

Asteroid orbital inversion using Markov-chain Monte Carlo methods

Karri Muinonen^{1,2}, Dagmara Oszkiewicz^{3,4,1},
Tuomo Pieniluoma¹, Mikael Granvik¹ & Jenni Virtanen²

¹Department of Physics, University of Helsinki, Finland

²Finnish Geodetic Institute, Masala, Finland

³Lowell Observatory, Flagstaff, Arizona, U.S.A.

⁴Northern Arizona University, Flagstaff, Arizona, U.S.A.

Solar System science before and after Gaia,
Pisa, Italy, May 4-6, 2011

Introduction

- Asteroid orbit determination is one of the oldest inverse problems
- Paradigm change from deterministic to probabilistic treatment near the turn of the millennium
- Uncertainties in orbital elements, ephemeris uncertainties, collision probabilities, classification
- Identification of asteroids, linkage of asteroid observations
- Incorporation of statistical orbital inversion methods into the Gaia/DPAC data processing pipeline
- Markov-chain Monte Carlo (MCMC, Oszkiewicz et al. 2009)
- OpenOrb open source software (Granvik et al. 2009)

Statistical inversion

- Observation equation
- A posteriori probability density function (p.d.f.)
- A priori p.d.f., Jeffreys or uniform
- Observational error p.d.f., multivariate normal

$$\psi = \Psi(\mathbf{P}, \mathbf{t}) + \varepsilon$$

$$p_p(\mathbf{P}) \propto p_{pr}(\mathbf{P})p(\psi|\mathbf{P})$$

$$p_{pr}(\mathbf{P}) \propto \sqrt{\det \Sigma^{-1}(\mathbf{P})}$$

$$p(\varepsilon; \Lambda) = \frac{1}{(2\pi)^{2N} \sqrt{\det \Lambda}} \exp \left[-\frac{1}{2} \varepsilon^T \Lambda^{-1} \varepsilon \right]$$

Statistical inversion

- A posteriori p.d.f. for orbital elements
- Linearization

$$p_p(\mathbf{P}) \propto \sqrt{\det \Sigma^{-1}(\mathbf{P})} \exp \left[-\frac{1}{2} \chi^2(\mathbf{P}) \right]$$

$$\chi^2(\mathbf{P}) = \Delta \Psi^T(\mathbf{P}) \Lambda^{-1} \Delta \Psi(\mathbf{P})$$

$$\Psi(\mathbf{P}, t) = \Psi(\mathbf{P}_{ls}, t) + \sum_{j=1}^6 \Delta P_j \frac{\partial \Psi}{\partial P_j}(\mathbf{P}_{ls}, t)$$

- A posteriori p.d.f. in the linear approximation

$$p_p(\mathbf{P}) \propto \sqrt{\det \Sigma^{-1}(\mathbf{P}_{ls})} \cdot \exp \left[-\frac{1}{2} \Delta \mathbf{P}^T \Sigma^{-1}(\mathbf{P}_{ls}) \Delta \mathbf{P} \right]$$

- Covariance matrix for orbital elements

$$\Sigma(\mathbf{P}_{ls}) = \left(\frac{\partial \mathbf{P}}{\partial \psi} \right)_{ls}^T \Sigma(\psi) \left(\frac{\partial \mathbf{P}}{\partial \psi} \right)_{ls}$$

MCMC ranging

- Initial orbital inversion using exiguous astrometric data (short observational time interval and/or a small number of observations)
- Ranging algorithm
 - Select two observation dates
 - Vary topocentric distances and values of R.A. and Decl.
 - From two Cartesian positions, compute elements and χ^2 against all the observations
- In MC ranging, systematic sampling and weighted sample elements
- How to sample using MCMC?

MCMC ranging

- Gaussian proposal p.d.f. in the space of two Cartesian positions
- Complex proposal p.d.f. in the space of the orbital elements (not needed!)
- Jacobians and cancellation of symmetric proposal p.d.f.s

$$a_r = \frac{p_p(\mathbf{P}') p_t(\mathbf{Q}_t; \mathbf{Q}') J_t}{p_p(\mathbf{P}_t) p_t(\mathbf{Q}'; \mathbf{Q}_t) J'}$$

$$J = \det \left| \frac{\partial \mathbf{Q}}{\partial \mathbf{P}} \right|$$

$$a_r = \frac{p_p(\mathbf{P}') J_t}{p_p(\mathbf{P}_t) J'}$$

If $a_r \geq 1$, then $\mathbf{P}_{t+1} = \mathbf{P}'$.

If $a_r < 1$, then $\begin{cases} \mathbf{P}_{t+1} = \mathbf{P}', & \text{with probability } a_r, \\ \mathbf{P}_{t+1} = \mathbf{P}_t, & \text{with probability } 1 - a_r. \end{cases}$

Examples

		Observations		
Data set	Parameter	2004 HA ₃₉	2004 QR	2002 CX ₂₂₄
Full	Nr of observations	57	19	20
	First observation	2004 Apr 25	2004 Aug 16	2001 Oct 21
	Last observation	2004 Sep 16	2004 Sep 23	2003 Oct 24
	Time interval [day]	144.7	38.3	733.2
Used	Nr of observations	5	10	6
	First observation	2004 May 2	2004 Aug 16	2002 Nov 7
	Last observation	2004 May 6	2004 Aug 18	2002 Nov 28
	Time interval [day]	4.75	2.04	20.98

Table 1: The full observational data sets and the data sets used in the computation.

Table 2. Least-squares orbits.

Object	2004 HA ₃₉	2004 QR	2002 CX ₂₂₄
<i>a</i> (AU)	2.1493040(34)	2.33155(74)	46.404(57)
<i>e</i>	0.5368719(72)	0.30163(21)	0.1329(49)
<i>i</i> (°)	36.2085(3)	6.3566(36)	16.8447(63)
Ω (°)	204.154337(44)	314.82(1)	42.2592(56)
ω (°)	67.32116(11)	330.701(26)	132.51(1.07)
M_0 (°)	345.02807(39)	28.4223(89)	256.12(1.73) ²

²The 1- σ standard deviations for the orbital elements are given after each element in the units of the last digit shown.

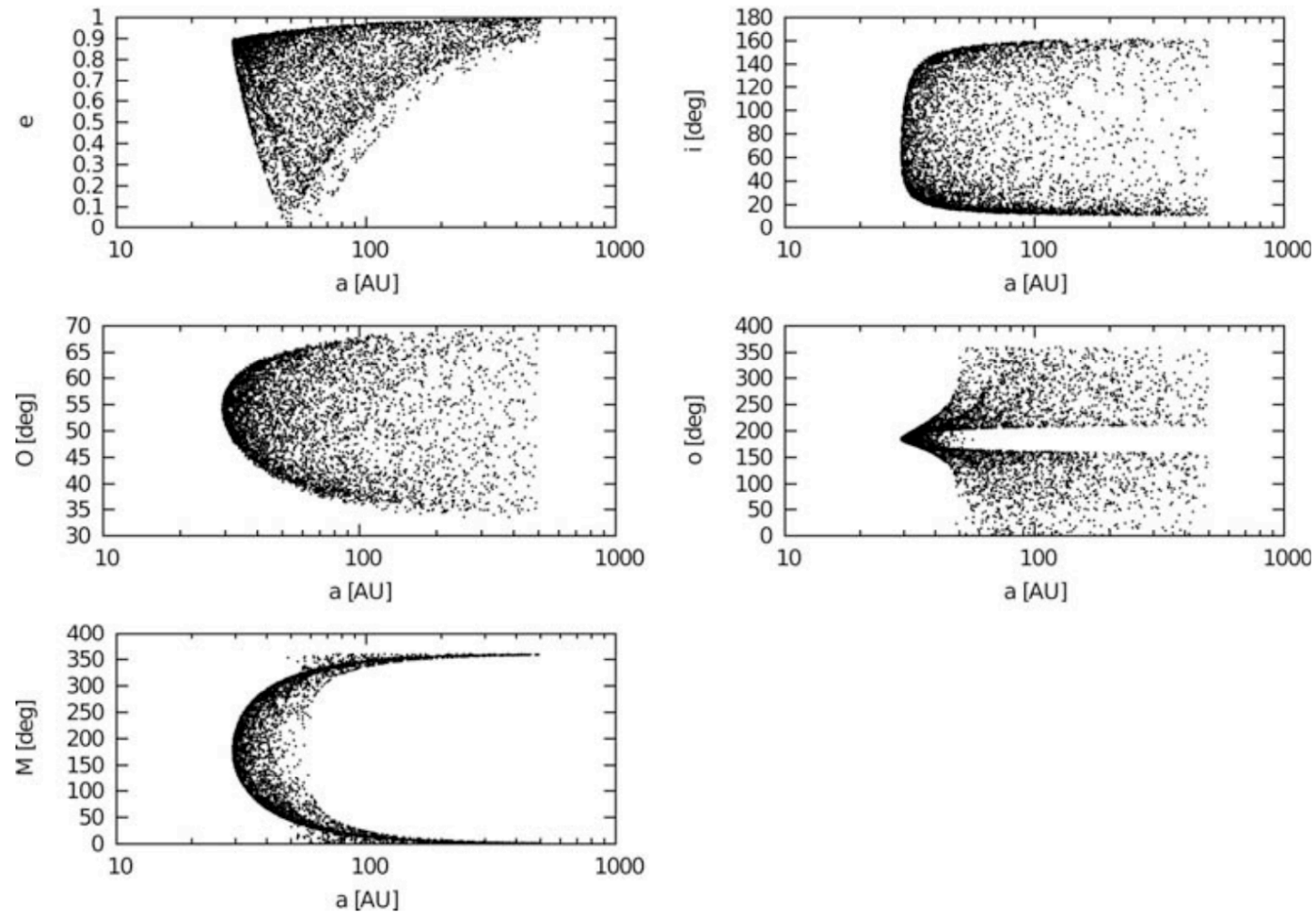


Fig. 1. A sample of 5000 different orbit solutions for TNO 2002 CX₂₂₄ (observational time interval 20.98 d) obtained with MCMC ranging. The following proposal standard deviations were used: $\sigma_{\alpha_A} = \sigma_{\delta_A} = \sigma_{\alpha_B} = \sigma_{\delta_B} = 0.5''$ for the angular coordinates, $\sigma_{\rho_A} = \sigma_{\rho_B} = 2.0$ AU for the ranges, and high correlation $Cor(\rho_A, \rho_B) = 0.999$ between the ranges.

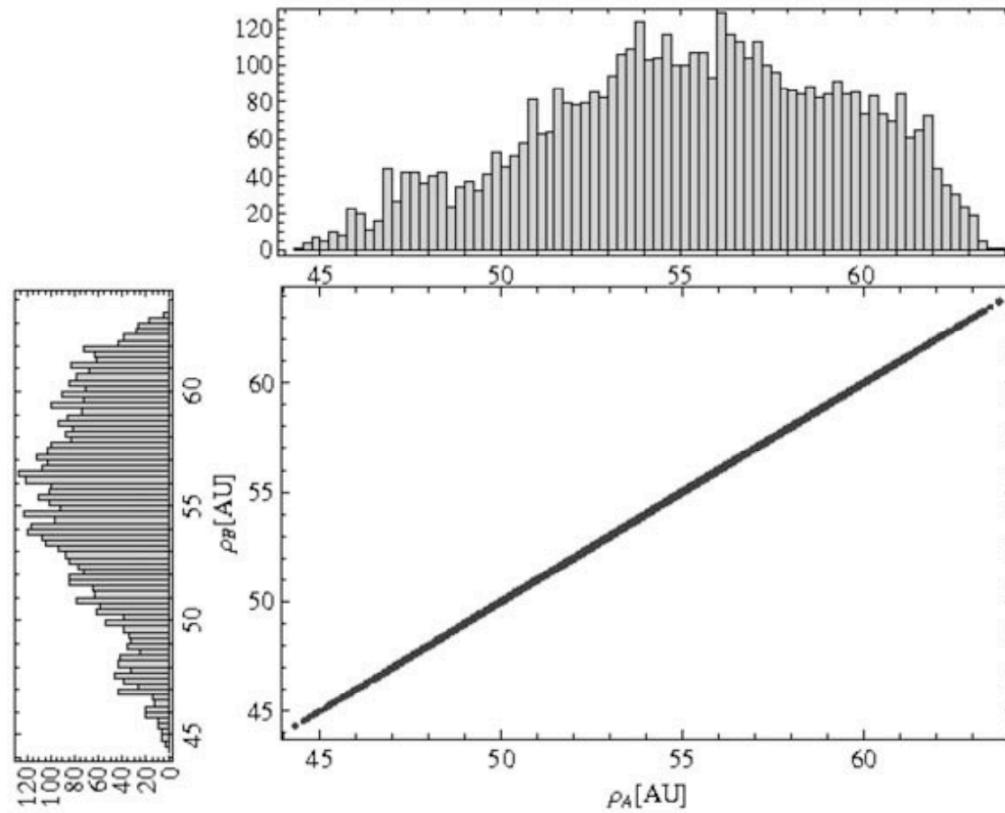
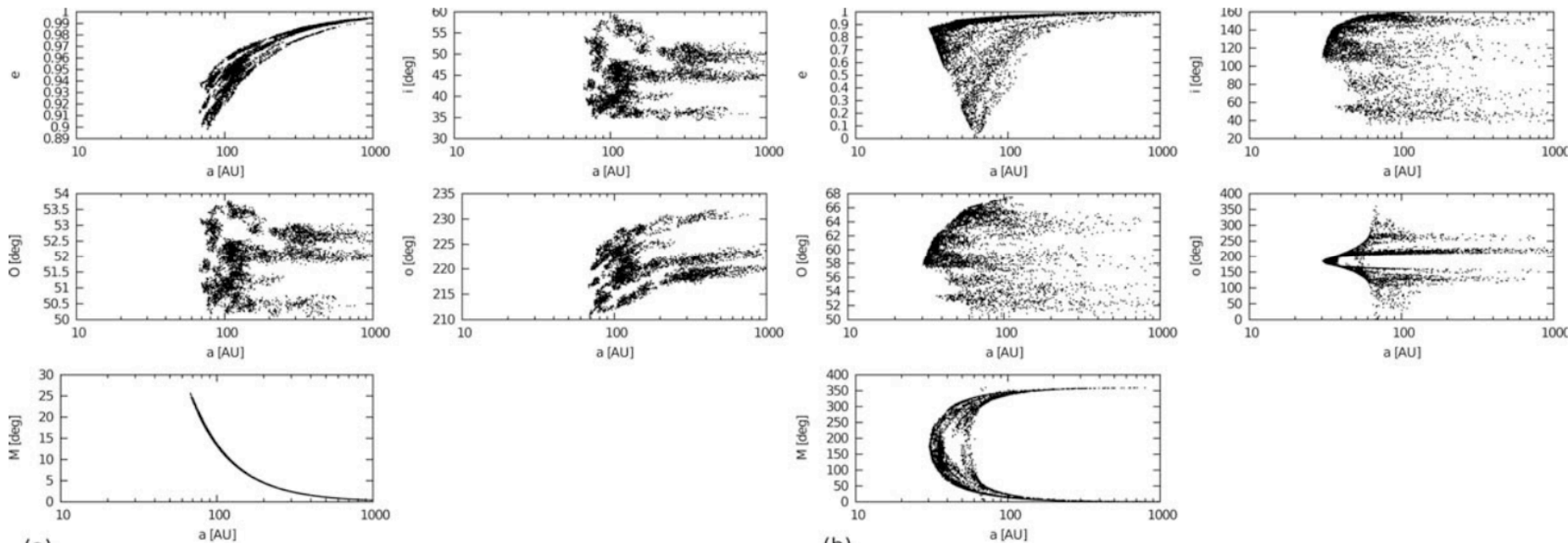
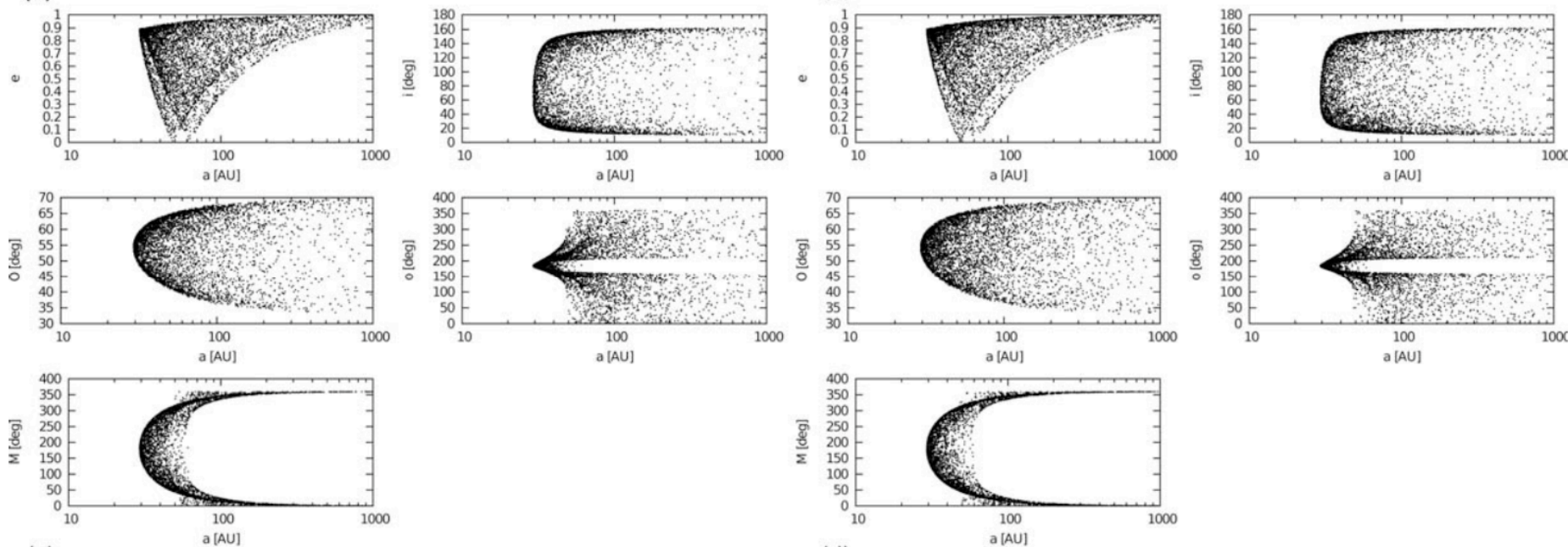


Fig. 2. Distribution of ranges for the observation dates A and B for TNO 2002 CX₂₂₄ (observational time interval 20.98 d) obtained with MCMC ranging (Parameters as in Fig. 1).



(a)



(b)

(c)

(d)

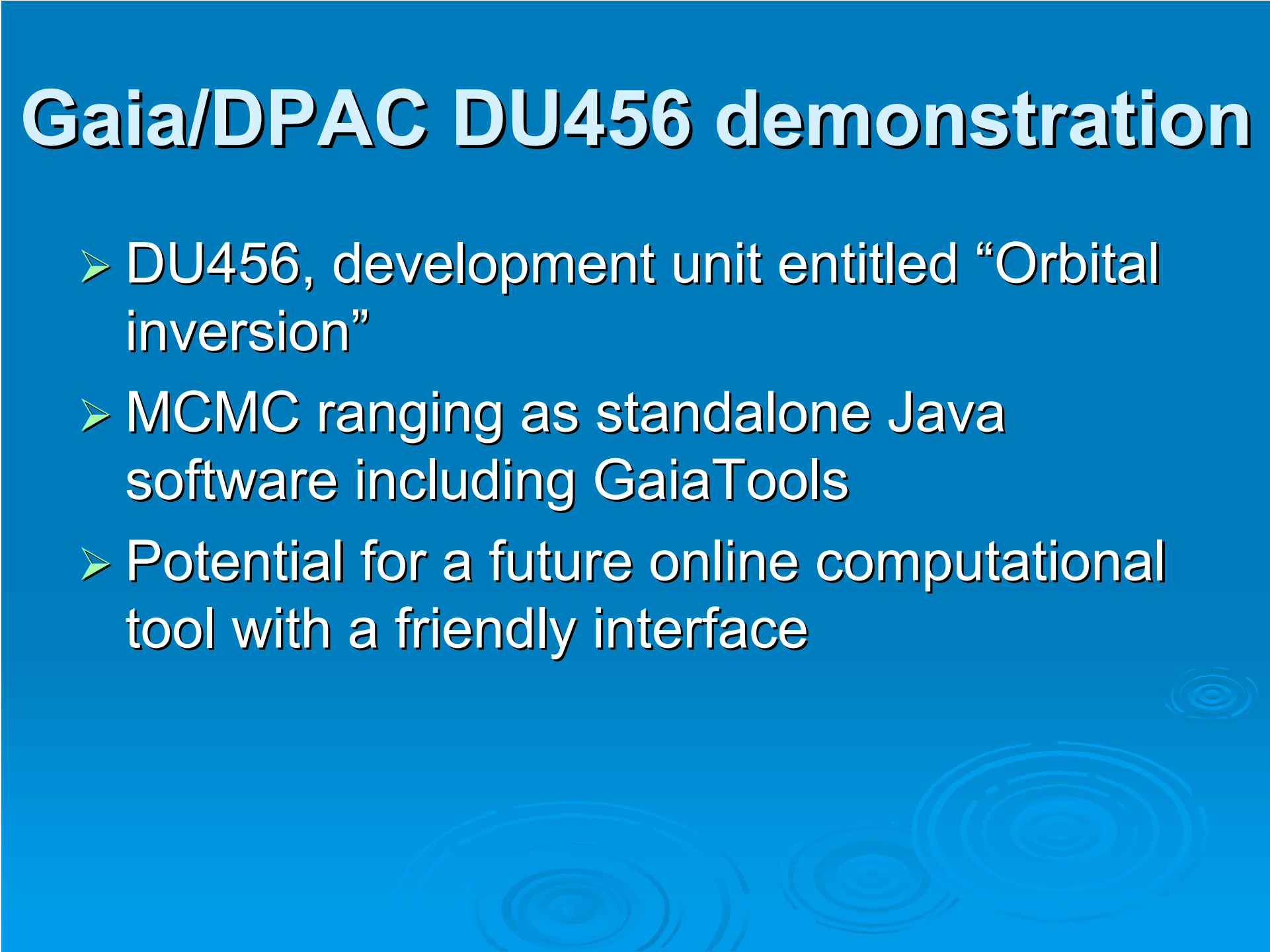
Table 3. Methods comparison.

Object	Parameter
2004 HA ₃₉	Computation time
	Acceptance ratio
2004 QR	Computation time
	Acceptance ratio
2002 CX ₂₂₄	Computation time
	Acceptance ratio

³Comparison of MCMC ranging and MC ranging methods.

MCMC ranging	MC ranging
0 min 19.417 s 0.17	0 min 48.663 s 0.05
0 min 6.344 s 0.54	1 min 19.170 s 0.03
0 min 4.525 s 0.45	0 min 14.155 s 0.09 ³

Gaia/DPAC DU456 demonstration

- DU456, development unit entitled “Orbital inversion”
 - MCMC ranging as standalone Java software including GaiaTools
 - Potential for a future online computational tool with a friendly interface
- 

Conclusion

- MCMC ranging more efficient than MC ranging
- Operational within the Gaia/DPAC pipeline and as stand-alone software
- MCMC-Gauss under development and to be incorporated into Gaia/DPAC
- MCMC-VoV on the drawing board