more detailed analysis of the individual wavelet components of the time series (Vigouroux et al., 1995) shows that there is a negative correlation also at +3 months time-lag which cannot be seen from a simple cross-correlation between the detrended data sets. Vigouroux et al. (1995) have also found the existence of a $5-6$ months periodicity in the data, similar to other studies (e.g. Bai & Sturrock, 1987), which is more pronounced during the maximum and declining portion of solar cycle 21 than during solar minimum as well as during the rise and maximum of cycle 22. In addition, there is a trend related to a longer periodicity (about 32 months) during solar cycle 22 which may demolish the anticorrelation at the +3 months time-lag. Finally, we note that there is an asymmetry in the cross-correlation spectra between the magnetic flux and S_c (Fig. 3b) and Mg c/w (Fig. 3c). Further investigations are required to clarify whether this asymmetry is related to the apparent differences between solar cycles 21 and 22 which was first pointed out by White et al. (1994).

Fig. 3d shows the cross-correlation between S_c and the Mg c/w. As can be seen, there is a strong positive correlation between the two data sets with zero time-lag, indicating that after eliminating the effect of sunspots from total irradiance, its variability is similar to that of UV irradiance as represented by the Mg c/w ratio. Besides the positive correlation at zero time-lag, an additional peak is seen in the cross-correlation spectrum at +10 months time-lag.

To study the time delays between the examined data sets in more detail, the cross-correlation has been calculated between the individual scales of the wavelet transforms corresponding to different frequency bands. The solid line of Fig. 4a shows the cross-correlation diagram between the second scale of the wavelet transforms, corresponding to $4-8$ months periodicity, of the magnetic field and Mg c/w ratio. The dashed line shows the autocorrelation of the magnetic

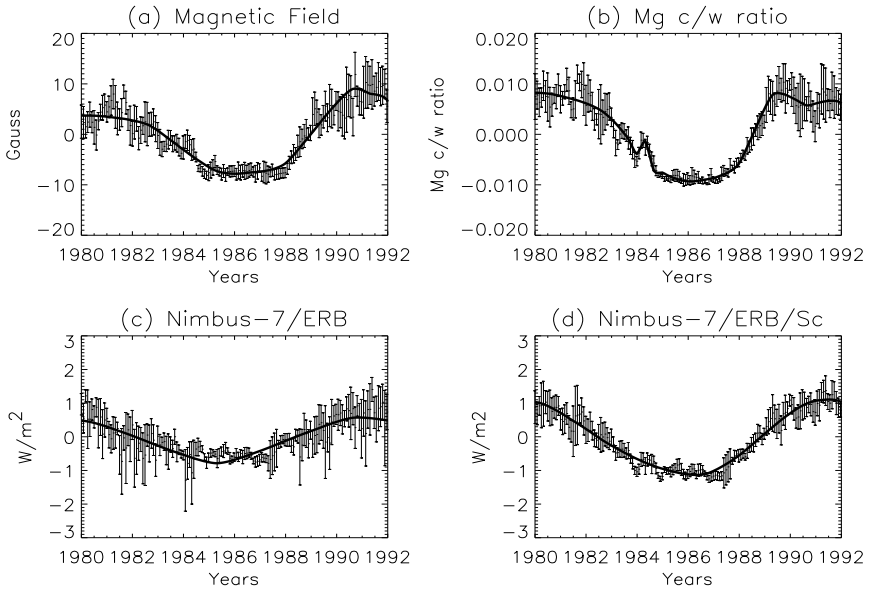

Figure 1. Models of long-term variations calculated by the wavelet transform (*solid line*) are superimposed on the 30-day averages of the original time-series together with their error-bars.

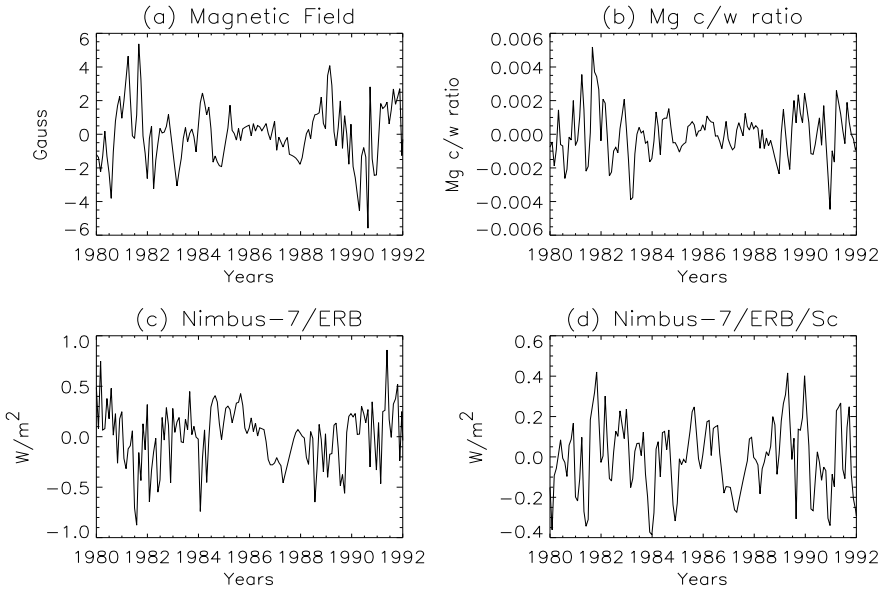

Figure 2. The residual time series after removing the solar cycle variability by means of the wavelet models.

which could be compared to the information contained in a frequency band). For further details on the wavelet analysis we refer the readers to the monograph by Meyer (1993).

Our first objective is to find a better technique to represent the long-term variations in total solar irradiance and its surrogates than the classical Fourier transform. For this purpose we use the wavelet technique. The advantages of the wavelet technique versus the Fourier transform have been discussed by Vigouroux & Delache (1993) and Vigouroux et al. (1995) so that we refer the readers to these papers. We just point out that one of the best benefits of the wavelet transform is that the thresholding procedure is time-depending.

Since our primary interest is to study irradiance variability and its relation to solar magnetic activity over months to years, any changes shorter than the solar rotational period have been considered as "noise" and removed from the data by calculating monthly averages in a similar way as published by Vigouroux & Delache (1994). The dispersion of the daily data within 30 days has been calculated as error-bars and variations within 30 days are considered as "noise" even if, in this case, the "noise" is solar, rather than instrumental, in its origin. The error-bars versus their corresponding 30 days averages have been studied in detail by Pap et al. (1995). It has been shown that the dispersion values are higher during the maximum of the solar activity cycle than during minimum activity conditions. Since the threshold is time-dependent and it hence will depend on the size of the error-bars of the original data, we can suppose that the wavelet processing will make less effort to follow the month to month variations during solar maximum than during solar minimum.

3. Month-to-month variations as revealed by the wavelet technique

Figs. 1a-d represent the monthly averages of the investigated time series with their respective error-bars. In the case of gaps in the data we have made a linear interpolation. The error-bars of the interpolated data have been calculated as the maxima between the error-bars of the two neighboring points in order to avoid giving too much weight to the interpolated data. Fig. 1a shows the full disk integrated magnetic flux (Harvey, 1994), whereas the Mg c/w ratio (Donnelly et al., 1994) is presented in Fig. 1b. Total solar irradiance observed by the Nimbus-7/ERB radiometer (Kyle et al., 1993) is plotted in Fig. 1c, its value corrected for sunspot darkening by means of the PSI function (hereafter S_c) (Fröhlich et al., 1994) is shown in Fig. 1d. The models of the solar cycle variability derived with wavelet analysis are presented by solid lines on each plot. In order to remove the long-term variation over the solar cycle from the time series, the appropriate wavelet models have been subtracted from the observed data. The residual time series, representing the month-to-month variability, are plotted in Figs. 2a-d.

To study the linear association between the month-to-month changes observed in various data sets, we have performed a cross-correlation analysis. The cross-correlation spectra between the time series are presented in Fig. 3. It has been shown that in the case of a random time series, which has the same number of data points as the residuals, 95% of the values in the cross-correlation spectrum lie between $\pm 2/\sqrt{(N)}$, where N=149 (Chatfield, 1984). Accordingly,

this latter one is allowed by the length of the time series at our disposal, that is the 11-year solar cycle. The short-term irradiance changes (from days to months) are related to the combined effect of dark sunspots and bright faculae (Fröhlich et al., 1991). The long-term changes over the solar cycle are attributed to the evolution of magnetic fields via the changing emission of faculae and the magnetic network (Foukal & Lean, 1988). However, the current empirical models developed from "proxy" data for sunspots and bright magnetic features cannot explain all the aspects of irradiance variability. It has been shown that considerable variation of total irradiance remains unexplained after removing the effect of sunspots, faculae and the magnetic network (Fröhlich & Pap, 1989). One may argue that the proxies, especially the ones used for describing the changing emission of bright magnetic elements, have to be improved. Unfortunately, as far as photospheric facula are concerned, there is no good synoptic data set on their occurrence. Some attempts have been made to estimate the effect of the bright magnetic elements either with the HeI index (Foukal & Lean, 1988; Fröhlich, 1994) or with the Mg II h & k core-to-wing ratio (Mg c/w) (Pap et al., 1994) in order to model total solar irradiance corrected for sunspot darkening by means of the Photometric Sunspot Index (PSI) (Fröhlich et al., 1994). But none of these models is really convincing. In contrast, it should be pointed out that techniques for trying to find the best linear fit between two or three parameters do not take into account the possible coupling between them (as reported also by Pap et al., 1990). For example, how the temporary storage and subsequent gradual release of the energy deficit in the sunspot-related dips in total irradiance can be taken into account with such simple models? In addition, it is necessary to know more about the possible time delay between the storage and release of this energy in order to explain the re-radiation mechanism.

The aim of this paper is to apply a relatively new technique, the wavelet analysis, to study the changes observed in total irradiance. We also use the wavelet technique to remove the long-term trend from total irradiance and its surrogates and to study the residual short-term variability. Cross-correlation between the detrended time series should give us some additional information on the possible coupling between total solar irradiance and its surrogates, such as the Mg II h & k core-to-wing ratio (Mg c/w), the Photometric Sunspot Index (PSI, not treated here) and the full disk magnetic flux.

2. The wavelet transform: a short description

Signal processing - reconstruction of time series from composition of defined basis functions - is now essential for scientific studies. It allows us to describe the data we have from various sources. But for an optimal description, the reconstructed time series have to be as near as possible to the original signals. In the case of studying non-stationary signals, when transient events appear but they cannot be predicted even on a statistical form, there is an urgent need to develop and apply techniques different from the Fourier transform. And the wavelets are part of these techniques. The wavelet transform can be defined as a time and scale transform. This transform consists of making successive projections on basis functions which are located in time and each basis function is a time-dilating of its previous one (and this corresponds to scale information

Studying solar irradiance variability with wavelet technique

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Abstract. The detection of variations in solar irradiance by satellite-based experiments during the last 17 years stimulated modelling efforts to help to identify their causes and to provide estimates for irradiance data when no satellite observations exist. Because of the lack of quantitative physical models of solar irradiance, the current empirical models have been developed with linear regression analysis, which can give only overall information about the physical origin of irradiance changes. In this paper we use wavelet and cross-correlation techniques to analyze the changes observed in total solar irradiance and its surrogates, including the Mg h & k core-to-wing ratio and the full disk integrated magnetic flux.

1. Introduction

The total radiation received from the Sun on the top of the Earth's atmosphere at 1 AU is called "solar constant". Observations of total irradiance from space within the last one and a half decades demonstrated that this important astrophysical quantity changes on time scales from minutes to years and decades. The most important discovery of the space-borne irradiance observations was the 0.1% peak-to-peak variation in total solar irradiance over the solar cycle (Willson & Hudson, 1988). Since the solar energy flux is one of the main natural driving forces of the terrestrial atmospheric and climate system, it is essential to understand and model the observed irradiance changes. One of the main interests is to reconstruct the irradiance changes back to the time of the Maunder Minimum (1645 – 1705), when little magnetic activity was seen on the solar surface (Ribes & Nesme–Ribes, 1993). One very interesting aspect of this solar anomaly is its counterpart in terms of the variation of the solar energy output and its impact on the Earth's climate. The dearth of sunspot activity during the Maunder Minimum was indeed coincident with a cold period in Europe and Atlantic region, known as the Little Ice Age.

It is now wellknown that the variations in total solar irradiance are directly correlated with the solar magnetic activity on both short and long time scales,